



Batterjee Medical College  
كلية البترجي الطبية

**General physics**

**Batterjee Medical College / Jeddah**

**Preparatory Year [Scientific Track]**

**PYPHY101**

**[2025 / 2026]**

## Table of contents

| <b>Chapter</b> | <b>Title</b>                                |
|----------------|---------------------------------------------|
| <b>Ch1</b>     | <b>Dimensional Analysis</b>                 |
| <b>Ch2</b>     | <b>Vectors</b>                              |
| <b>Ch3</b>     | <b>Kinematics in One and Two Dimensions</b> |
| <b>Ch4</b>     | <b>Dynamics: Newton's Laws of Motion</b>    |
| <b>Ch5</b>     | <b>Biomechanics</b>                         |
| <b>Ch6</b>     | <b>Fluid Dynamics</b>                       |
| <b>Ch7</b>     | <b>Light and optics</b>                     |
| <b>Ch8</b>     | <b>Nuclear physics</b>                      |
| <b>Ch9</b>     | <b>Electricity</b>                          |

# **Chapter One:**

## **Dimensional Analysis**

**Intended Learning Outcomes:**

**At the end of this chapter, the students will be able to:**

- Recognize the methodology of dimensional analysis and the SI unit.

## Dimensional Analysis

In 1960, an international committee established a set of standards for the fundamental quantities of science. It is called the SI (Système International), and its units of length, mass, and time are the meter, kilogram, and second, respectively. Other SI standards established by the committee are those for temperature (the kelvin), electric current (the ampere), luminous intensity (the candela), and the amount of substance (the mole).

| The fundamental Quantities and Units |           |          |
|--------------------------------------|-----------|----------|
| Quantity                             | Dimension | SI unit  |
| Length                               | L         | meter    |
| Mass                                 | M         | kilogram |
| Time                                 | T         | second   |
| Temperature                          | $\theta$  | Kelvin   |
| Electric Current                     | I         | ampere   |
| Luminous intensity                   | /         | candela  |
| Amount of Material                   | N         | mole     |

| Physical quantity | Formula                 |
|-------------------|-------------------------|
| Area              | length x width          |
| Volume            | Length x width x height |
| Density           | mass / volume           |
| Velocity          | displacement / time     |
| Acceleration      | velocity / time         |
| Force             | mass x acceleration     |
| Pressure          | force / area            |
| Work              | force x distance        |
| Plane Angle       | arc length / radius     |

### The quantities in physics are divided to:

- Basic quantities in physics (Length, Mass, Time, Temperature, Electric current, Luminous intensity, and Amount of material)
- Derived quantities in physics (All other quantities could expressed in terms of the basic quantities)

- **The basic quantities in mechanics are: the length, the mass, and the time.**
- The dimension of each of the following quantities is L: Length, width, height, thickness, distance, depth, arc length, radius, diameter, wavelength, circumference, displacement.

### Example 1

Find the dimension and the SI unit of area “A” ?

#### Solution:

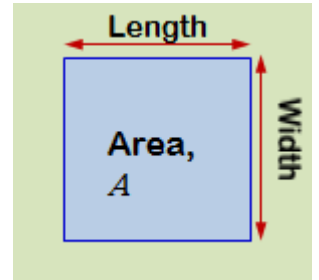
Formula:  $\text{Area} = \text{Length} \times \text{Width}$

The length and the width have the same dimension: L. So the dimension of the area is

$$A = L \times L = L^2$$

Therefore, the SI unit of area is  $\text{m}^2$ .

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### Example 2

Find the dimension and the SI unit of volume “V”?

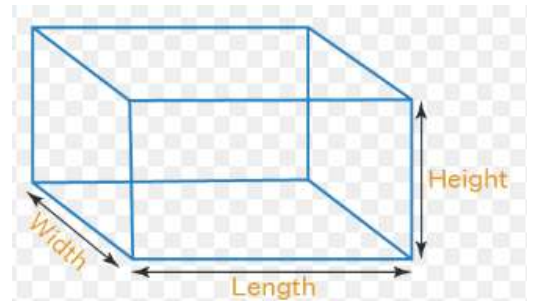
#### Solution:

Formula:  $\text{Volume} = \text{Length} \times \text{Width} \times \text{Height}$

The length, the width, and the height have the same dimension: L. So the dimension of the volume is

$$\text{Volume} = L \times L \times L = L^3$$

Therefore, the SI unit of volume is  $\text{m}^3$ .



### Example 3

Find the dimension and the SI unit of density “ $\rho$ ” ?

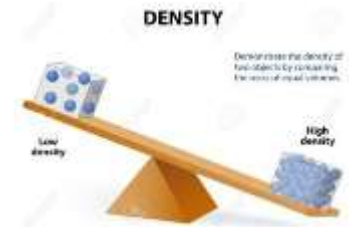
**Solution:**

Formula:  $Density = \frac{Mass}{Volume}$

So, the dimension of the density is  $\rho = \frac{M}{L^3} = M.L^{-3}$

Therefore, the SI unit of density is  $kg.m^{-3}$ .

---



### Example 4

Find the dimension and the SI unit of velocity “ $v$ ” ?

**Solution:**

Formula:  $velocity = \frac{distance}{time}$

So, the dimension of the velocity is  $v = \frac{L}{T} = L.T^{-1}$

Therefore, the SI unit of velocity is  $m.s^{-1}$ .

---

### Example 5

Find the dimension and the SI unit of acceleration “ $a$ ” ?

**Solution:**

Formula:  $acceleration = \frac{velocity}{time}$

So, the dimension of the acceleration is  $a = \frac{L}{T^2} = L.T^{-2}$

Therefore, the SI unit of acceleration is  $m.s^{-2}$

### Example 6

Find the dimension and the SI unit of plane angle “ $\theta$ ” ?

**Solution:**

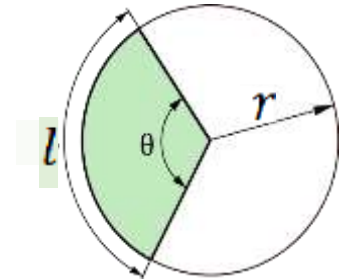
Formula:  $\theta = \frac{\text{arclength}}{\text{radius}} = \frac{L}{r}$

So, the dimension of the angle is  $\theta = \frac{L}{L} = 1$

Then the dimension of the angle is dimensionless

This quantity is unit less (because it is dimensionless)

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### Exercise 1

Find the dimension and the SI unit of:

1. Force?
2. Work?
3. Pressure?

**The answer:** (see the table below)

*Force*  $\rightarrow$  *Dimension* =  $\frac{M \cdot L}{T^2}$  , *SI unit* =  $\frac{Kg \cdot m}{s^2}$

*Work*  $\rightarrow$  *Dimension* =  $\frac{M \cdot L^2}{T^2}$  , *SI unit* =  $\frac{Kg \cdot m^2}{s^2}$

*Pressure*  $\rightarrow$  *Dimension* =  $\frac{M}{L \cdot T^2}$  , *SI unit* =  $\frac{Kg}{m \cdot s^2}$

### Exercise 2

Find the dimension and the SI unit of the mathematical constant  $\pi$ . Consider

$$\textit{Area of the circle} = \pi \times (\textit{radius})^2$$

**The answer:**  $\pi \rightarrow$  *Dimension = 1 (Dimensionless quantity)*

### Exercise 3

Find the dimension and the SI unit of the variable  $k$  in the following equation

$$\textit{Force} = \frac{k \times (\textit{velocity})^2}{\textit{radius}}$$

**The answer:**

$k \rightarrow$  *dimension = M, SI unit = Kg*

### Dimensions and Units of Some Physical Quantities

| Physical quantity | Formula                 | Dimension                          | SI Unit            |
|-------------------|-------------------------|------------------------------------|--------------------|
| Area              | length x width          | $L \times L = L^2$                 | $m^2$              |
| Volume            | length x width x height | $L \times L \times L = L^3$        | $m^3$              |
| Density           | mass / volume           | $M / L^3 = ML^{-3}$                | $kg.m^{-3}$        |
| Velocity          | displacement / time     | $L / T = LT^{-1}$                  | $m.s^{-1}$         |
| Acceleration      | velocity / time         | $(L / T) / T = L / T^2 = LT^{-2}$  | $m.s^{-2}$         |
| Force             | mass x acceleration     | $M \times LT^{-2} = MLT^{-2}$      | $kg.m.s^{-2}$      |
| Pressure          | force / area            | $MLT^{-2} / L^2 = M L^{-1} T^{-2}$ | $kg.m^{-1}.s^{-2}$ |
| Work              | force x distance        | $MLT^{-2} \times L = M L^2T^{-2}$  | $kg.m^2.s^{-2}$    |
| Plane Angle       | arc length / radius     | $L / L = 1$                        | unit less          |

### Example 6

Find the values of  $n$  and  $m$  for the following equation  $a = r^n \cdot v^m$ . Where:

$a$  : is the acceleration

$r$ : is the radius

$v$ : is the velocity

#### Solution:

Knowing the dimensions of  $a$ ,  $r$ , and  $v$ , we see that the dimensional equation must be

$$a = r^n \cdot v^m$$

$$\frac{L}{T^2} = L^n \cdot \left(\frac{L}{T}\right)^m$$

$$\frac{L}{T^2} = L^n \cdot \frac{L^m}{T^m}$$

Write the equation in one line (shift all the denominators to the numerator)

$$L \cdot T^{-2} = L^n \cdot L^m \cdot T^{-m}$$

Or

$$L \cdot T^{-2} = L^{n+m} \cdot T^{-m}$$

After equating the powers of  $L$  and  $T$

$$-m = -2 \rightarrow m = 2 \quad \text{and} \quad n + m = 1$$

$$\text{Where,} \quad n + m = 1 \rightarrow n + 2 = 1 \rightarrow n = -1,$$

and we can write the acceleration expression as

$$a = r^{-1} \cdot v^2 \quad \text{or} \quad a = \frac{v^2}{r}$$

#### Exercise 4

Suppose we are told that the hydrostatic pressure  $P$  is proportional to some power of depth  $h$ , say  $h^n$ , some power of density  $\rho$ , say  $\rho^m$ , and the gravity acceleration  $g$ . Using the dimensional analysis rules, determine the values of  $n$  and  $m$ .

(Hint: find the values of  $n$  and  $m$  in the following equation:  $P = h^n \cdot \rho^m \cdot g$ )

**The answer:** (  $n = 1$  and  $m = 1$ , so  $P = h \cdot \rho \cdot g$  )

## **Chapter Two:**

### **Vectors**

**Intended Learning Outcomes:**

**At the end of this chapter, the students will be able to:**

- Recognize vector quantities.
- Recognize vector properties.
- Resolve the vector into rectangular components.
- Recognize unit vectors.

## Vectors

Quantities in physics are divided to two major parts:

- a) A scalar quantity: is completely specified by a single value with an appropriate unit and has no direction. Like mass, time, temperature.
- b) A vector quantity: is completely specified by a number and appropriate units plus a direction. Like velocity, force, acceleration.

The length of the line shows its magnitude and the arrowhead points to the direction.

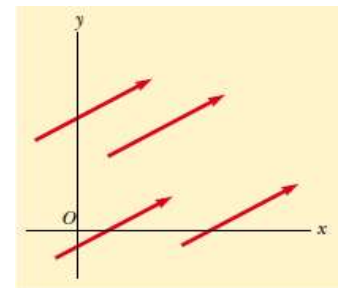


### Some properties of vectors

#### 1) Equality of two vectors.

Two vectors  $\vec{A}$  and  $\vec{B}$  are defined to be equal if they have the same magnitude and point in the same direction.

$\vec{A} = \vec{B}$  Only if  $A = B$  and if  $A$  and  $B$  point in the same direction.



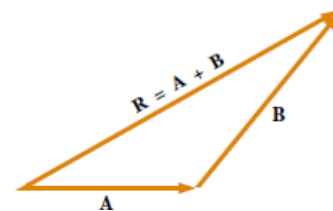
These four vectors are equal because they have equal lengths and point in the same direction.

#### 2) Adding vectors \*\*

When two or more vectors are added together all vectors involved must have the same units.

The rules for adding vectors are conveniently described by graphical methods. In geometric method:

- a) Draw vector  $\vec{A}$  with magnitude represented by a convenient length scale.
- b) Draw vector  $\vec{B}$  to the same scale, with its tail starting from tip of  $\vec{A}$ .
- c) The resultant vector  $\vec{R} = \vec{A} + \vec{B}$  is the vector drawn from the tail of  $\vec{A}$  to the tip of  $\vec{B}$ .



When two vectors are added, the sum is independent of the order of the addition.

$$\vec{A} + \vec{B} = \vec{B} + \vec{A}$$

Commutative law of addition.

$$\vec{A} + (\vec{B} + \vec{C}) = (\vec{A} + \vec{B}) + \vec{C}$$

Associative law of addition.

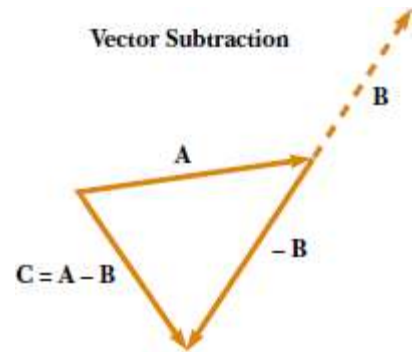
### 3) Negative of a vectors \*\*

The negative of the vector  $\vec{A}$  is defined as the vector that when added to  $\vec{A}$  gives zero for the vector sum. That is,  $\vec{A} + (-\vec{A}) = 0$ . The vectors  $\vec{A}$  and  $-\vec{A}$  have the same magnitude but point in opposite directions.

### 4) Subtracting vectors \*\*

The operation of vector subtraction makes use the definition of the negative of a vector. We define the operation  $\vec{A} - \vec{B}$  as vector  $-\vec{B}$  added to vector  $\vec{A}$ :

$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$



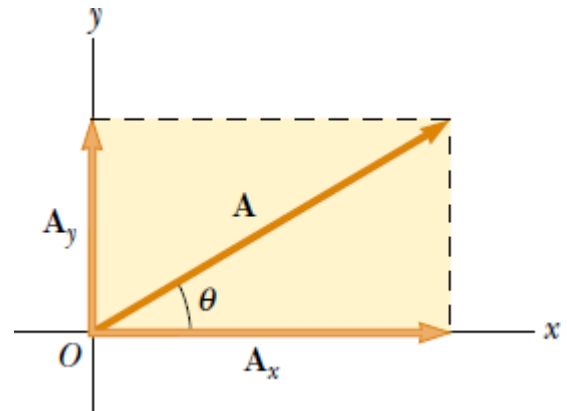
### 5) Multiplying a vector by a scalar \*\*

If vector  $\vec{A}$  multiply by a positive scalar quantity  $m$ , the product  $m\vec{A}$  is a vector has the same direction of  $\vec{A}$  and magnitude  $mA$ .

## Components of vector

Consider a vector  $\vec{A}$  lying in the x-y plane and making angle  $\theta$  with the positive x-axis as showing in the following Figure.

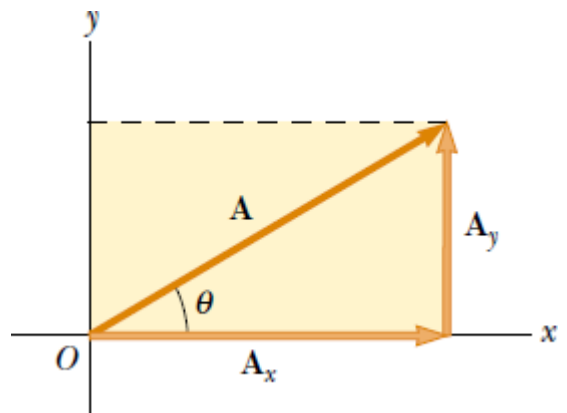
This vector can be expressed as the sum of two other vectors  $\vec{A}_x$  and  $\vec{A}_y$ .



From the following Figure, we see that the three vectors form a right triangle and that

$$\vec{A} = \vec{A}_x + \vec{A}_y. \quad (1)$$

We shall often refer to the components of a vector  $\vec{A}$ , written  $A_x$  and  $A_y$  (without the arrow). **The components  $A_x$  represents the projection of  $\vec{A}$  along the x-axis, and the component  $A_y$  represents the projection of  $\vec{A}$  along the y-axis.** These components can be positive or negative.



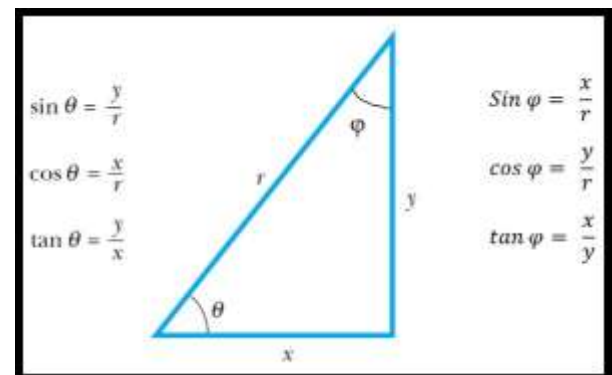
## Finding the vector components

From the last two Figures and the definition of sine and cosine, we see that  $\cos \theta = \frac{A_x}{A}$  and that  $\sin \theta = \frac{A_y}{A}$ .

Hence, the components of  $\vec{A}$  are:

$$A_x = A \cos \theta \quad (2)$$

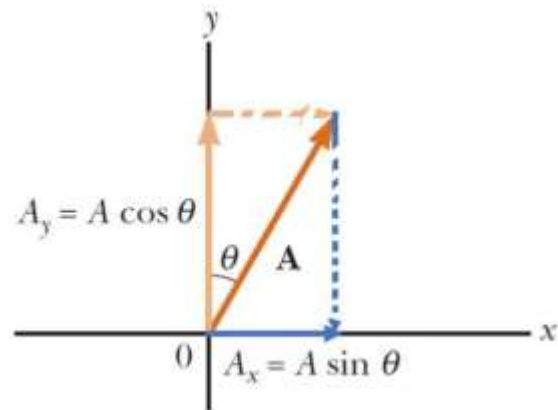
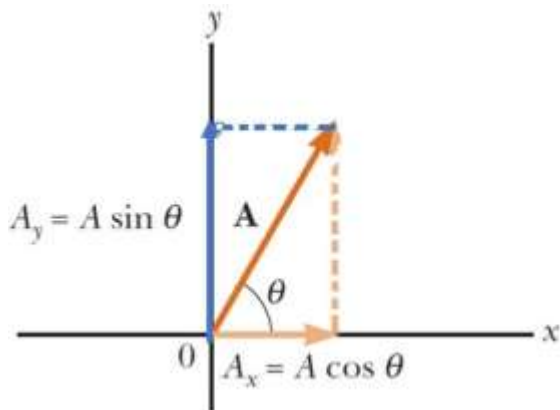
$$A_y = A \sin \theta \quad (3)$$



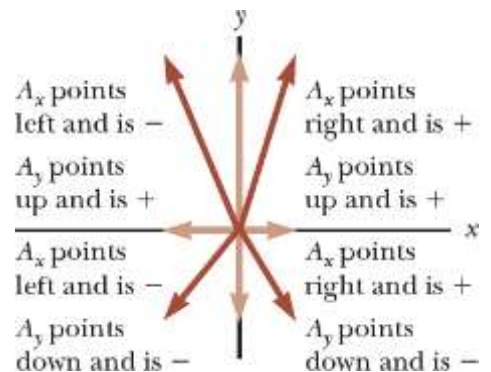
• Strategy:

If you want to calculate the components from the vector and the angle

$A$     $\sqrt{\quad}$   
 $\theta$     $\sqrt{\quad}$   
 $A_x$    ?    $\rightarrow$    The component that is close to the angle =  $A \cdot \cos \theta$   
 $A_y$    ?   The component that is far away from the angle =  $A \cdot \sin \theta$



The sign of the components  $A_x$  and  $A_y$  depends on the position of  $A$ .



### Example 1

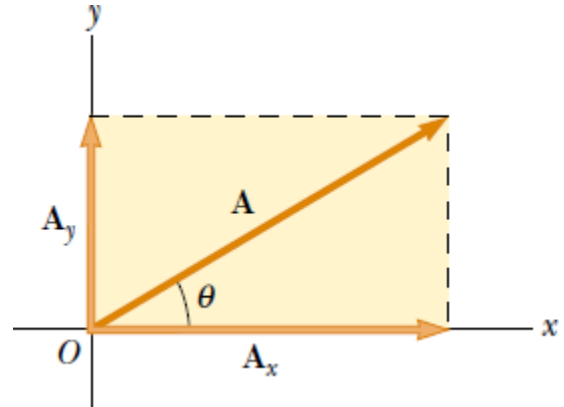
Find the components of the vector  $\vec{A}$  in the following Figure, if  $A = 2$  and  $\theta = 30^\circ$ .

#### Solution:

The x component  $A_x$  is close to the angle (takes the cosine), and the y component  $A_y$  is far away from the angle (takes the sin), so

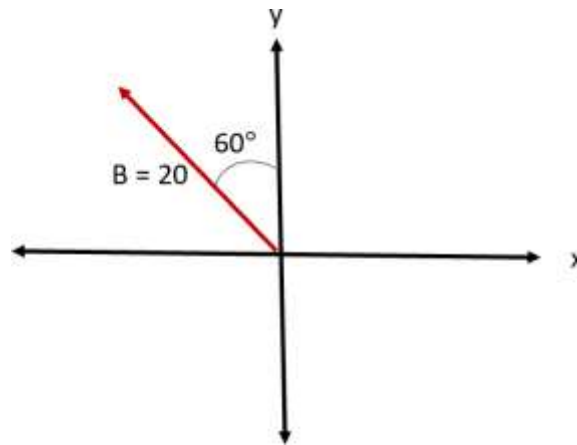
$$A_x = A \cos \theta = 2 \cos 30^\circ = 2 (0.866) = 1.73$$

$$A_y = A \sin \theta = 2 \sin 30^\circ = 2 (0.5) = 1.0$$

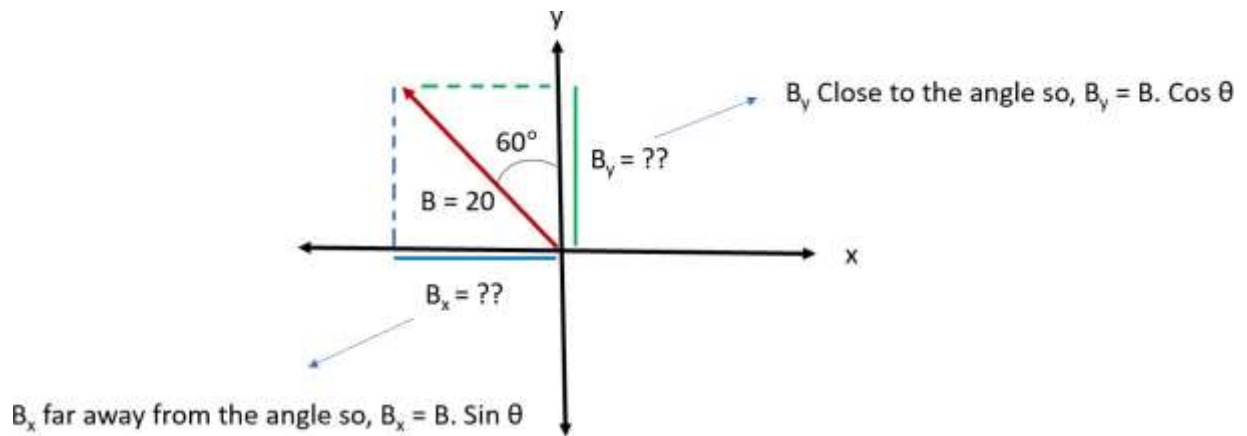


### Example 2

Find the components of the vector  $\vec{B}$  in the following Figure, if  $B = 20$  and  $\theta = 60^\circ$ .



**Solution:**



$$B_x = B \sin \theta = 20 \sin 60^\circ = 20 (0.866) = 17.3$$

→  $-17.3$  Because it is in the negative axis side.

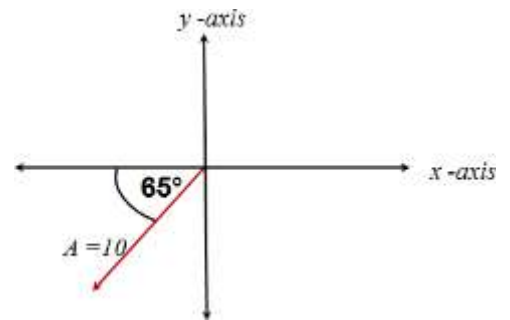
$$B_y = B \cos \theta = 20 \cos 60^\circ = 20 (0.5) = 10$$

→  $+10$  Because it is in the positive axis side.

### Exercise 1

Find the  $x$  and  $y$  components for vector  $A$  shown in the following figures.

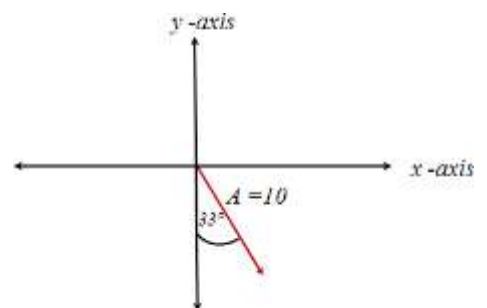
The answer:  $A_x = -4.2$  and  $A_y = -9.1$



### Exercise 2

Find the  $x$  and  $y$  components for vector  $A$  shown in the following figures.

The answer:  $A_x = 5.5$  and  $A_y = -8.4$

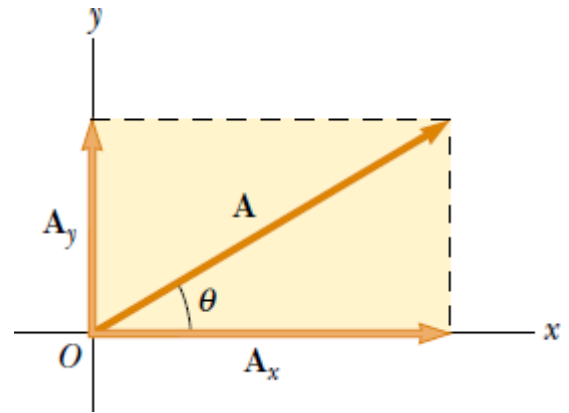


## Finding the vector magnitude and direction from the vector components

The vector components form two sides of a right triangle with a hypotenuse of length  $A$ . Thus, it follows that the magnitude and direction of  $A$  are related to its components through the expressions

$$A = \sqrt{A_x^2 + A_y^2} \quad (4)$$

$$\theta = \tan^{-1} \left( \frac{|A_y|}{|A_x|} \right) \quad (5)$$



### • Strategy:

If you want to calculate the magnitude and the direction (angle) of a vector from the components:

$A$  ?

$\theta$  ?

$A_x$  ✓

$A_y$  ✓



The magnitude of  $A$  is given by  $A = \sqrt{A_x^2 + A_y^2}$

The axis reference determines the direction (the angle):

If the angle is taken with respect to x axis  $\theta_x = \tan^{-1} \left( \frac{|A_y|}{|A_x|} \right)$

If the angle is taken with respect to y axis  $\theta_y = \tan^{-1} \left( \frac{|A_x|}{|A_y|} \right)$

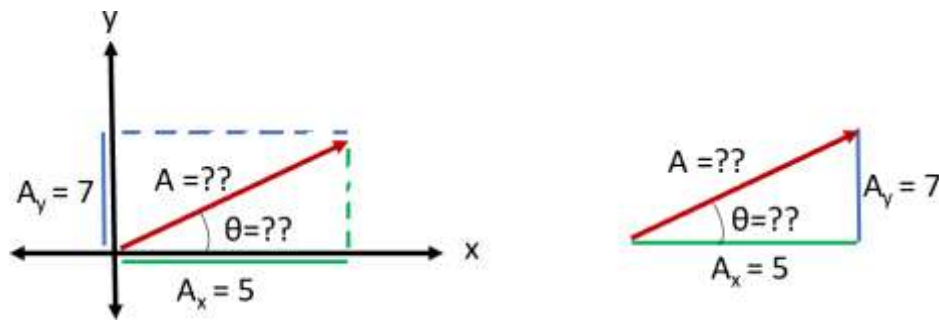
In order to calculate the angle,  $|A_x|$  and  $|A_y|$  are taken with positive signs.

### Example 3

Find the magnitude and the direction (the angle with respect to x-axis) for vector A, if the vector components of this vector are  $A_x = 5$  and  $A_y = 7$ .

### Solution:

We have  $A_x = 5$  and  $A_y = 7$ , let us find 1. The magnitude of A, and 2. The direction of A (the angle  $\theta$  with respect to x-axis)



1. The magnitude ( $A = ??$ )

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{5^2 + 7^2} = 8.6$$

2. The direction of A ( the angle with respect to x axis) ( $\theta = ??$ )

$$\tan \theta = \frac{A_y}{A_x}$$

So,

$$\theta = \tan^{-1} \frac{A_y}{A_x}$$

$$\theta_x = \tan^{-1} \left( \frac{|A_y|}{|A_x|} \right) = \tan^{-1} \left( \frac{7}{5} \right) = 54.5^\circ$$

### Exercise 3

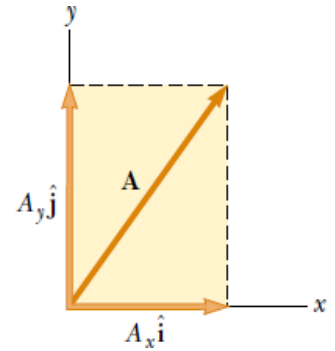
Find the magnitude and the direction (the angle with respect to +y axis) for vector  $A$ , if the components of this vector are:  $A_x = -3$  and  $A_y = 4$

(see the solution at the end of the chapter)

**The answer:**  $A = 5$  and  $\theta_y = 36.9^\circ$

## Unit vectors\*\*

- A unit vector is a dimensionless vector having a magnitude of exactly 1.
- Unit vectors are used to specify a given direction and have no other physical significance.
- They are used solely as a convenience in describing a direction in space. We shall use the symbols  $\hat{i}$ ,  $\hat{j}$ , and  $\hat{k}$  to represent unit vectors pointing in the positive x, y and z direction, respectively.
- The hats on the symbols are a standard notation for unit vectors.



Consider vector  $\vec{A}$  in the x-y plane, we can write it in the form:

$$\vec{A} = A_x \hat{i} + A_y \hat{j} \quad \text{Remember equation} \quad (1)$$

$$\vec{A} = A_x \hat{i} + A_y \hat{j} \quad (6)$$

### Example 4\*\*

If you have the following two vectors:

$$\vec{A} = -2\hat{i} + 7\hat{j} \quad \text{and} \quad \vec{B} = 5\hat{i} + 6\hat{j}.$$

Find the components of vector  $\vec{C}$ , where  $\vec{C} = \vec{A} + \vec{B}$ .

**Solution:**

$$\vec{C} = \vec{A} + \vec{B} = (A_x \hat{i} + A_y \hat{j}) + (B_x \hat{i} + B_y \hat{j}) = (A_x + B_x) \hat{i} + (A_y + B_y) \hat{j}$$

$$\vec{C} = (-2 + 5) \hat{i} + (7 + 6) \hat{j} = 3\hat{i} + 13\hat{j}.$$

Thus,  $C_x = 3$  and  $C_y = 13$

**Example 5\*\***

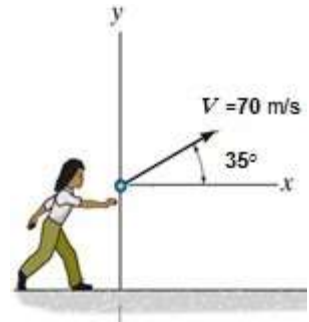
A ball is thrown with an initial velocity of  $70 \text{ m/s}$ , at an angle of  $35^\circ$  with the horizontal. Write the velocity vector  $V$  in terms of unit vectors. (Find the vertical and horizontal components of the velocity first).

**Solution:**

The velocity  $v$  in terms of unit vectors will take the following form

$$\vec{v} = v_x \hat{i} + v_y \hat{j}$$

Now we need to find  $v_x$  and  $v_y$  and then substitute them in the equation mentioned above.



The horizontal component of the velocity is  $v_x$

$$v_x = v \cos \theta = 70 \cos 35^\circ = 57.34$$

The vertical component of the velocity is  $v_y$

$$v_y = v \sin \theta = 70 \sin 35^\circ = 40.15$$

The velocity  $v$  in terms of unit vectors is

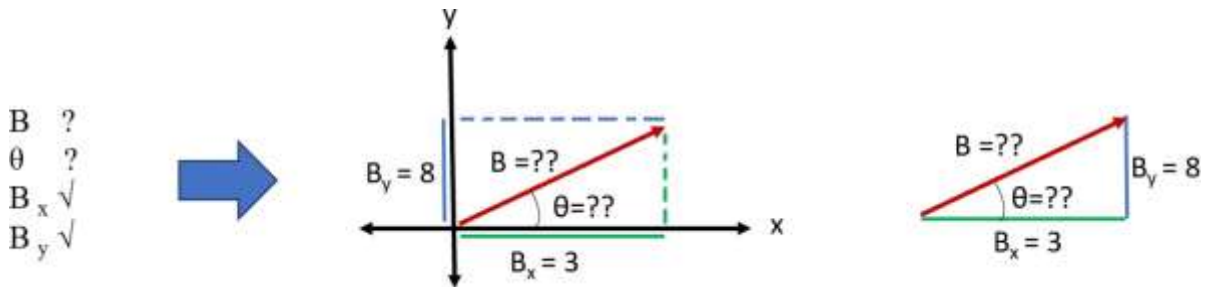
$$\vec{v} = v_x \hat{i} + v_y \hat{j}$$

$$\vec{v} = 57.34 \hat{i} + 40.15 \hat{j}$$

### Example 6\*\*

Find the magnitude and the direction (the angle with respect to  $x$ -axis) for vector  $B$ , where  $B$  is given by:

$$\vec{B} = 3\hat{i} + 8\hat{j}$$



1. The magnitude ( $B = ??$ )

$$B = \sqrt{B_x^2 + B_y^2} = \sqrt{3^2 + 8^2} = 8.5$$

2. The direction of  $B$  ( the angle with respect to  $x$  axis) ( $\theta = ??$ )

$$\tan \theta = \frac{B_y}{B_x}$$

So,

$$\theta = \tan^{-1} \frac{B_y}{B_x}$$

$$\theta = \tan^{-1} \left( \frac{|B_y|}{|B_x|} \right) = \tan^{-1} \left( \frac{8}{3} \right) = 69.4^\circ$$

**Exercise 4\*\***

Find the magnitude and the direction (the angle with respect to  $y$ -axis) for vector  $K$ , where  $K$  is given by:

$$\vec{K} = 6 \hat{i} + 10 \hat{j}$$

**The answer:**  $k = 11.7$  and  $\theta_y = 31^\circ$

## The scalar product (Dot product)\*\*

We write this scalar product of vectors **A** and **B** as **A.B** (Because of the dot symbol, the scalar product is often called the dot product.)

In general, the scalar product of any two vectors **A** and **B** is a scalar quantity equal to the product of the magnitudes of the two vectors and the cosine of the angle  $\theta$  between them:

$$\mathbf{A} \cdot \mathbf{B} = AB \cos \theta \quad (1)$$

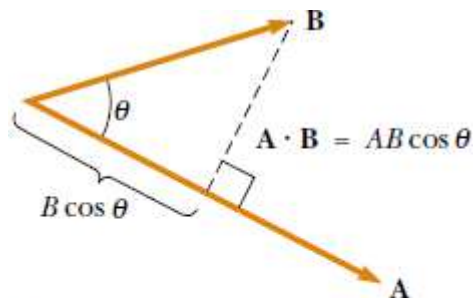


**Figure 1** The scalar product  $\mathbf{A} \cdot \mathbf{B}$  equals the magnitude of **A** multiplied by  $B \cos \theta$ , which is the projection of **B** onto **A**.

Note that **A** and **B** need not have the same units, as is the case with any multiplication. Figure 1 show two vectors **A** and **B** and the angle  $\theta$  between them that is used in the definition of the dot product. In Figure 1,  $B \cos \theta$  is the projection of **B** onto **A**. Therefore, Equation 1 means that **A.B** is the product of the magnitude of **A** and the projection of **B** onto **A**.

The unit vectors  $\hat{i}$ ,  $\hat{j}$  lie in the + x and y direction, respectively, of a right-handed coordinate system. Therefore, it follows from the definition of  $\vec{A} \cdot \vec{B}$  (or  $\mathbf{A} \cdot \mathbf{B}$ ) that the scalar product of these unit vectors are:

$$\begin{aligned}\hat{i} \cdot \hat{i} &= \hat{j} \cdot \hat{j} = 1 \\ \hat{i} \cdot \hat{j} &= \hat{j} \cdot \hat{i} = 0\end{aligned}$$

$\mathbf{A}$  and  $\mathbf{B}$  can be expressed in component vector form as

$$\begin{aligned}\mathbf{A} &= A_x \hat{i} + A_y \hat{j} \\ \mathbf{B} &= B_x \hat{i} + B_y \hat{j}\end{aligned}$$

Using the information given shows that the scalar product of  $\mathbf{A}$  and  $\mathbf{B}$  reduces to

$$\mathbf{A} \cdot \mathbf{B} = A_x B_x + A_y B_y + A_z B_z$$

In the special case in which  $\mathbf{A} = \mathbf{B}$ , we see that

$$\mathbf{A} \cdot \mathbf{A} = A_x^2 + A_y^2 + A_z^2 = A^2$$

**Example 7\*\***

If you have the following two vectors:

$$\vec{A} = 5\hat{i} + 2\hat{j} \quad \text{and} \quad \vec{B} = 2\hat{i} - 8\hat{j}.$$

Find the dot product (scalar product)  $\vec{A} \cdot \vec{B}$

**Solution:**

$$\begin{array}{r} \vec{A} = 5\hat{i} + 2\hat{j} \\ \vec{B} = 2\hat{i} - 8\hat{j} \end{array}$$


---


$$\vec{A} \cdot \vec{B} = 10 - 16 = -6$$

**Or**

$$\vec{A} \cdot \vec{B} = (5\hat{i} + 2\hat{j}) \cdot (2\hat{i} - 8\hat{j})$$

$$\vec{A} \cdot \vec{B} = [5\hat{i} \cdot 2\hat{i}] + [5\hat{i} \cdot (-8\hat{j})] + [2\hat{j} \cdot 2\hat{i}] + [2\hat{j} \cdot (-8\hat{j})]$$

Remember that

$$\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = 1$$

$$\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{i} = 0$$

$$\vec{A} \cdot \vec{B} = [10] + [0] + [0] + [-16] = -6$$

## More examples \*\*

### Example The Scalar Product

The vectors **A** and **B** are given by  $\mathbf{A} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$  and  $\mathbf{B} = -\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$ .

(A) Determine the scalar product  $\mathbf{A} \cdot \mathbf{B}$ .

**Solution** Substituting the specific vector expressions for **A** and **B**, we find,

$$\begin{aligned}\mathbf{A} \cdot \mathbf{B} &= (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) \cdot (-\hat{\mathbf{i}} + 2\hat{\mathbf{j}}) \\ &= -2\hat{\mathbf{i}} \cdot \hat{\mathbf{i}} + 2\hat{\mathbf{i}} \cdot 2\hat{\mathbf{j}} - 3\hat{\mathbf{j}} \cdot \hat{\mathbf{i}} + 3\hat{\mathbf{j}} \cdot 2\hat{\mathbf{j}} \\ &= -2(1) + 4(0) - 3(0) + 6(1) \\ &= -2 + 6 = 4\end{aligned}$$

where we have used the facts that  $\hat{\mathbf{i}} \cdot \hat{\mathbf{i}} = \hat{\mathbf{j}} \cdot \hat{\mathbf{j}} = 1$  and  $\hat{\mathbf{i}} \cdot \hat{\mathbf{j}} = \hat{\mathbf{j}} \cdot \hat{\mathbf{i}} = 0$

(B) Find the angle  $\theta$  between **A** and **B**.

**Solution** The magnitudes of **A** and **B** are

$$\begin{aligned}A &= \sqrt{A_x^2 + A_y^2} = \sqrt{(2)^2 + (3)^2} = \sqrt{13} \\ B &= \sqrt{B_x^2 + B_y^2} = \sqrt{(-1)^2 + (2)^2} = \sqrt{5}\end{aligned}$$

Using Equation 1 and the result from part (a) we find that

$$\begin{aligned}\cos \theta &= \frac{\mathbf{A} \cdot \mathbf{B}}{AB} = \frac{4}{\sqrt{13}\sqrt{5}} = \frac{4}{\sqrt{65}} \\ \theta &= \cos^{-1} \frac{4}{8.06} = 60.2^\circ\end{aligned}$$

### Example Work Done by a Constant Force

A particle moving in the  $xy$  plane undergoes a displacement  $\Delta \mathbf{r} = (2.0\hat{\mathbf{i}} + 3.0\hat{\mathbf{j}})$  m as a constant force  $\mathbf{F} = (5.0\hat{\mathbf{i}} + 2.0\hat{\mathbf{j}})$  N acts on the particle.

(A) Calculate the magnitudes of the displacement and the force.

**Solution** We use the Pythagorean theorem:

$$\Delta r = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{(2.0)^2 + (3.0)^2} = 3.6 \text{ m}$$

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(5.0)^2 + (2.0)^2} = 5.4 \text{ N}$$

(B) Calculate the work done by **F**. [ $W = \mathbf{F} \cdot \Delta \mathbf{r}$ ]

**Solution**

Using Equation 1

$$\begin{aligned}W &= \mathbf{F} \cdot \Delta \mathbf{r} = [(5.0\hat{\mathbf{i}} + 2.0\hat{\mathbf{j}}) \text{ N}] \cdot [(2.0\hat{\mathbf{i}} + 3.0\hat{\mathbf{j}}) \text{ m}] \\ &= (5.0\hat{\mathbf{i}} \cdot 2.0\hat{\mathbf{i}} + 5.0\hat{\mathbf{i}} \cdot 3.0\hat{\mathbf{j}} + 2.0\hat{\mathbf{j}} \cdot 2.0\hat{\mathbf{i}} + 2.0\hat{\mathbf{j}} \cdot 3.0\hat{\mathbf{j}}) \text{ N} \cdot \text{m} \\ &= [10 + 0 + 0 + 6] \text{ N} \cdot \text{m} = 16 \text{ J}\end{aligned}$$

## The Vector Product and Torque \*\*

Consider a force  $\mathbf{F}$  acting on a rigid object at the vector position  $\mathbf{r}$ , the *magnitude* of the torque due to this force relative to the origin is  $rF \sin \phi$ , where  $\phi$  is the angle between  $\mathbf{r}$  and  $\mathbf{F}$ . The axis about which  $\mathbf{F}$  tends to produce rotation is perpendicular to the plane formed by  $\mathbf{r}$  and  $\mathbf{F}$ .

The torque vector  $\boldsymbol{\tau}$  is related to the two vectors  $\mathbf{r}$  and  $\mathbf{F}$ . We can establish a mathematical relationship between  $\boldsymbol{\tau}$ ,  $\mathbf{r}$ , and  $\mathbf{F}$  using a mathematical operation called the **vector product**, or **cross product**:

$$\boldsymbol{\tau} \equiv \mathbf{r} \times \mathbf{F}$$

We now give a formal definition of the vector product. Given any two vectors  $\mathbf{A}$  and  $\mathbf{B}$ , the **vector product**  $\mathbf{A} \times \mathbf{B}$  is defined as a third vector  $\mathbf{C}$ , which has a magnitude of  $AB \sin \theta$ , where  $\theta$  is the angle between  $\mathbf{A}$  and  $\mathbf{B}$ . That is, if  $\mathbf{C}$  is given by

$$\mathbf{C} = \mathbf{A} \times \mathbf{B}$$

then its magnitude is

$$C \equiv AB \sin \theta$$

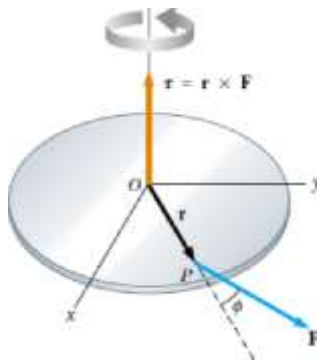
The quantity  $AB \sin \theta$  is equal to the area of the parallelogram formed by  $\mathbf{A}$  and  $\mathbf{B}$ . The *direction* of  $\mathbf{C}$  is perpendicular to the plane formed by  $\mathbf{A}$  and  $\mathbf{B}$ , and the best way to determine this direction is to use the right-hand rule.

The four fingers of the right hand are pointed along  $\mathbf{A}$  and then “wrapped” into  $\mathbf{B}$  through the angle  $\theta$ . The direction of the upright thumb is the direction of  $\mathbf{A} \times \mathbf{B} = \mathbf{C}$ . Because of the notation,  $\mathbf{A} \times \mathbf{B}$  is often read “**A cross B**”; hence, the term *cross product*.

Some properties of the vector product that follow from its definition are as follows:

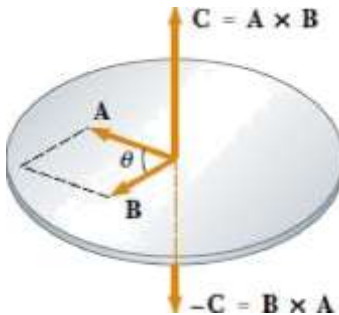
1. Unlike the scalar product, the vector product is *not* commutative. Instead, the order in which the two vectors are multiplied in a cross product is important:

$$\mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A}$$



The torque vector lies in a direction perpendicular to the plane formed by the position vector  $\mathbf{r}$  and the applied force vector  $\mathbf{F}$ .

Right-hand rule



The vector product  $\mathbf{A} \times \mathbf{B}$  is a third vector having a magnitude  $AB \sin \theta$  equal to the area of the parallelogram shown. The direction of  $\mathbf{C}$  is perpendicular to the plane formed by  $\mathbf{A}$  and  $\mathbf{B}$ , and this direction is determined by the right-hand rule.

Therefore, if you change the order of the vectors in a cross product, you must change the sign. You can easily verify this relationship with the right-hand rule.

2. If  $\mathbf{A}$  is parallel to  $\mathbf{B}$  ( $\theta = 0^\circ$  or  $180^\circ$ ), then  $\mathbf{A} \times \mathbf{B} = 0$ ; therefore, it follows that  $\mathbf{A} \times \mathbf{A} = 0$ .

3. If  $\mathbf{A}$  is perpendicular to  $\mathbf{B}$ , then  $|\mathbf{A} \times \mathbf{B}| = AB$ .

4. The vector product obeys the distributive law:

$$\mathbf{A} \times (\mathbf{B} + \mathbf{C}) = \mathbf{A} \times \mathbf{B} + \mathbf{A} \times \mathbf{C}$$

5. The derivative of the cross product with respect to some variable such as  $t$  is

$$\frac{d}{dt}(\mathbf{A} \times \mathbf{B}) = \frac{d\mathbf{A}}{dt} \times \mathbf{B} + \mathbf{A} \times \frac{d\mathbf{B}}{dt}$$

where it is important to preserve the multiplicative order of  $\mathbf{A}$  and  $\mathbf{B}$ .

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$$\mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A}$$

$$\mathbf{A} \times \mathbf{A} = 0$$

$$\mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A}$$

$$\mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A}$$

Signs are interchangeable in cross products. For example,  $\mathbf{A} \times (-\mathbf{B}) = -\mathbf{A} \times \mathbf{B}$  and  $\mathbf{i} \times (-\mathbf{j}) = -\mathbf{i} \times \mathbf{j}$ .

The cross product of any two vectors  $\mathbf{A}$  and  $\mathbf{B}$  can be expressed in the following determinant form:

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} \hat{i} - \begin{vmatrix} A_x & A_z \\ B_x & B_z \end{vmatrix} \hat{j} + \begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix} \hat{k}$$

Expanding these determinants gives the result

$$\mathbf{A} \times \mathbf{B} = (A_y B_z - A_z B_y) \hat{i} - (A_x B_z - A_z B_x) \hat{j} + (A_x B_y - A_y B_x) \hat{k}$$

Given the definition of the cross product, we can now assign a direction to the torque vector. If the force lies in the  $xy$  plane, the torque is represented by a vector parallel to the  $z$  axis. The force in the first figure above creates a torque that tends to rotate the object counterclockwise about the  $z$  axis; thus the direction of  $\tau$  is out of the page and  $\tau$  is therefore in the positive  $z$  direction. If we reversed the direction of  $\mathbf{F}$  in the first figure above, then  $\tau$  would be in the negative  $z$  direction.

Two vectors, lying in the  $xy$  plane are given by the equations  $\mathbf{A} = 2\hat{i} - 5\hat{j}$  and  $\mathbf{B} = -\hat{i} + 2\hat{j}$ . Find  $\mathbf{A} \times \mathbf{B}$  and verify

YOU OUP Using Equations 11.7a through 11.7d, we obtain

$$\begin{aligned} \mathbf{A} \times \mathbf{B} &= (2\hat{i} + 3\hat{j}) \times (-\hat{i} + 2\hat{j}) \\ &= \hat{i} \times (-\hat{i}) + 3\hat{j} \times (-\hat{i}) + 2\hat{i} \times 2\hat{j} + 3\hat{j} \times 2\hat{j} \\ &= 0 - 3\hat{k} + 4\hat{k} + 0 = \hat{k} \end{aligned}$$

(We have omitted the terms containing  $\hat{i} \times \hat{i}$  and  $\hat{j} \times \hat{j}$  because, as Equation 11.7a shows, they are equal to zero.)

A force of  $\mathbf{P} = (12.0\hat{i} + 3.00\hat{j})$  N is applied to an object that is pivoted about a fixed axis aimed along the  $z$  coordinate axis. If the force is applied at a point located at  $\mathbf{r} = (4.00\hat{i} + 5.00\hat{j})$  m, find the torque vector  $\tau$ .

SOLUTION The torque vector is defined by means of a cross product in Equation 11.7:

$$\begin{aligned} \tau &= \mathbf{r} \times \mathbf{P} \\ &= [(4.00\hat{i} + 5.00\hat{j}) \text{ m}] \times [(12.0\hat{i} + 3.00\hat{j}) \text{ N}] \\ &= (4.00)(3.00)\hat{k} + (5.00)(12.0)\hat{k} \\ &= (12.0 + 60.0)\hat{k} \text{ N}\cdot\text{m} \\ &= 72.0\hat{k} \text{ N}\cdot\text{m} \end{aligned}$$

We can show that  $\mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A}$ , because

$$\begin{aligned} \mathbf{B} \times \mathbf{A} &= (-\hat{i} + 2\hat{j}) \times (2\hat{i} - 5\hat{j}) \\ &= -\hat{i} \times 2\hat{i} + 5\hat{j} \times \hat{i} + 2\hat{j} \times 2\hat{i} - 10\hat{j} \times \hat{j} \\ &= 0 - 5\hat{k} - 4\hat{k} + 0 = -9\hat{k} \end{aligned}$$

Therefore,  $\mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A}$

As an alternative method for finding  $\mathbf{A} \times \mathbf{B}$ , we could use Equation 11.8, with  $A_x = 2$ ,  $A_y = 3$ ,  $A_z = 0$  and  $B_x = -1$ ,

$$\mathbf{A} \times \mathbf{B} = (0)\hat{i} - (0)\hat{j} + [(2)(2) - (3)(-1)]\hat{k} = 7\hat{k}$$

$$\begin{aligned} \tau &= [12.0\hat{i} \times \hat{j} + 10.0\hat{j} \times \hat{i}] \text{ N}\cdot\text{m} \\ &= [12.0\hat{k} - 10.0\hat{k}] \text{ N}\cdot\text{m} \\ &= 2.0\hat{k} \text{ N}\cdot\text{m} \end{aligned}$$

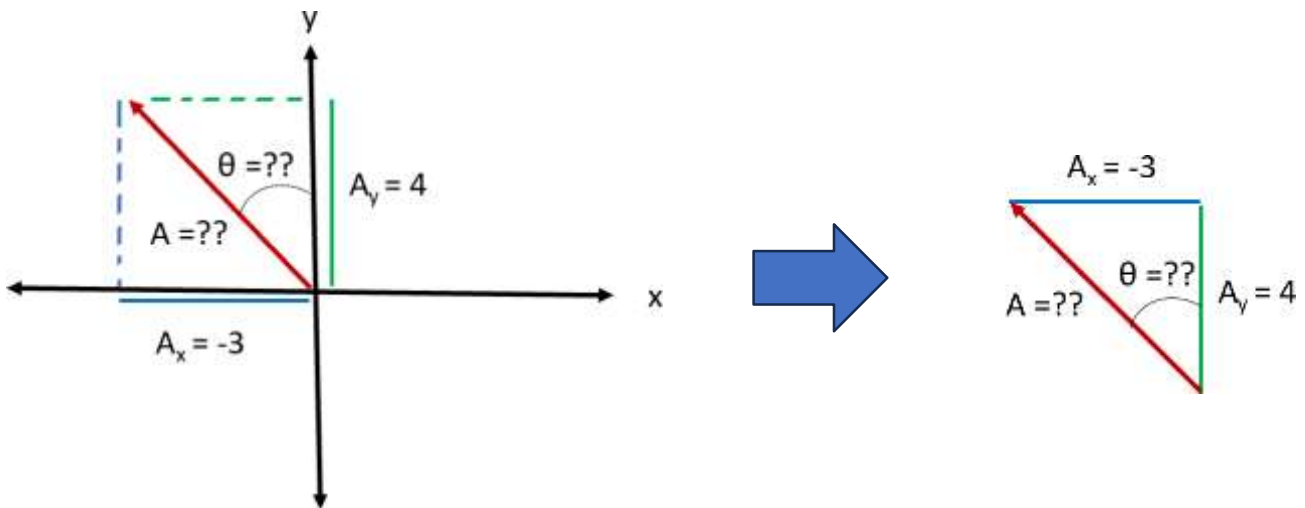
Notice that  $\mathbf{r}$  and  $\mathbf{F}$  are in the  $xy$  plane. As expected, the torque vector is perpendicular to this plane, having only a  $z$  component.

### Exercise 3 / Solution

Find the magnitude and the direction (the angle with respect to +y axis) for vector A, if the components of this vector are:  $A_x = -3$  and  $A_y = 4$

#### Solution:

We have  $A_x = -3$  and  $A_y = 4$ , let us find 1. The magnitude of A, and 2. The direction of A (the angle  $\theta$  with respect to x-axis)



1. ( $A = ??$ )

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(-3)^2 + 4^2} = 5$$

2. ( $\theta = ??$ )

$$\theta_y = \tan^{-1} \left( \frac{|A_x|}{|A_y|} \right) = \tan^{-1} \left( \frac{3}{4} \right) = 36.9^\circ$$

**Chapter Three:**  
**Kinematics in One and Two Dimensions**

**Intended Learning Outcomes:**

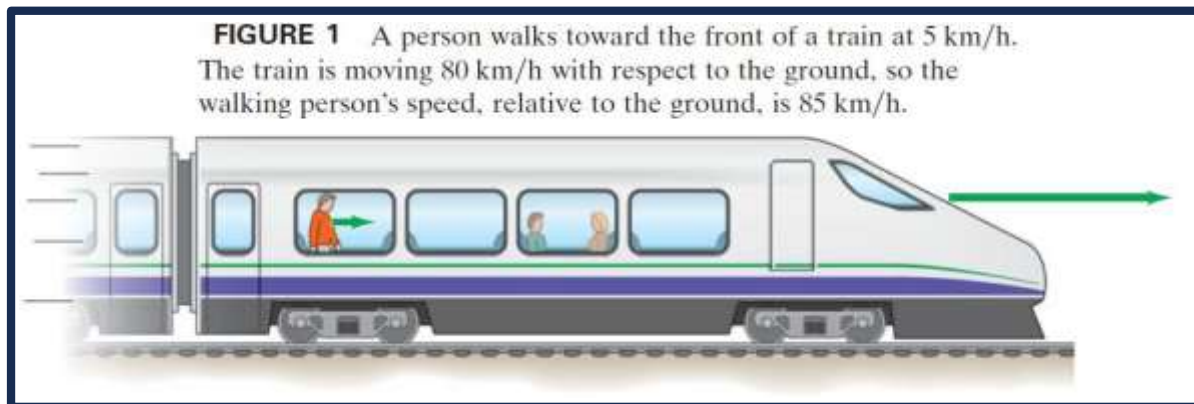
**At the end of this chapter, the students will be able to:**

- Define distance, displacement, speed, and velocity.
- Solve mathematical problems related to kinematics in one and two dimensions.

## Describing Motion: Kinematics in One Dimension

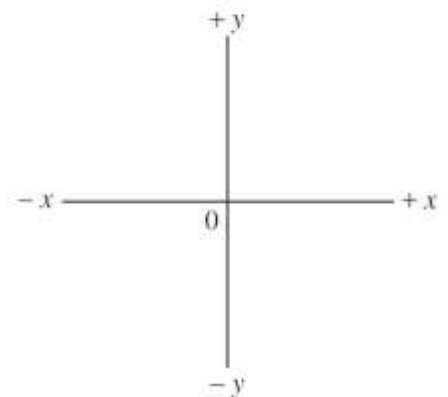
### Reference Frames and Displacement

Any measurement of position, distance, or speed must be made with respect to a reference frame, or frame of reference. For example, while you are on a train traveling at 80 km/h, suppose a person walks past you toward the front of the train at a speed of, say, 5 km/h (Fig. 1). This 5 km/h is the person's speed with respect to the train as frame of reference. With respect to the ground, that person is moving at a speed of  $80 \text{ km/h} + 5 \text{ km/h} = 85 \text{ km/h}$ . It is always important to specify the frame of reference when stating a speed. In everyday life, we usually mean "with respect to the Earth" without even thinking about it, but the reference frame must be specified whenever there might be confusion.



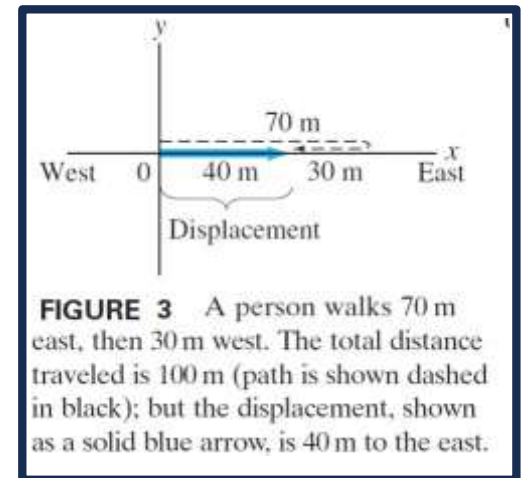
When specifying the motion of an object, it is important to specify not only the speed but also the direction of motion. Often we can specify a direction by using north, east, south, and west, and by "up" and "down." In physics, we often draw a set of coordinate axes, as shown in Fig. 2, to represent a frame of reference. We can always place the origin 0, and the directions of the x and y axes, as we like for convenience. The x and y axes are always perpendicular to each other. The origin is where  $x = 0$ ,  $y = 0$ . Objects positioned to the right of the origin of coordinates (0) on the x axis have an x coordinate which we almost always choose to be positive; then points to the left of 0 have a negative x coordinate. The position along the y axis is usually considered positive when above 0, and negative when below 0, although the reverse convention can be used

**FIGURE 2** Standard set of xy coordinate axes, sometimes called "rectangular coordinates."



if convenient. Any point on the plane can be specified by giving its x and y coordinates. In three dimensions, a z-axis perpendicular to the x and y axes is added. For one-dimensional motion, we often choose the x axis as the line along which the motion takes place. Then the position of an object at any moment is given by its x coordinate. If the motion is vertical, as for a dropped object, we usually use the y axis.

We need to make a distinction between the distance an object has traveled and its displacement, which is defined as the change in position of the object. That is, displacement is how far the object is from its starting point. To see the distinction between total distance and displacement, imagine a person walking 70 m to the east and then turning around and walking back (west) a distance of 30 m (see Fig. 3). The total distance traveled is 100 m, but the displacement is only 40 m since the person is now only 40 m from the starting point. Displacement is a quantity that has both magnitude and direction. Such quantities are called vectors, and are represented by arrows in diagrams. For example, in Fig. 3, the blue arrow represents the displacement whose magnitude is 40 m and whose direction is to the right (east).



Displacement is the change in position of an object. (how far the object is from its starting point).

In symbols, displacement  $\Delta x$  is defined to be  $\Delta x = x_f - x_i$

Distance traveled is the total length of the path traveled between two positions.

## Average velocity

An important aspect of the motion of a moving object is how fast it is moving—its speed or velocity.

The term “speed” refers to how far an object travels in a given time interval, regardless of direction. If a car travels 240 kilometers (km) in 3 hours (h), we say its average speed was 80 km/h. In general, the average speed of an object is defined as the total distance traveled along its path divided by the time it takes to travel this distance:

$$\text{average speed} = \frac{\text{distance traveled}}{\text{change in time}}$$

The terms “velocity” and “speed” are often used interchangeably in ordinary language. But in physics we make a distinction between the two. Speed is simply a positive number, with units. Velocity, on the other hand, is used to signify both the magnitude (numerical value) of how fast an object is moving and also the direction in which it is moving. Velocity is therefore a vector.

There is a second difference between speed and velocity: namely, the average velocity is defined in terms of displacement, rather than total distance traveled:

$$\text{average velocity} = \frac{\text{displacement}}{\text{change in time}} = \frac{\text{final position} - \text{initial position}}{\text{change in time}}$$

Average speed and average velocity have the same magnitude when the motion is all in one direction. In other cases, they may differ: recall the walk we described earlier, in Fig. 3, where a person walked 70 m east and then 30 m west. The total distance traveled was 70 m + 30 m = 100 m, but the displacement was 40 m. Suppose this walk took 70 s to complete. Then the average speed was:

$$\frac{\text{distance}}{\text{change in time}} = \frac{100 \text{ m}}{70 \text{ s}} = 1.4 \text{ m/s}$$

The magnitude of the average velocity, on the other hand, was:

$$\frac{\text{displacement}}{\text{change in time}} = \frac{40 \text{ m}}{70 \text{ s}} = 0.57 \text{ m/s}$$

## Instantaneous velocity

If you drive a car along a straight road for 150 km in 2 h, the magnitude of your average velocity is 75 km/h. It is unlikely, though, that you were moving at precisely 75 km/h at every instant. To describe this situation we need the concept of instantaneous velocity, which is the velocity at any instant of time. More precisely, the instantaneous velocity at any moment is defined as the average velocity over an infinitesimally short time interval. We can write the definition of instantaneous velocity  $v$ , for one-dimensional motion as

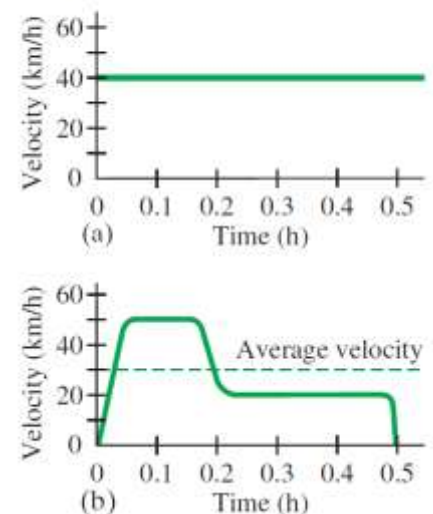
$$v = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t}$$

The notation  $\lim_{\Delta t \rightarrow 0}$  means the ratio  $\frac{\Delta x}{\Delta t}$  is to be evaluated in the limit of  $\Delta t$  approaching zero.

For instantaneous velocity we use the symbol  $v$ , whereas for average velocity we use  $\bar{v}$  with a bar above. In the rest of this booklet, when we use the term “velocity” it will refer to instantaneous velocity. When we want to speak of the average velocity, we will make this clear by including the word “average.”

Note that the instantaneous speed always equals the magnitude of the instantaneous velocity. Why? Because distance traveled and the magnitude of the displacement become the same when they become infinitesimally small. If an object moves at a uniform (that is, constant) velocity during a particular time interval, then its instantaneous velocity at any instant is the same as its average velocity (see Fig. 4 a). But in many situations this is not the case. For example, a car may start from rest, speed up to 50 km/h, remain at that velocity for a time, then slow down to 20 km/h in a traffic jam, and finally stop at its destination after traveling a total of 15 km in 30 min. This trip is plotted on the graph of Fig. 4 b. Also shown on the graph is the average velocity (dashed line), which is  $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{15 \text{ km}}{0.5 \text{ h}} = 30 \text{ km/h}$ .

**FIGURE 4** Velocity of a car as a function of time: (a) at constant velocity; (b) with velocity varying in time.



## Acceleration

An object whose velocity is changing is said to be accelerating. For instance, a car whose velocity increases in magnitude from zero to 80 km/h is accelerating. Acceleration specifies how rapidly the velocity of an object is changing. Average acceleration is defined as the change in velocity divided by the time taken to make this change:

$$\text{average acceleration} = \frac{\text{change of velocity}}{\text{time interval}}$$

In symbols, the average acceleration,  $\bar{a}$ , over a time interval  $\Delta t = t_2 - t_1$ , during which the velocity changes by  $\Delta v = v_2 - v_1$ , is defined as

$$\bar{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

We saw that velocity is a vector (it has magnitude and direction), so acceleration is a vector too. But for one dimensional motion, we need only use a plus or minus sign to indicate acceleration direction relative to a chosen coordinate axis. (Usually, right is +, left is -.)

The instantaneous acceleration,  $a$ , can be defined in analogy to instantaneous velocity as the average acceleration over an infinitesimally short time interval at a given instant:

$$a = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t}$$

Here  $\Delta v$  is the very small change in velocity during the very short time interval  $\Delta t$ .

## Deceleration

When an object is slowing down, we can say it is decelerating. But be careful: deceleration does not mean that the acceleration is necessarily negative. The velocity of an object moving to the right along the positive x-axis is positive; if the object is slowing down (as in Fig. 5–1), the acceleration is negative. But the same car moving to the left (decreasing x), and slowing down, has positive acceleration that points to the right, as shown in Fig. 5–2. We have a deceleration whenever the magnitude of the velocity is decreasing; thus the velocity and acceleration point in opposite directions when there is deceleration.

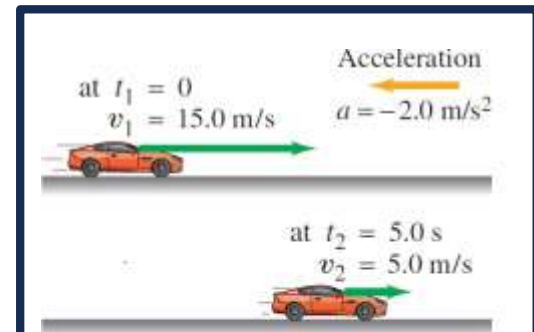


**FIGURE 5 – 1**

The position of the car at times  $t_1$  and  $t_2$ , as well as the car's velocity represented by the green arrows. The acceleration vector (orange) points to the left because the car slows down as it moves to the right.

**FIGURE 5 – 2** The car is moving to the left and decelerating.

The acceleration is  
 $a = (v_2 - v_1)/\Delta t$ , or  

$$a = \frac{(-5.0 \text{ m/s}) - (-15.0 \text{ m/s})}{5.0 \text{ s}}$$

$$= \frac{-5.0 \text{ m/s} + 15.0 \text{ m/s}}{5.0 \text{ s}} = +2.0 \text{ m/s}^2.$$

Figure 5-2 shows the same car moving to the left. At time  $t_1$ , the velocity is  $v_1 = -15.0 \text{ m/s}$  (green arrow pointing left). At time  $t_2$ , the velocity is  $v_2 = -5.0 \text{ m/s}$  (shorter green arrow pointing left). An orange arrow labeled 'a' points to the right.

**Remember that:**

- Distance traveled is the total length of the path traveled between two positions. (scalar quantity)
- Displacement is the change in position of an object. (how far the object is from its starting point). In symbols, displacement  $\Delta x$  is defined to be  $\Delta x = x_f - x_i$  (vector quantity)
- The average speed (scalar quantity) of an object is defined as the total distance traveled along its path divided by the time it takes to travel this distance:

$$\text{average speed} = \frac{\text{distance traveled}}{\text{change in time}}$$

- The average velocity is defined as the displacement divided by the time interval during which that displacement occurs:

$$\text{average velocity} = \frac{\text{displacement}}{\text{change in time}} = \frac{\text{final position} - \text{initial position}}{\text{change in time}}$$

- The instantaneous velocity at any moment is defined as the average velocity over an infinitesimally short time interval.

$$v = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t}$$

- The instantaneous speed always equals the magnitude of the instantaneous velocity.
- If an object moves at a constant velocity during a particular time interval, then its instantaneous velocity at any instant is the same as its average velocity.

- Average acceleration is defined as the change in velocity divided by the time taken to make this change:

$$\text{average acceleration} = \frac{\text{change of velocity}}{\text{time interval}}$$

- The instantaneous acceleration is the average acceleration over an infinitesimally short time interval at a given instant:

$$a = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t}$$

- When an object's velocity and acceleration are in the same direction, the object is speeding up. On the other hand, when the object's velocity and acceleration are in opposite directions, the object is slowing down. Remembering that  $F \propto a$  is a useful way to identify the direction of the acceleration by associating it with a force.

### Motion at Constant Acceleration- horizontal motion

Imagine a moving object that can be modeled as a particle. If it begins from position  $x_0$  and initial velocity  $v_0$  and moves in a straight line with a constant acceleration  $a$ , its subsequent position and velocity are described by the following kinematic equations (assuming  $x_i = 0$ ):

$$v = v_0 + a t$$

$$x = v_0 t + \frac{1}{2} a t^2$$

$$v^2 = v_0^2 + 2 a x$$

You will gain a great deal of experience in the use of these equations by solving a number of exercises and problems. Many times you will discover that more than one method can be used to obtain a solution. Remember that these equations of kinematics cannot be used in a situation in which the acceleration varies with time. They can be used only when the acceleration is constant.

#### Example 1

A body starts from rest and accelerates at  $3 \text{ m/s}^2$  for 4 s. Calculate (a) its final velocity, (b) the distance traveled.

#### Solution:

Considering  $v_0 = 0$  (starts from rest),  $a = 3 \text{ m/s}^2$

(a)  $v = ?$

$$v = v_0 + a t$$

$$v = 0 + 3 \times 4 = 12 \text{ m/s}$$

(b)  $x = ?$

$$x = v_0 t + \frac{1}{2} a t^2$$

$$x = 0 \times (3) + \frac{1}{2} \times 3 \times (4)^2 = 24 \text{ m}$$

$$v = v_0 + a t$$
$$x = v_0 t + \frac{1}{2} a t^2$$
$$v^2 = v_0^2 + 2 a x$$

### Example 2

A car moving with velocity  $10 \text{ m/s}$  accelerates uniformly at  $2 \text{ m/s}^2$ . Calculate its velocity after traveling 200 m.

#### Solution:

$$v_0 = 10 \text{ m/s}, \quad a = 2 \text{ m/s}^2, \quad x = 200 \text{ m}, \quad v = ?$$

$$v^2 = v_0^2 + 2 a x$$

$$v^2 = (10)^2 + 2 \times 2 \times 200$$

$$v^2 = 900$$

$$v = \sqrt{900} = 30 \text{ m/s}$$

$$\begin{aligned} v &= v_0 + a t \\ x &= v_0 t + \frac{1}{2} a t^2 \\ v^2 &= v_0^2 + 2 a x \end{aligned}$$

### Exercise 1

In an electron gun, an electron is accelerated uniformly from rest to a velocity of  $4 \times 10^7 \text{ m/s}$  in a distance of 0.10 m. Calculate the acceleration.

**The answer:**  $a = 8 \times 10^{15} \text{ m/s}^2$

### Exercise 2

A car slows down from 28 m/s to rest in a distance of 88 m. What was its acceleration, assumed constant?

**The answer:**  $a = -4.5 \text{ m/s}^2$

### Exercise 3

A car accelerates from 14 m/s to 21 m/s in 6 s. What was its acceleration? How far did it travel in this time? Assume constant acceleration.

**The answer:**  $a = 1.2 \text{ m/s}^2$  and  $x = 102 \text{ m}$

## Motion under gravity- vertical motion

Objects that move vertically near the surface of the Earth, either falling or having been projected vertically up or down, move with the constant downward acceleration due to gravity, whose magnitude is  $g = 9.8 \text{ m/s}^2$  if air resistance can be ignored. If we neglect air resistance and assume the free-fall acceleration does not vary with altitude over short vertical distances, the motion of a freely falling object moving vertically is equivalent to the motion of a particle under constant acceleration in one dimension. Therefore, the previous kinematic equations for the particle under constant acceleration can be applied. The only modification for freely falling objects that we need to make in these equations is to note that the motion is in the vertical direction (the  $y$  direction) rather than in the horizontal direction ( $x$ ) and that the acceleration is downward and has a magnitude of  $9.8 \text{ m/s}^2$ . Therefore, we choose  $a_y = -g = -9.8 \text{ m/s}^2$ , where the negative sign means that the acceleration of a freely falling object is downward.

$$\begin{array}{lll} v = v_0 + a_y t & \text{or} & v = v_0 - g t \\ y = v_0 t + \frac{1}{2} a_y t^2 & \text{or} & y = v_0 t - \frac{1}{2} g t^2 \\ v^2 = v_0^2 + 2 a_y y & \text{or} & v^2 = v_0^2 - 2 g y \end{array}$$

## Acceleration at the Top of the Motion

A common misconception is that the acceleration of a projectile at the top of its trajectory is zero. Although the velocity at the top of the motion of an object thrown upward momentarily goes to zero, the acceleration is still that due to gravity at this point. If the velocity and acceleration were both zero, the projectile would stay at the top.

**Remember that:** at a given location on the Earth and in the absence of air resistance, all objects fall with the same constant acceleration.

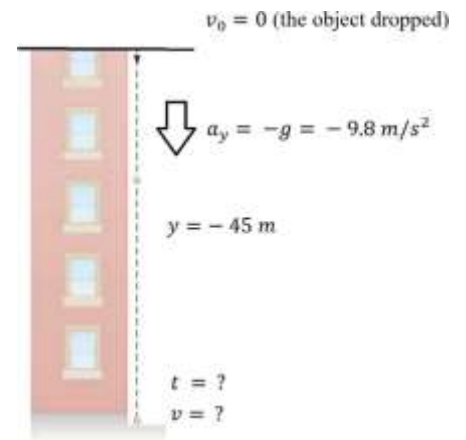
### Example 3

An object is dropped from a height of 45 m. Calculate

(a) the time taken to reach the ground, and

(b) its maximum velocity (before it reaches the ground).

Consider  $g = 9.8 \text{ m/s}^2$



### Solution:

We have  $v_0 = 0$  (the object dropped),

$y = -45 \text{ m}$  ( below the reference line to negative direction )

$$g = 9.8 \text{ m/s}^2$$

(a)  $t = ?$

$$y = v_0 t - \frac{1}{2} g t^2$$

$$-45 = 0 - \frac{1}{2} (9.8) t^2$$

$$-45 = -4.9 t^2$$

$$t^2 = \frac{-45}{-4.9} = 9.18$$

$$t = \sqrt{9.18} = 3.03 \text{ s}$$

(b)  $v = ?$

$$v = v_0 - g t$$

$$v = 0 - (9.8) 3.03$$

$$v = -29.7 \text{ m/s} \text{ ( negative sign indicates the direction to negative y axis)}$$

|                                 |
|---------------------------------|
| $v = v_0 - g t$                 |
| $y = v_0 t - \frac{1}{2} g t^2$ |
| $v^2 = v_0^2 - 2 g y$           |

#### Example 4

A cricket ball is thrown vertically upwards with a velocity of  $20 \text{ m/s}$ . Calculate

(a) the maximum height reached,

(b) the time taken to return to Earth.

Consider  $g = 9.8 \text{ m/s}^2$

#### Solution:

(a) At the maximum height the velocity is zero  $v = 0 \text{ m/s}$ . The maximum height reached  $y$  is taken from the ground level.

$$v_0 = 20 \text{ m/s},$$

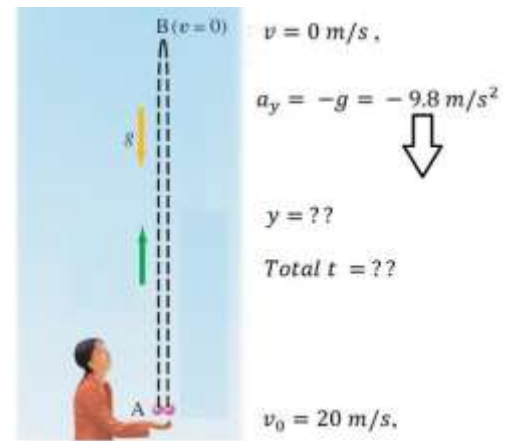
$$v = 0 \text{ m/s}, \quad y = ??$$

$$v^2 = v_0^2 - 2 g y$$

$$0^2 = 20^2 - 2 (9.8) y$$

$$0 = 400 - 19.6 y$$

$$y = \frac{400}{19.6} = 20.4 \text{ m}$$



$$\begin{aligned} v &= v_0 - g t \\ y &= v_0 t - \frac{1}{2} g t^2 \\ v^2 &= v_0^2 - 2 g y \end{aligned}$$

(b) The time taken to return to earth. To do that, we can find the time to reach its maximum height, (when its velocity is zero) and double it.

$$v_0 = 20 \text{ m/s},$$

$$v = 0 \text{ m/s}, \quad t = ?$$

$$v = v_0 - g t$$

$$0 = 20 - (9.8) t$$

$$9.8 t = 20$$

$$t = \frac{20}{9.8} = 2.04 \text{ s}$$

$$\begin{aligned} v &= v_0 - g t \\ y &= v_0 t - \frac{1}{2} g t^2 \\ v^2 &= v_0^2 - 2 g y \end{aligned}$$

So, the total time to return to Earth  $= 2 \times 2.04 = 4.08 \text{ s}$

(b)// Another solution

On its return to earth, after time  $t$ , we have  $y = 0$ . So

$$v_0 = 20 \text{ m/s},$$

$$v = 0 \text{ m/s}, \quad t = ?$$

$$y = v_0 t - \frac{1}{2} g t^2$$

$$0 = 20 t - \frac{1}{2} (9.8) t^2$$

$$0 = 20 t - 4.9 t^2$$

$$4.9 t^2 - 20 t = 0$$

$$t (4.9 t - 20) = 0$$

We have two solutions:

$$t = 0 \text{ (which is before throwing the ball, also } y = 0 \text{ )}$$

Or

$$4.9 t - 20 = 0$$

$$4.9 t = 20$$

$$t = \frac{20}{4.9} = 4.08 \text{ s}$$

$$\begin{aligned} v &= v_0 - g t \\ y &= v_0 t - \frac{1}{2} g t^2 \\ v^2 &= v_0^2 - 2 g y \end{aligned}$$

#### Exercise 4

A ball is dropped from a cliff top and takes 3s to reach the ground below. Calculate (a) the height of the cliff and (b) the ball's velocity before it reaches the ground. *Consider  $g = 9.8 \text{ m/s}^2$ .*

**The answer:**  $y = -44.1 \text{ m}$  and  $v = -29.4 \text{ m/s}$

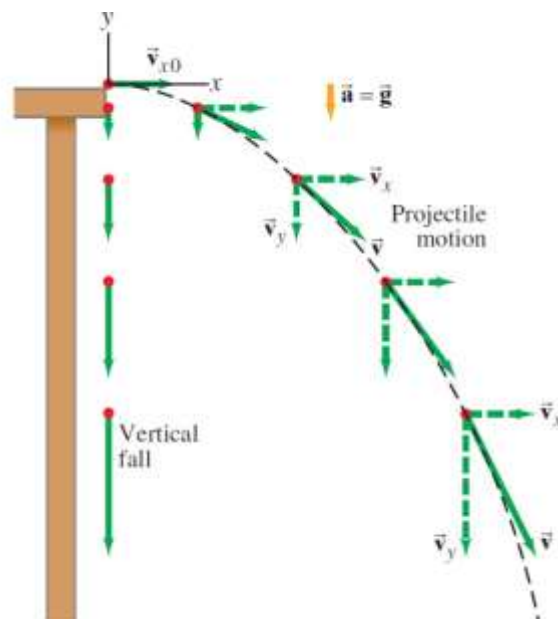
## Kinematics in Two Dimensions; Projectile Motion \*\*

Galileo was the first to describe projectile motion accurately. He showed that it could be understood by analyzing the horizontal and vertical components of the motion separately. For convenience, we assume that the motion begins at time  $t = 0$  at the origin of an  $xy$  coordinate system. Let us look at a (tiny) ball rolling off the end of a horizontal table with an initial velocity in the horizontal ( $x$ ) direction,  $v_{x0}$ . See Fig. 6, where an object falling vertically is also shown for comparison. The velocity vector  $\vec{v}$  at each instant points in the direction of the ball's motion at that instant and is thus always tangent to the path. Following Galileo's ideas, we treat the horizontal and vertical components of velocity and acceleration separately, and we can apply the previous kinematic equations to the  $x$  and  $y$  components of the motion.

First we examine the vertical ( $y$ ) component of the motion. At the instant the ball leaves the table's top  $t = 0$ , it has only an  $x$  component of velocity. Once the ball leaves the table (at  $t = 0$ ), it experiences a vertically downward acceleration  $g$ , the acceleration due to gravity. Thus  $v_y$  is initially zero ( $v_{y0} = 0$ ) but increases continually in the downward direction (until the ball hits the ground). Let us take  $y$  to be positive upward. Then the acceleration due to gravity is in the  $-y$  direction, so  $a_y = -g$ . From previous kinematic equations (using  $y$  in place of  $x$ ) we can write  $v_y = v_{y0} + a_y t = -gt$  since we set  $v_{y0} = 0$ . The vertical displacement is written in terms of  $y$ :  $y = v_{y0} t + \frac{1}{2} a_y t^2$ . Given  $v_{y0} = 0$ , and  $a_y = -g$  then

$$y = -\frac{1}{2} g t^2 .$$

**FIGURE 6** Projectile motion of a small ball projected horizontally with initial velocity  $\vec{v} = \vec{v}_{x0}$ . The dashed black line represents the path of the object. The velocity vector  $\vec{v}$  is in the direction of motion at each point, and thus is tangent to the path. The velocity vectors are green arrows, and velocity components are dashed. (A vertically falling object starting from rest at the same place and time is shown at the left for comparison;  $v_y$  is the same at each instant for the falling object and the projectile.)



In the horizontal direction, on the other hand, there is no acceleration (we are ignoring air resistance). With  $a_x = 0$ , the horizontal component of velocity,  $v_x$ , remains constant, equal to its initial value,  $v_{x0}$ , and thus has the same magnitude at each point on the path. The horizontal displacement (with  $a_x = 0$ ) is given by  $x = v_{x0} t + \frac{1}{2} a_x t^2 = v_{x0} t$ .

One result of this analysis, which Galileo himself predicted, is that **an object projected horizontally will reach the ground in the same time as an object dropped vertically**. This is because the vertical motions are the same in both cases, as shown in Fig. 6. Figure 7 is a multiple-exposure photograph of an experiment that confirms this.

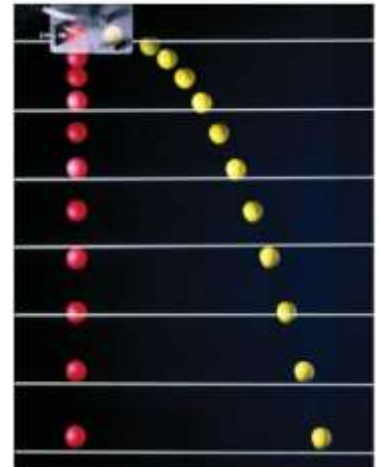


**FIGURE 7** Multiple-exposure photograph showing positions of two balls at equal time intervals. One ball was dropped from rest at the same time the other ball was projected horizontally outward. The vertical position of each ball is seen to be the same at each instant.

If an object is projected at an upward angle, as in Fig. 8, the analysis is similar, except that now there is an initial vertical component of velocity,  $v_{y0}$ . Because of the downward acceleration of gravity, the upward component of velocity  $v_y$  gradually decreases with time until the object reaches the highest point on its path, at which point  $v_y = 0$ . Subsequently, the object moves downward (Fig. 8) and  $v_y$  increases in the downward direction, as shown (that is, becoming more negative). As before,  $v_x$  remains constant.

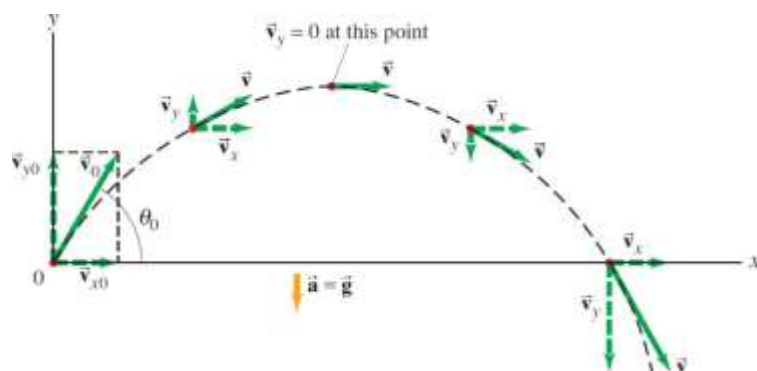


**FIGURE 8** Path of a projectile launched with initial velocity  $\vec{v}_0$  at angle  $\theta_0$  to the horizontal. Path is shown dashed in black, the velocity vectors are green arrows, and velocity components are dashed. The figure does not show where the projectile hits the ground (at that point, projectile motion ceases).

## Solving Projectile Motion Problems\*\*

We now work through several examples of projectile motion quantitatively. We use the previous kinematic equations separately for the vertical and horizontal components of the motion. These equations are shown separately for the x and y components of the motion in Table 1, for the general case of two-dimensional motion at constant acceleration. Note that x and y are the respective displacements, that  $v_x$  and  $v_y$  are the components of the velocity, and that  $a_x$  and  $a_y$  are the components of the acceleration, each of which is constant. The subscript 0 means “at t = 0.”

**TABLE 1 General Kinematic Equations for Constant Acceleration in Two Dimensions**

| x component (horizontal)             | y component (vertical)            |
|--------------------------------------|-----------------------------------|
| $v_x = v_{x0} + a_x t$               | $v_y = v_{y0} - gt$               |
| $x = v_{x0} t + \frac{1}{2} a_x t^2$ | $y = v_{y0} t - \frac{1}{2} gt^2$ |
| $v_x^2 = v_{x0}^2 + 2a_x x$          | $v_y^2 = v_{y0}^2 - 2g y$         |

**For the projectile motion**

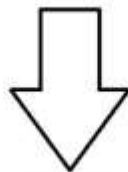


**TABLE 2 Kinematic Equations for Projectile Motion**  
(y positive upward;  $a_x = 0$ ,  $a_y = -g = -9.80 \text{ m/s}^2$ )

| Horizontal Motion<br>( $a_x = 0$ , $v_x = \text{constant}$ ) | Vertical Motion<br>( $a_y = -g = \text{constant}$ ) |
|--------------------------------------------------------------|-----------------------------------------------------|
| $v_x = v_{x0}$                                               | $v_y = v_{y0} - gt$                                 |
| $x = v_{x0} t$                                               | $y = v_{y0} t - \frac{1}{2} gt^2$                   |
|                                                              | $v_y^2 = v_{y0}^2 - 2g y$                           |

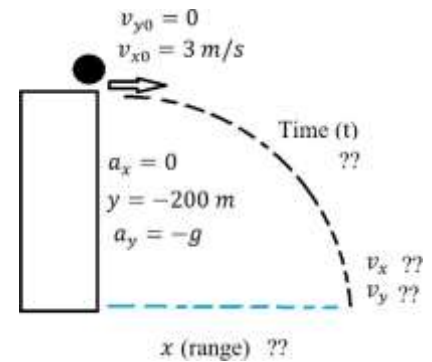
If the projection angle  $\theta$  is chosen relative to the x axis (Fig. 8), then

$$v_{x0} = v_0 \cos \theta \quad \text{and} \quad v_{y0} = v_0 \sin \theta$$

In many cases, the vertical component determines the time of flight (and the vertical distances), and the horizontal component determines the range.

### Example 5\*\*

A stone is projected horizontally with a velocity of 3 m/s, from the top of a vertical cliff 200 m high. Calculate (a) how long it takes to reach the ground (time), (b) its distance from the foot of the cliff (horizontal distance; range), and (c) its vertical and horizontal components of velocity when it hits the ground. Consider  $g = 9.8 \text{ m/s}^2$



### Solution:

We have

| Known                                      | Unknown   |
|--------------------------------------------|-----------|
| $v_{x0} = 3 \text{ m/s} = \text{constant}$ | time (t)  |
| $v_{y0} = 0$                               | x (range) |
| $a_x = 0$                                  | $v_x$     |
| $y = -200 \text{ m}$                       | $v_y$     |
| $a_y = -g$                                 |           |
| $g = 9.8 \text{ m/s}^2$                    |           |

| Horizontal Motion<br>( $a_x = 0$ , $v_x = \text{constant}$ ) | Vertical Motion<br>( $a_y = -g = \text{constant}$ ) |
|--------------------------------------------------------------|-----------------------------------------------------|
| $v_x = v_{x0}$                                               | $v_y = v_{y0} - gt$                                 |
| $x = v_{x0}t$                                                | $y = v_{y0}t - \frac{1}{2}gt^2$                     |
|                                                              | $v_y^2 = v_{y0}^2 - 2gy$                            |

(a) The vertical motion decides the time of flight.

$$y = v_{y0}t - \frac{1}{2}gt^2$$

$$-200 = 0 - \frac{1}{2}(9.8)t^2$$

$$-200 = -4.9t^2$$

$$t^2 = \frac{-200}{-4.9} = 40.8$$

$$t = \sqrt{40.8} = 6.4 \text{ s}$$

(b) The range  $x$  (horizontal distance)

$$x = v_{x0} t = 3 \times 6.4 = 19.2 \text{ m} \quad (\text{Remember that } v_{x0} = \text{constant} = 3 \text{ m/s})$$

(c) The vertical component of velocity when the stone hits the ground is required

$$v_y = ??$$

$$v_y = v_{y0} - g t$$

$$v_y = 0 - 9.8 \times 6.4 = -62.7 \text{ m/s}.$$

The negative sign indicates the direction of the negative axis.

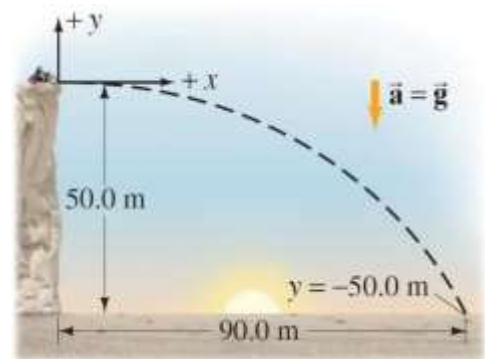
(you can have the same answer by using this equation  $v_y^2 = v_{y0}^2 - 2 g y$ )

$$v_x = ??$$

The horizontal component remains at  $3 \text{ m/s}$  ( $v_x = v_{x0} = 3 \text{ m/s} = \text{constant}$ )

### Exercise 5 (Projectile Motion)\*\*

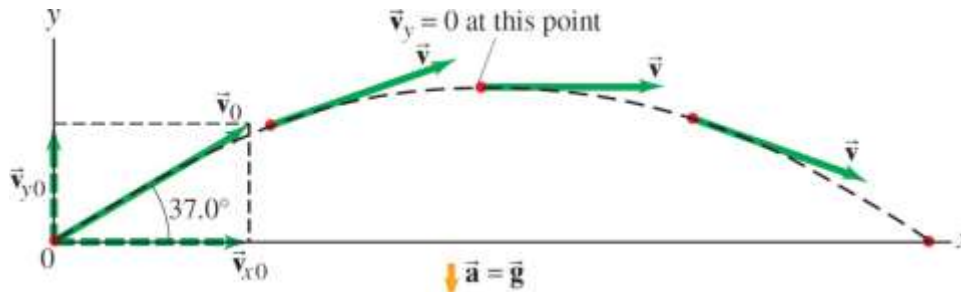
A movie stunt driver on a motorcycle speeds horizontally off a 50 m-high cliff. How fast must the motorcycle leave the cliff top to land on level ground below, 90 m from the base of the cliff where the cameras are? Consider  $g = 9.8 \text{ m/s}^2$



**The answer:**  $v_{x0} = 28.2 \text{ m/s}$

### Example 6\*\*\*

A kicked football leaves the ground at an angle  $\theta = 37^\circ$  with a velocity of 20 m/s as shown in the figure below. Calculate (a) the maximum height, (b) the time of travel before the football hits the ground, and (c) how far away it hits the ground. Assume the ball leaves the foot at ground level. Consider  $g = 9.8 \text{ m/s}^2$



### Solution:

The initial velocity components

$$v_{x0} = v_0 \cos \theta = 20 \times \cos 37 = 16 \text{ m/s}$$

$$v_{y0} = v_0 \sin \theta = 20 \times \sin 37 = 12 \text{ m/s}$$

We have

| Known                                       | Unknown   |
|---------------------------------------------|-----------|
| $v_{x0} = 16 \text{ m/s} = \text{constant}$ | Time (t)  |
| $v_{y0} = 12 \text{ m/s}$                   | x (range) |
| $a_x = 0$                                   | $v_x$     |
| $a_y = -g$                                  | $v_y$     |
| $g = 9.8 \text{ m/s}^2$                     |           |

| Horizontal Motion<br>( $a_x = 0$ , $v_x = \text{constant}$ ) | Vertical Motion<br>( $a_y = -g = \text{constant}$ ) |
|--------------------------------------------------------------|-----------------------------------------------------|
| $v_x = v_{x0}$                                               | $v_y = v_{y0} - gt$                                 |
| $x = v_{x0}t$                                                | $y = v_{y0}t - \frac{1}{2}gt^2$                     |
|                                                              | $v_y^2 = v_{y0}^2 - 2gy$                            |

(a) The maximum height

$y = ??$  ( *the maximum y is when  $v_y = 0$*  )

$$v_y^2 = v_{y0}^2 - 2 g y$$

$$0 = 12^2 - 2 \times 9.8 \times y$$

$$144 = 19.6 \times y$$

$$y = \frac{144}{19.6} = 7.35 \text{ m}$$

(b) The time of travel (finding t when y = 0)

$$y = v_{y0} t - \frac{1}{2} g t^2$$

$$0 = 12 t - \frac{1}{2} \times 9.8 \times t^2$$

$$12 t - 4.9 \times t^2 = 0$$

$$(12 - 4.9 t) t = 0$$

$$t = 0 \text{ ( at the beginning )}$$

$$12 - 4.9 t = 0$$

$$12 = 4.9 t$$

$$t = \frac{12}{4.9} = 2.45 \text{ s}$$

(c) The total distance traveled in the x direction

$$x = v_{x0} t = 16 \times 2.45 = 39.2 \text{ m}$$

**Exercise 6 (Projectile Motion)\*\***

A shell is fired from a gun with a velocity of 600 m/s at an angle of  $40^\circ$  to the ground (to the horizontal). Calculate (a) the time of flight, (b) the range, (c) the maximum height reached.

*Consider  $g = 9.8 \text{ m/s}^2$*

**The answer:**    (a)  $t = 78.8 \text{ s}$                       (b)  $x = 36248 \text{ m}$                       (c)  $y = 7602 \text{ m}$

**Chapter Four:**  
**Dynamics: Newton's Laws of Motion**

**Intended Learning Outcomes:**

**At the end of this chapter, the students will be able to:**

- Explain Newton's laws of motion
- Solve mathematical problems related to Newton's laws.

## Forces

- A force is a push or pull acting upon an object as a result of its interaction with another object. We can hit the ball at different speeds and direct it into different parts of the opponent's court. This means that we can control the magnitude of the applied force and also its direction, so the force is a vector quantity, just like velocity and acceleration.
- Forces can be used to change the speed of the object or/and the direction of the object.
- The unit of force in the metric system is:

$$\text{Newton (N) where } 1\text{N} = \frac{\text{kg.m}}{\text{s}^2}$$

## Mass and Inertia

Imagine hitting a golf ball off a tee with a driver (the golf club). If you're a good golfer, the ball will sail over two hundred yards down the fairway. Now imagine teeing up a bowling ball and striking it with the same club (an experiment we don't recommend). Your club would probably break, you might sprain your wrist, and the bowling ball, at best, would fall off the tee, take half a roll, and come to rest. From this thought experiment, we conclude that although both balls resist changes in their state of motion, the bowling ball offers much more effective resistance. The tendency of an object to continue in its original state of motion is called inertia. Although inertia is the tendency of an object to continue its motion in the absence of a force, mass is a measure of the object's resistance to changes in its motion due to a force. The greater the mass of a body, the less it accelerates under the action of a given applied force. The SI unit of mass (Scalar quantity) is the kilogram.

- Inertia is the tendency of an object to resist changes in its state of motion.

## Newton's First Law (Law of Inertia)

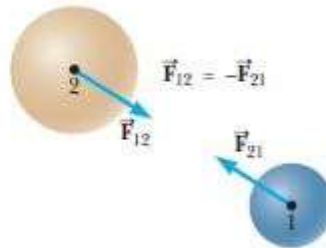
In the absence of external forces, an object at rest remains at rest and an object in motion continues in motion with a constant velocity.

## Newton's Third Law

"For every action there is an equal and opposite reaction"

Consider the task of driving (hitting hard) a nail into a block of wood, for example, as illustrated in the Figure below. To accelerate the nail and drive it into the block, the hammer must exert a net force on the nail. Newton recognized, however, that a single isolated force (such as the force exerted by the hammer on the nail) couldn't exist. Instead, forces in nature always exist in pairs. According to Newton, as the nail is driven into the block by the force exerted by the hammer, the hammer is slowed down and stopped by the force exerted by the nail. Newton described such paired forces with his third law:

If object 1 and object 2 interact, the force  $\vec{F}_{12}$  exerted by object 1 on object 2 is equal in magnitude but opposite in direction to the force  $\vec{F}_{21}$  exerted by object 2 on object 1.



Newton's third law constantly affects our activities in everyday life. Without it, no locomotion of any kind would be possible, whether on foot, on a bicycle, or in a motorized vehicle. When walking, for example, we exert a frictional force against the ground. The reaction force of the ground against our foot propels us forward. In the same way, the tires on a bicycle exert a frictional force against the ground, and the reaction of the ground pushes the bicycle forward.

## Newton's Second Law

Newton's first law explains what happens to an object that has no net force acting on it: The object either remains at rest or continues moving in a straight line with constant speed. Newton's second law answers the question of what happens to an object that does have a net force acting on it. Imagine pushing a block of ice across a frictionless horizontal surface. When you exert some horizontal force on the block, it moves with an acceleration of, say,  $2 \text{ m/s}^2$ . If you apply a force twice as large, the acceleration doubles to  $4 \text{ m/s}^2$ . Pushing three times as hard triples the acceleration, and so on. From such observations, we conclude that the acceleration of an object is directly proportional to the net force acting on it.

Mass also affects acceleration. Suppose you stack identical blocks of ice on top of each other while pushing the stack with constant force. If the force applied to one block produces an acceleration of  $2 \text{ m/s}^2$ , then the acceleration drops to half that value,  $1 \text{ m/s}^2$ , when two blocks are pushed, to one-third the initial value when three blocks are pushed, and so on. We conclude that the acceleration of an object is inversely proportional to its mass. These observations are summarized in Newton's second law: The acceleration  $\vec{a}$  of an object is directly proportional to the net force  $\sum \vec{F}$  acting on it and inversely proportional to its mass. Mathematically

$$\vec{a} = \frac{\sum \vec{F}}{m} \quad \text{or} \quad \sum \vec{F} = m \cdot \vec{a}$$

The net force on an object is the vector sum of all forces acting on the object. In solving a problem using Newton's second law, it is imperative to determine the correct net force on an object. There may be many forces acting on an object, but there is only one acceleration.

- If the net force exerted on an object is zero, the acceleration of the object is zero and its velocity remains constant.

Note that the previous equation is a vector expression and hence is equivalent to two-component equations:

$$\sum F_x = m \cdot a_x \qquad \sum F_y = m \cdot a_y$$

## Weight - the Force of Gravity

All objects are attracted to the Earth. The attractive force exerted by the Earth on an object is called the gravitational force  $\vec{F}_g$ . This force is directed toward the center of the Earth, and its magnitude is called the weight of the object. The gravitational force exerted on an object is equal to the product of its mass (a scalar quantity) and the acceleration of gravity  $g$

$$\vec{F}_g = m \vec{g}$$

The weight of an object is the magnitude of the gravitational force acting on the object:

$$W = F_g = m g$$

### Example 1

What is the weight of a child with a mass of 40 kg on the earth? *Consider  $g = 9.8 \text{ m/s}^2$*

**Solution:**

$$W = F_g = m g = 40 \times 9.8 = 392 \text{ N}$$

## Resultant force $\sum \vec{F}$ (net force)

A resultant force is a single force which can replace a set of forces acting on an object. It has exactly the same effect on the object as the original set.

### Example 2

For the following system of forces shown in each figure below, find

- (a) The resultant (net) force on the x-axis  $\sum F_x = ??$ ,
- (b) The resultant (net) force on the y-axis  $\sum F_y = ??$ , and
- (c) The magnitude and the direction of the resultant (net) force  $\sum \vec{F} = ??$

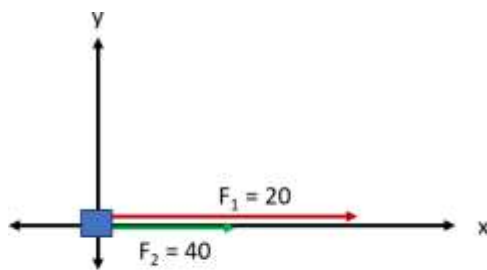


Figure. 1

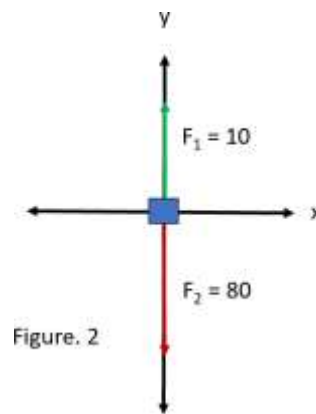


Figure. 2

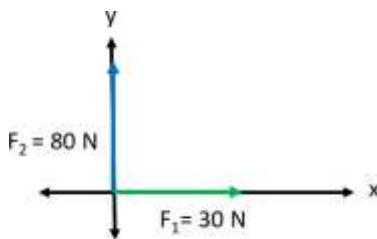


Figure. 3

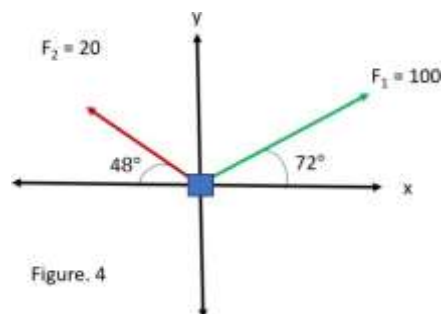


Figure. 4

## Solution

### Figure 1

$$\sum F_x = F_1 + F_2 = 20 + 40 = 60 \text{ N in the same direction ( to the } + x \text{ axis )}$$

$$\sum F_y = 0$$

$$\sum \vec{F} = 60 \text{ N to the } + x \text{ axis direction}$$

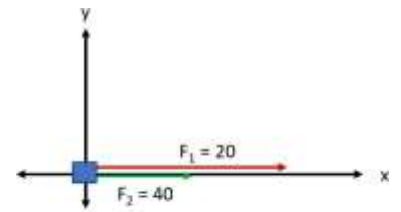


Figure. 1

### Figure 2

$$\sum F_x = 0$$

$$\sum F_y = F_1 - F_2 = 10 - 80 = -70 \text{ N to negative y axis}$$

$$\sum \vec{F} = -70 \text{ N to negative y axis direction}$$

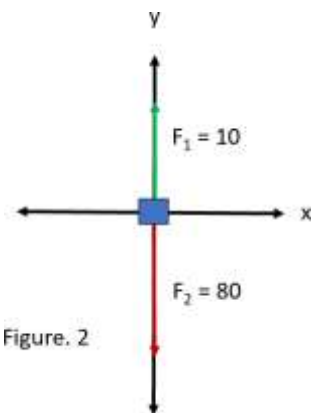


Figure. 2

### Figure 3

The resultant (net) force of these perpendicular forces is a force lying between them, making an angle  $\theta$  with respect to the x-axis (or y-axis).

(like examples 3 and 4 in chapter 2; vectors)

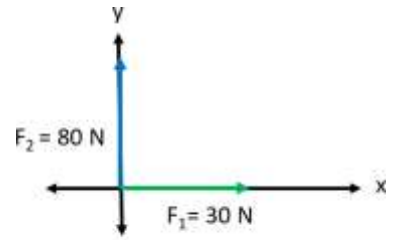
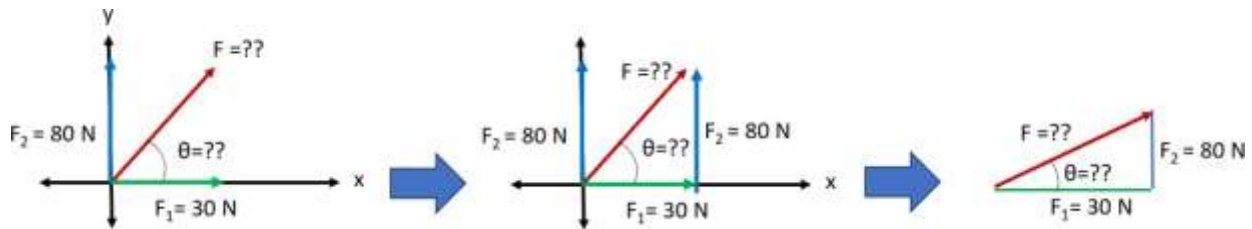


Figure. 3



$$\Sigma F_x = 30 \text{ N}$$

$$\Sigma F_y = 80 \text{ N}$$

$$\Sigma \vec{F} = \sqrt{(F_x)^2 + (F_y)^2} = \sqrt{(30)^2 + (80)^2} = 85.4 \text{ N}$$

The direction of  $F$  ( the angle with respect to x-axis) ( $\theta = ??$ )

$$\tan \theta = \frac{F_2}{F_1}$$

So,

$$\theta = \tan^{-1} \frac{F_2}{F_1}$$

$$\theta_x = \tan^{-1} \left( \frac{80}{30} \right) = 69.4^\circ$$

**Figure 4**

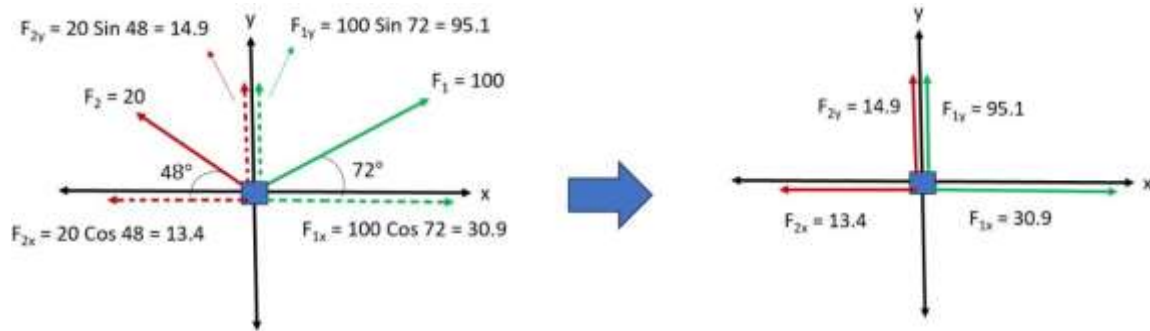
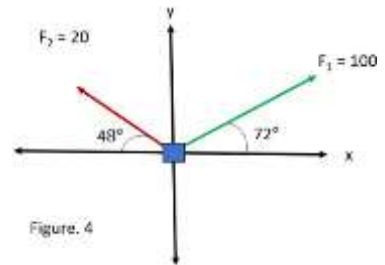
For such a system, we must

1. Analyze each vector (find the x and the y components for each vector.

2. Find the resultant force on each axis

( $\sum F_x = ??$ , and  $\sum F_y = ??$ )

3. Find the resultant (net) force of the perpendicular forces.  $\sum \vec{F} = ??$



The x and the y components for each vector are

$$F_{1x} = F_1 \cos 72 = 100 \cdot \cos 72 = 30.9 \text{ (to the + x axis)}$$

$$F_{1y} = F_1 \sin 72 = 100 \cdot \sin 72 = 95.1 \text{ (to the + y axis)}$$

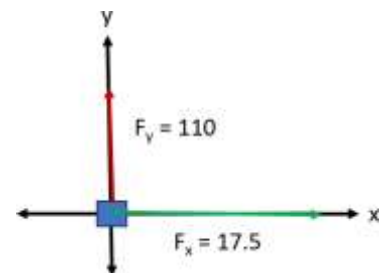
$$F_{2x} = F_2 \cos 48 = 20 \cdot \cos 48 = 13.4 \text{ (to the negative x axis)}$$

$$F_{2y} = F_2 \sin 48 = 20 \cdot \sin 48 = 14.9 \text{ (to the + y axis)}$$

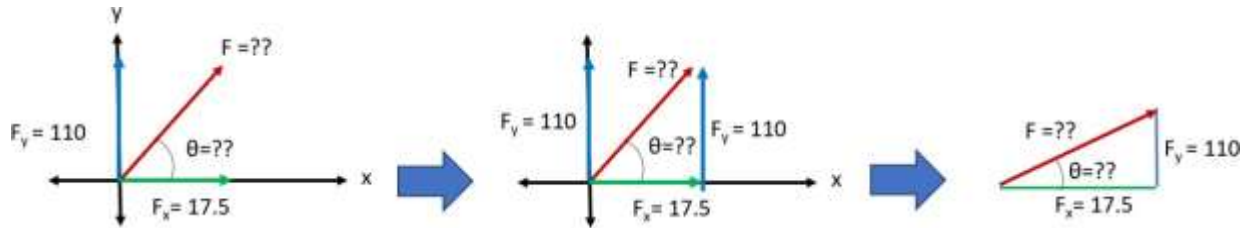
Now; the net forces on the x and y axis

$$\sum F_x = 30.9 - 13.4 = 17.5 \text{ N to the + x axis}$$

$$\sum F_y = 95.1 + 14.9 = 110 \text{ N to the + y axis}$$



The resultant (net) force  $\Sigma \vec{F}$  of these perpendicular forces is a force lying between them, making an angle  $\theta$  with respect to the x-axis (or y-axis). (like the previous example)



$$\Sigma \vec{F} = \sqrt{(F_x)^2 + (F_y)^2} = \sqrt{(17.5)^2 + (110)^2} = 111.4 \text{ N}$$

The direction of  $F$  ( the angle with respect to x-axis) ( $\theta = ??$ )

$$\tan \theta = \frac{F_y}{F_x}$$

So,

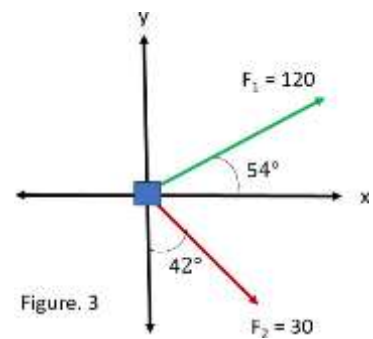
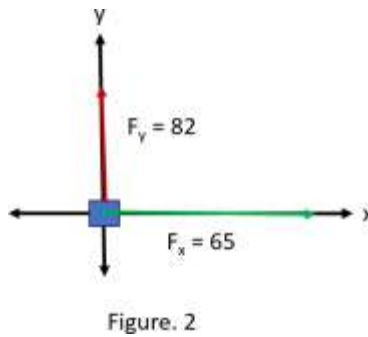
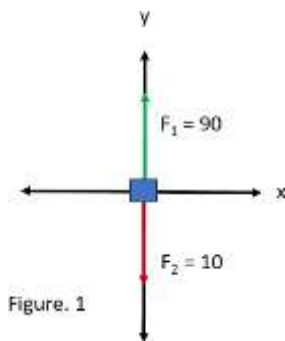
$$\theta = \tan^{-1} \frac{F_y}{F_x}$$

$$\theta_x = \tan^{-1} \left( \frac{110}{17.5} \right) = 81^\circ$$

### Exercise 1

For the following system of forces shown in each figure below, find

- (a) The resultant (net) force on the x-axis  $\sum F_x = ??$ ,
- (b) The resultant (net) force on the y-axis  $\sum F_y = ??$ , and
- (c) The magnitude and the direction of the resultant (net) force  $\sum \vec{F} = ??$



**The answer:**

Figure 1  $\rightarrow \sum F_x = 0, \sum F_y = 80 \text{ N}, \sum \vec{F} = 80 \text{ N to the } + y \text{ axis direction}$

Figure 2  $\rightarrow \sum F_x = 65 \text{ N}, \sum F_y = 82 \text{ N}, \sum \vec{F} = 104.6 \text{ N}, \theta_x = 51.6^\circ$

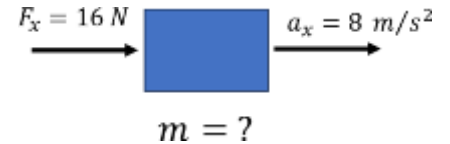
Figure 3  $\rightarrow \sum F_x = 90.6 \text{ N}, \sum F_y = 74.8 \text{ N}, \sum \vec{F} = 117.4 \text{ N}, \theta_x = 39.6^\circ$

### Example 3

A resultant force of 16 N (in the + x-axis direction) causes a mass to accelerate at a rate of  $8\text{ m.s}^{-2}$  to the + x-axis direction as shown in the Figure below. Determine the mass.

#### Solution:

First, draw a free-body diagram for that object, showing all the forces acting on that object. Include any unknown variables that you have to solve for.



$$\sum F_x = m \cdot a_x$$

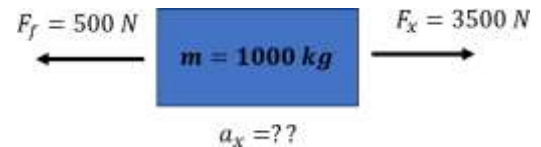
$$16 = m \times 8 \quad \text{Then} \quad m = \frac{16}{8} = 2\text{ kg}$$

### Example 4

A box of mass 1000 kg is pulled by a force of 3500 N (in the + x-axis direction) and experiences a constant frictional force of 500 N as shown in the Figure below. Calculate the acceleration of the box.

#### Solution:

First, draw a free-body diagram for that object, showing all the forces acting on that object. Include any unknown variables that you have to solve for.



$$\sum F_x = m \cdot a_x$$

From the Figure, the net force on the x-axis  $\sum F_x$  is  $\sum F_x = F_x - F_f$

$$F_x - F_f = m \times a_x$$

$$3500 - 500 = 1000 \times a_x$$

$$3000 = 1000 \times a_x$$

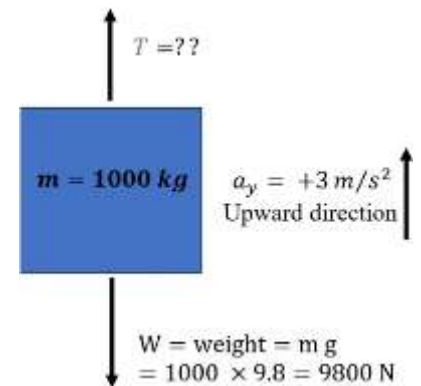
$$a_x = \frac{3000}{1000} = 3\text{ m/s}^2$$

### Example 5

An elevator has a mass of 1000 kg. It accelerates upward at  $3 \text{ m/s}^2$ . What is the force tension  $T$  exerted by the cable on the elevator? Consider  $g = 9.8 \text{ m/s}^2$

#### Solution:

First, draw a free-body diagram for that object, showing all the forces acting on that object. Include any unknown variables that you have to solve for.



For the vertical motion

$$\sum F_y = m \cdot a_y$$

The net force on the y-axis  $\sum F_y = T - W$

So

$$\sum F_y = m \cdot a_y$$

$$T - W = m \cdot a_y$$

$$T - 9800 = 1000 \times 3$$

$$T = 3000 + 9800 = 12800 \text{ N}$$

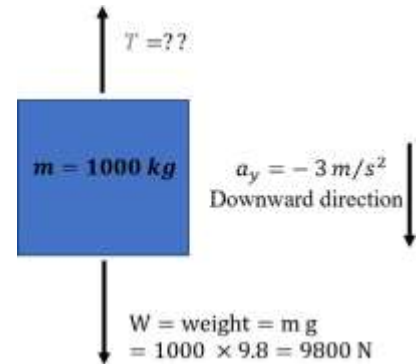
(Note that  $T$  is greater than the weight ( $mg$ ) since the elevator is accelerating upward)

### Example 6

An elevator has a mass of 1000 kg. It accelerates downward at  $3 \text{ m/s}^2$ . What is the force tension  $T$  exerted by the cable on the elevator? Consider  $g = 9.8 \text{ m/s}^2$

#### Solution:

First, draw a free-body diagram for that object, showing all the forces acting on that object. Include any unknown variables that you have to solve for.



For the vertical motion

$$\sum F_y = m \cdot a_y$$

The net force on the y-axis  $\sum F_y = T - W$

So

$$\sum F_y = m \cdot a_y$$

$$T - W = m \cdot a_y$$

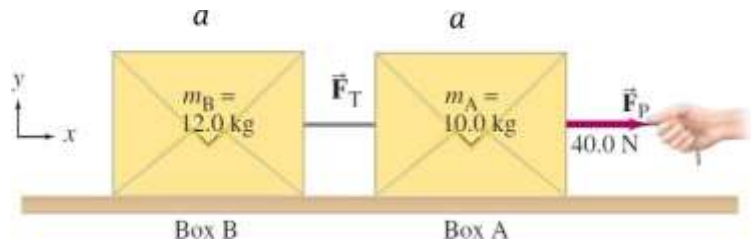
$$T - 9800 = 1000 \times -3$$

$$T = -3000 + 9800 = 6800 \text{ N}$$

(Note that  $T$  is less than the weight ( $mg$ ) since the elevator is accelerating downward)

### Example 7. Two boxes connected by a cord.

Two boxes, A and B, are connected by a lightweight cord and are resting on a smooth table. The boxes have masses of 12 kg and 10 kg. A horizontal force  $\vec{F}_p$  of 40 N is applied to the 10 kg box, as shown in the Figure below. Find (a) the acceleration of each box  $a$  (they have the same acceleration), and (b) the tension force  $F_T$  in the cord connecting the boxes.



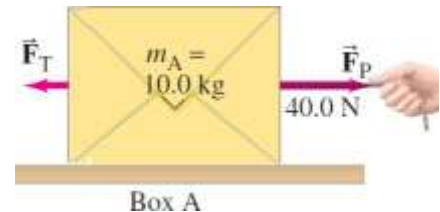
#### Solution:

We have two boxes so we draw a free-body diagram for each. To draw them correctly, we must consider the forces on each box by itself, so that Newton's second law can be applied to each. The person exerts a force  $F_p$  on box A. Box A exerts a force  $F_T$  on the connecting cord, and the cord exerts an opposite but equal magnitude force  $F_T$  back on box A (Newton's third law). The two horizontal forces on box A are shown below.

$$\sum F_x = m \cdot a_x$$

$$F_p - F_T = m_A a$$

$$40 - F_T = 10 a \text{ --- (1)}$$

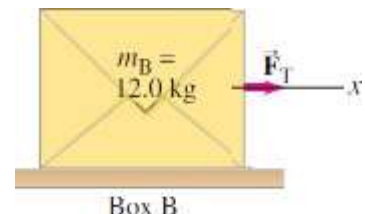


For box B, the only horizontal force is  $F_T$ . so

$$\sum F_x = m \cdot a_x$$

$$F_T = m_B a$$

$$F_T = 12 a \text{ --- (2)}$$



Substituting equation 2 in 1, then

$$40 - 12 a = 10 a$$

$$40 = 10 a + 12 a$$

$$40 = 22 a$$

$$a = \frac{40}{22} = 1.82 \text{ m/s}^2$$

From equation 2, the tension in the cord is  $F_T$

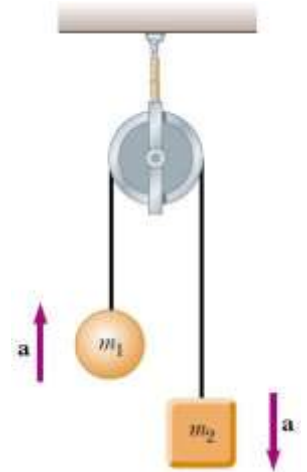
$$F_T = 12 a$$

$$F_T = 12 \times 1.82 = 21.8 \text{ N}$$

### Example 8. The Atwood Machine

When two objects of unequal mass are hung vertically over a frictionless pulley of negligible mass, as in the Figure below, the arrangement is called an Atwood machine. The device is sometimes used in the laboratory to measure the free-fall acceleration. Determine the magnitude of the acceleration of the two objects  $a$  and the tension  $T$  in the lightweight cord. Consider

$$m_1 = 1000 \text{ kg}, m_2 = 1150 \text{ kg}, \text{ and } g = 9.8 \text{ m/s}^2$$



#### Solution:

Again we have two objects, so we draw a free-body diagram for each, and we will need to apply Newton's second law to each of them separately.

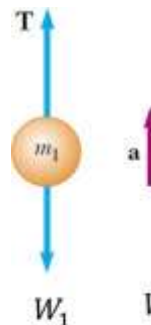
For  $m_1$  :

$$\sum F_y = m \cdot a_y$$

$$T - W_1 = m_1 a$$

$$T - 9800 = 1000 a$$

$$T = 9800 + 1000 a \quad \dots \dots \dots (1)$$



$$W_1 = m_1 g$$

$$W_1 = 1000 \times 9.8 = 9800 \text{ N}$$

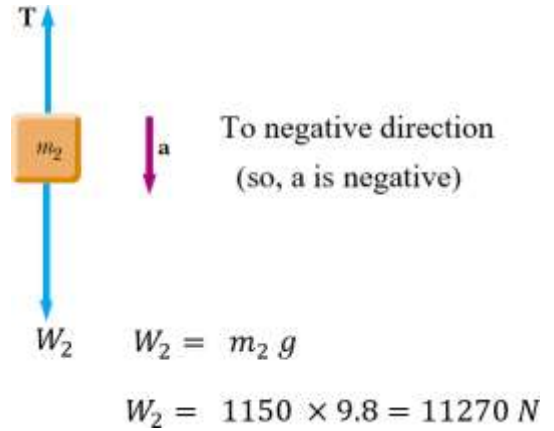
For  $m_2$ :

$$\sum F_y = m \cdot a_y$$

$$T - W_2 = m_2 a$$

$$T - 11270 = -1150 a$$

$$T = 11270 - 1150 a \dots \dots \dots (2)$$



Substituting equation 2 in 1, then

$$11270 - 1150 a = 9800 + 1000 a$$

$$11270 - 9800 = 1150 a + 1000 a$$

$$1470 = 2150 a$$

$$a = \frac{1470}{2150} = 0.68 \text{ m/s}^2$$

The tension T in the cable can be obtained from either of the two equations (1 or 2).

Using equation 1

$$T = 9800 + 1000 a$$

$$T = 9800 + 1000 \times 0.68$$

$$T = 10480 \text{ N}$$

### Exercise 2

What net force is needed to give a 1000 kg car an acceleration of  $3 \text{ m/s}^2$ ?

**The answer:**  $\Sigma F_x = 3000 \text{ N}$

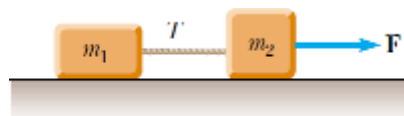
### Exercise 3

An elevator has a mass of 800 kg. It accelerates upward at  $5 \text{ m/s}^2$ . What is the force tension  $T$  exerted by the cable on the elevator? Consider  $g = 9.8 \text{ m/s}^2$

**The answer:**  $T = 11840 \text{ N}$

### Exercise 4

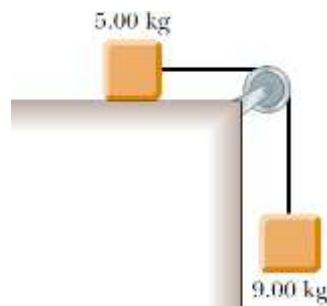
Two blocks connected by a rope of negligible mass are being dragged by a horizontal force  $F$  as shown in the Figure below. Suppose that  $F = 68 \text{ N}$ ,  $m_1 = 12 \text{ kg}$ , and  $m_2 = 18 \text{ kg}$ . (a) Draw a free-body diagram for each block. (b) Determine the tension  $T$  and the magnitude of the acceleration of the system.



**The answer:**  $a = 2.27 \text{ m/s}^2$  and  $T = 27.24 \text{ N}$

### Exercise 5

A 5 kg object placed on a frictionless, horizontal table is connected to a string that passes over a pulley and then is fastened to a hanging 9 kg object, as shown in the Figure below. Draw free-body diagrams of both objects. Find the acceleration of the two objects and the tension in the string. Consider  $g = 9.8 \text{ m/s}^2$



**The answer:**  $a = 6.3 \text{ m/s}^2$  and  $T = 31.5 \text{ N}$

## Objects in Equilibrium

Objects that are either at rest or moving with constant velocity are said to be in equilibrium. Because  $\vec{a} = 0$ , Newton's second law applied to an object in equilibrium gives

$$\sum \vec{F} = 0$$

This statement signifies that the vector sum of all the forces (the net force) acting on an object in equilibrium is zero. Last equation is equivalent to the set of component equations given by

$$\sum F_x = 0 \quad \text{and} \quad \sum F_y = 0$$

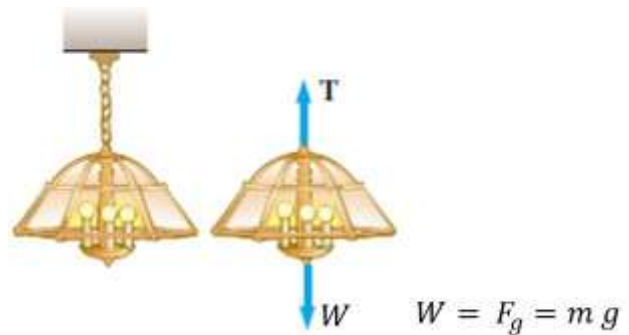
Consider a lamp suspended from a light chain fastened to the ceiling, as in the Figure below. The free-body diagram for the lamp shows that the forces acting on the lamp are the downward gravitational force (weight)  $W$  and the upward force  $T$  exerted by the chain. If we apply the second law to the lamp, noting that  $\vec{a} = 0$ , we see that because there are no forces in the  $x$  direction,  $\sum F_x = 0$  provides no helpful information. The condition  $\sum F_y = 0$  Since  $a_y = 0$

Gives

$$\sum F_y = m \cdot a_y$$

$$T - W = 0$$

$$T = W$$



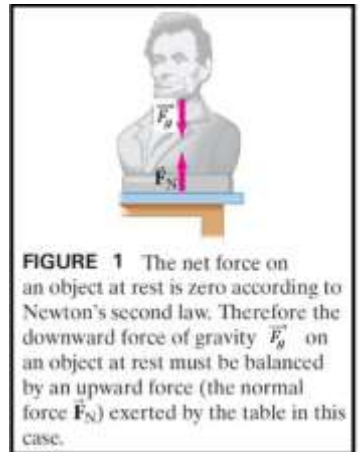
## The Normal Force\*\*

The force of gravity acts on an object when it is falling. When an object is at rest on the Earth, the gravitational force on it does not disappear, as we know if we weigh it on a spring scale. The same force, given by the previous equation, continues to act. Why, then, doesn't the object move? From Newton's second law, the net force on an object that remains at rest is zero.

There must be another force on the object to balance the gravitational force. For an object resting on a table, the table exerts this upward force; see Figure 1. The table is compressed slightly beneath the object, and due to its elasticity, it pushes up on the object as shown. The force exerted by the table is often called a contact force, since it occurs when two objects are in contact. (The force of your hand pushing on a cart is also a contact force.) When a contact force acts perpendicular to the common surface of contact, it is referred to as the **normal force** ("normal" means perpendicular); hence it is labeled  $\vec{F}_N$  in Figure 1. The two forces shown in Figure 1 are both acting on the statue, which remains at rest, so the vector sum of these two forces must be zero (Newton's second law).

Hence  $\vec{F}_g$  and  $\vec{F}_N$  must be of equal magnitude and in opposite directions.

The **normal force** is the force exerted by a surface on an object in contact with it, perpendicular to the surface.



## Example 9\*\*

A box of mass 10 kg is resting on a smooth horizontal table. Determine the weight of the box and the normal force exerted on it by the table. Consider  $g = 9.8 \text{ m/s}^2$

### Solution:

The weight of the box is

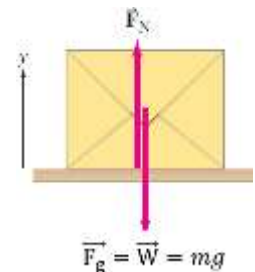
$$\vec{F}_g = \vec{W} = mg = 10 \times 9.8 = 98 \text{ N}$$

To find the normal force

$$\sum F_y = m \cdot a_y \text{ where } a_y = 0 \text{ since the box is at rest.}$$

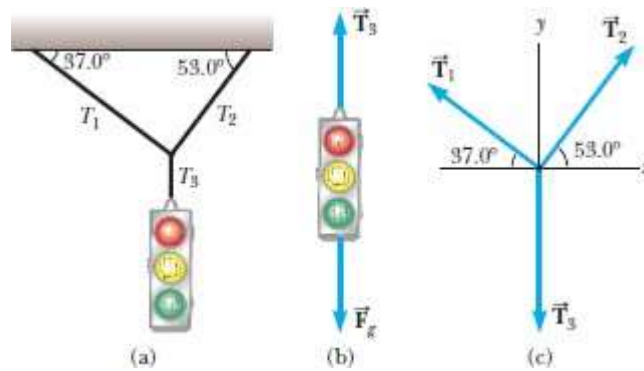
$$F_N - W = 0$$

$$F_N - 98 = 0 \quad \text{Then} \quad F_N = 98 \text{ N in upward direction}$$



### Example 10

A traffic light weighing  $1 \times 10^2 \text{ N}$  hangs from a vertical cable tied to two other cables that are fastened to a support, as in the Figure shown below. The upper cables make angles of  $37^\circ$  and  $53^\circ$  with the horizontal. Find the tension in each of the three cables.



### Solution

we construct two free-body diagrams, one for the traffic light, shown in Figure ( b ), and one for the knot that holds the three cables together, as in Figure ( c ). This knot is a convenient object to choose because all the forces of interest act along lines passing through the knot

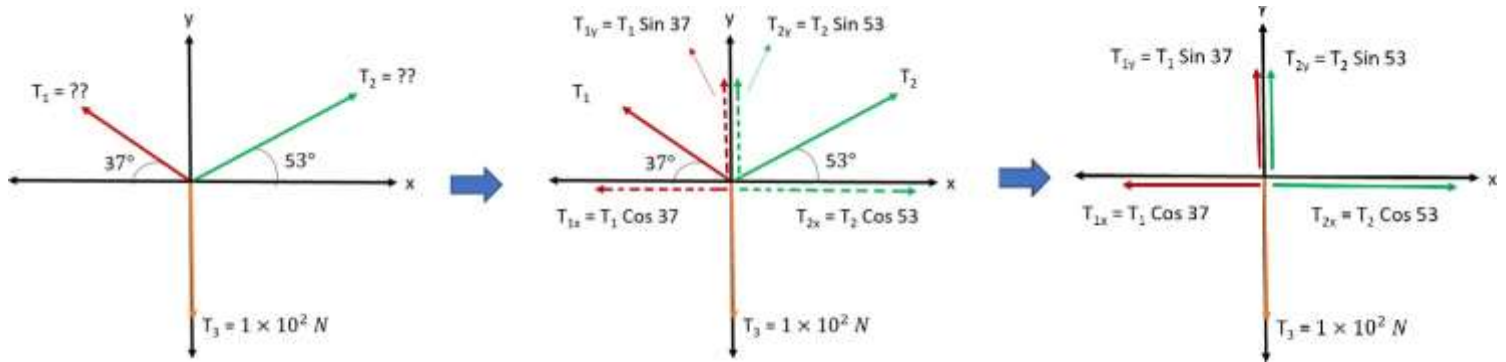
Find  $T_3$  from Figure ( b ), using the condition of equilibrium

$$\sum F_y = 0 \rightarrow T_3 - F_g = 0$$

$$\text{So, } T_3 = F_g = 1 \times 10^2 \text{ N}$$



Next, we choose the coordinate axes shown in Figure (c) and resolve the forces acting on the knot into their components:



Apply the conditions for equilibrium to the knot, using the components in the table:

$$\sum F_x = 0 \quad \text{and} \quad \sum F_y = 0$$

The first equilibrium condition  $\sum F_x = 0$

$$\sum F_x = T_2 \cos 53^\circ - T_1 \cos 37^\circ = 0$$

$$T_2 \cos 53^\circ = T_1 \cos 37^\circ$$

$$0.6 T_2 = 0.8 T_1$$

$$T_2 = \left( \frac{0.8}{0.6} \right) T_1 = 1.33 T_1 \quad \dots \dots \dots (1)$$

The second equilibrium condition  $\sum F_y = 0$

$$\sum F_y = T_1 \sin 37^\circ + T_2 \sin 53^\circ - 1 \times 10^2 = 0$$

$$0.6 T_1 + 0.8 T_2 - 1 \times 10^2 = 0 \dots \dots \dots (2)$$

Substitute the result for  $T_2$  (equation 1) into Equation (2):

$$0.6 T_1 + 0.8 T_2 - 1 \times 10^2 = 0. \text{ So}$$

$$0.6 T_1 + 0.8 \cdot (1.33 T_1) - 1 \times 10^2 = 0$$

$$0.6 T_1 + 1.06 T_1 = 1 \times 10^2, \quad \text{so} \quad 1.66 T_1 = 1 \times 10^2, \quad \text{so} \quad T_1 = \frac{1 \times 10^2}{1.66}$$

$$T_1 = 60.2 \text{ N}$$

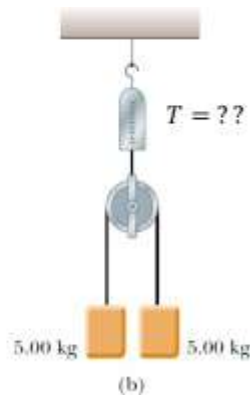
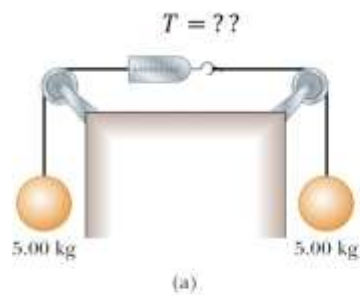
Using equation 1 to find  $T_2$

$$T_2 = 1.33 T_1 = 1.33 \times 60.2 = 80.1 \text{ N}$$

□

### Exercise 6

The systems shown in the Figure below are in equilibrium. If the spring scales are calibrated in newtons, what do they read? (Find the tension  $T$  in each Figure). Consider  $g = 9.8 \text{ m/s}^2$ .



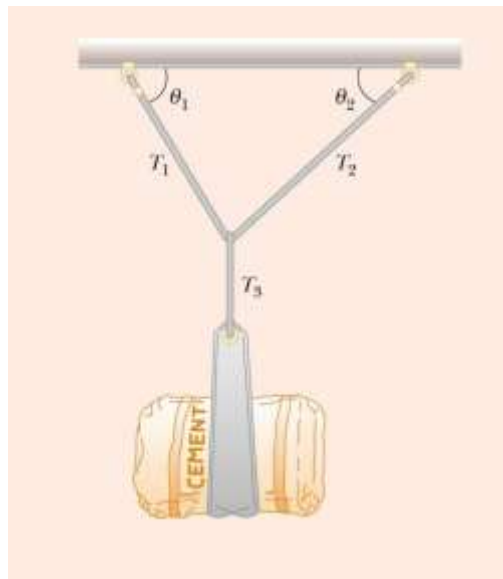
**The answer:**

Figure a → The scale reads the tension  $T$ ,  $T = 49 \text{ N}$

Figure b → The scale reads the tension  $T$ ,  $T = 98 \text{ N}$

### Exercise 7

A bag of cement of weight 325 N hangs from three wires as suggested in the Figure below. Two of the wires make angles  $\theta_1 = 60^\circ$  and  $\theta_2 = 25^\circ$  with the horizontal. If the system is in equilibrium, find the tensions  $T_1, T_2$ , and  $T_3$  in the wires.



**The answer:**  $T_3 = 325 \text{ N}$ ,  $T_1 = 295 \text{ N}$ , and  $T_2 = 162 \text{ N}$

## **Chapter Five:**

## **Biomechanics**

**Intended Learning Outcomes:**

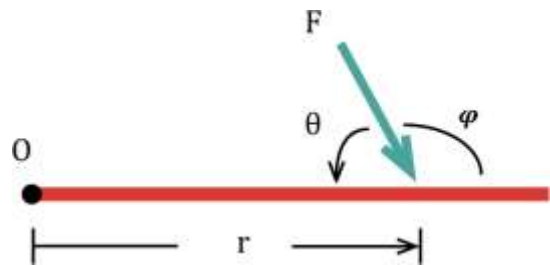
**At the end of this chapter, the students will be able to:**

- Define the moment of force
- Recognize the static equilibrium conditions
- Recognize the static equilibrium in the human body.

## Torque (moment of force)

- A torque is the tendency of a force to rotate an object about an axis or pivot.  
Mathematically, torque is defined as the product of the lever arm distance and the perpendicular force to the lever arm, which tends to produce rotation.

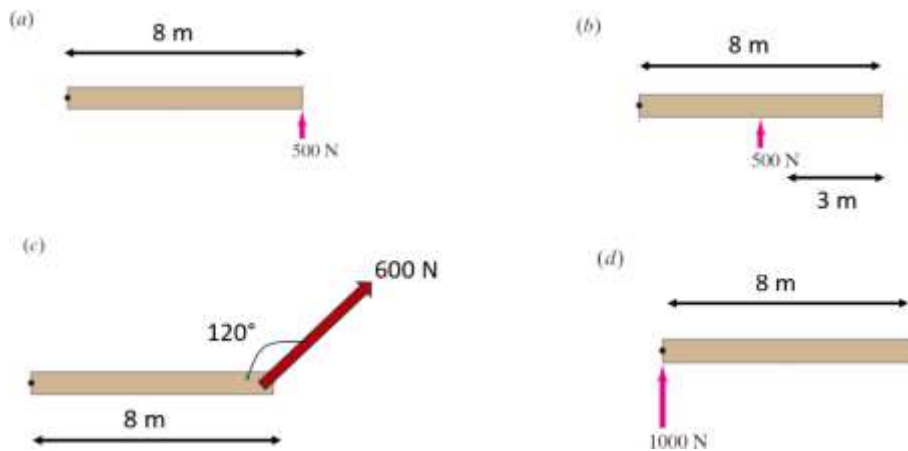
$$\tau = r \cdot F \cdot \sin \theta$$



- $\theta$  is the angle between  $F$  and  $r$ .
- Remember that, mathematically,  $\sin \theta = \sin (180 - \theta) = \sin \phi$
- The torque ( $\tau$ ) is a vector quantity
- The SI unit of torque is N. m
- The direction of rotation is either in a clockwise direction (c.w) or in the counterclockwise direction (c.c.w)

### Example 1

Calculate the torque for each force in the Figure below



### Solution:

$$a) \tau = r \cdot F \cdot \sin \theta = 8 \times 500 \times \sin 90 = 4000 \text{ N.m in c.c.w}$$

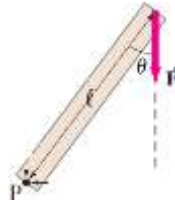
$$b) \tau = r \cdot F \cdot \sin \theta = 5 \times 500 \times \sin 90 = 2500 \text{ N.m in c.c.w}$$

$$c) \tau = r \cdot F \cdot \sin \theta = 8 \times 600 \times \sin 120 = 4157 \text{ N.m in c.c.w}$$

$$d) \tau = r \cdot F \cdot \sin \theta = 0 \times 1000 \times \sin 90 = 0$$

### Exercise 1

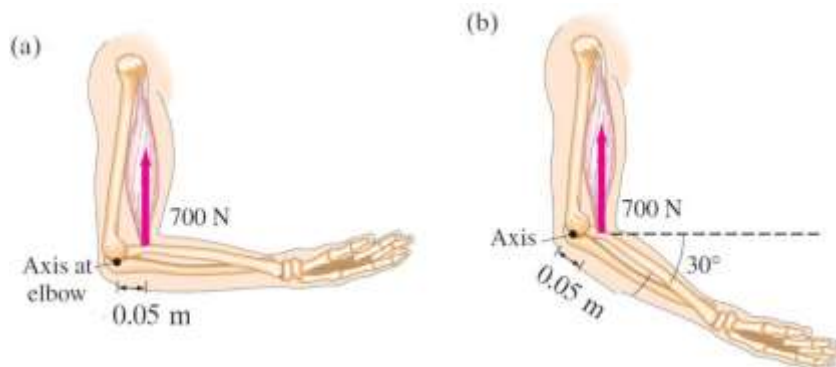
Consider a force  $F = 80\text{ N}$  applied to a beam as shown in the figure below. The length of the beam is  $l = 5\text{ m}$ , and  $\theta = 37^\circ$ . Calculate the torque due to the force  $F$  on the beam.



The answer:  $\tau = 241\text{ N.m}$  in c.w

### Exercise 2

The biceps muscle exerts a vertical force on the lower arm, bent as shown in the Figure below. For each case, calculate the torque about the axis of rotation through the elbow joint, assuming the muscle is attached  $0.05\text{ m}$  from the elbow as shown.



The answer: a)  $\tau = 35\text{ N.m}$  in c.c.w and b)  $\tau = 30.3\text{ N.m}$  in c.c.w

## The two conditions for equilibrium

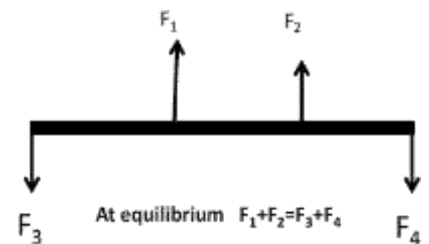
An object in mechanical equilibrium must satisfy the following two conditions:

### Translational Equilibrium

A body is said to be in translational equilibrium if the vector sum of all external forces acting on the body is zero. Or, in translational equilibrium, the sum of forces in one direction must equal the sum of forces in the opposite direction

Mathematically;

$$\sum \vec{F} = 0$$



### Example 2

If the system of forces applied to the object shown in the figure below satisfies the translational equilibrium, calculate the value of  $F_3$ .



**Solution:**

$$\sum \vec{F} = 0, \text{ or}$$

$$\sum \vec{F} \text{ in one direction} = \sum \vec{F} \text{ in opposite direction}$$

$$F_1 + F_3 = F_2$$

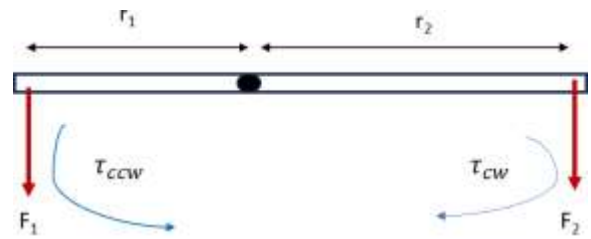
$$200 + F_3 = 400 \Rightarrow F_3 = 200\text{ N}$$

## Rotational equilibrium

A body is said to be in rotational equilibrium if the sum of torques due to all the external forces acting on the body is zero. Or, for any object in rotational equilibrium, the sum of the clockwise moments about any pivot must equal the sum of the anti-clockwise moments about that pivot.

Mathematically;

$$\sum \vec{\tau} = 0$$



At rotational equilibrium

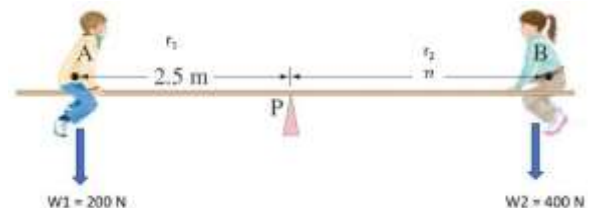
$$\tau_{cw} = \tau_{ccw}$$

$$r_2 \cdot F_2 \cdot \sin 90 = r_1 \cdot F_1 \cdot \sin 90$$

$$r_2 \cdot F_2 = r_1 \cdot F_1$$

### Example 3

Two children of weight  $w_1$  and  $w_2$  are balanced on a board (satisfy the rotational equilibrium) pivoted about its center. If  $w_1 = 200 \text{ N}$ ,  $w_2 = 400 \text{ N}$ , and  $r_1 = 1 \text{ m}$ , what is  $r_2$ ?



### Solution

Because the board satisfies the rotational equilibrium, then

$$\tau_1 = r_1 \cdot w_1 \cdot \sin \theta = 2.5 \times 200 \times \sin 90 = 500 \text{ N.m} \quad \text{in c. c. w}$$

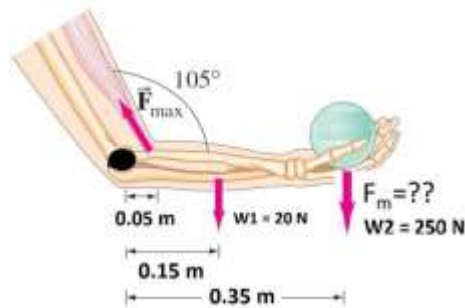
$$\tau_2 = r_2 \cdot w_2 \cdot \sin \theta = r_2 \times 400 \times \sin 90 = r_2 \cdot 400 \text{ N.m} \quad \text{in c. w}$$

$$\tau_{cw} = \tau_{ccw}$$

$$r_2 \times 400 = 500 \quad \Rightarrow \quad r_2 = \frac{500}{400} = 1.25 \text{ m}$$

### Example 4

If 250 N weight is the maximum weight that a person can hold in a hand when the arm is positioned with a  $105^\circ$  angle at the elbow as shown in the Figure below. What is the maximum force ( $F_m = ? ?$ ) that the biceps muscle exerts on the forearm? Assume the forearm and hand have a total weight of 20 N which is 15 cm from the elbow, and that the biceps muscle attaches 0.05 m from the elbow.

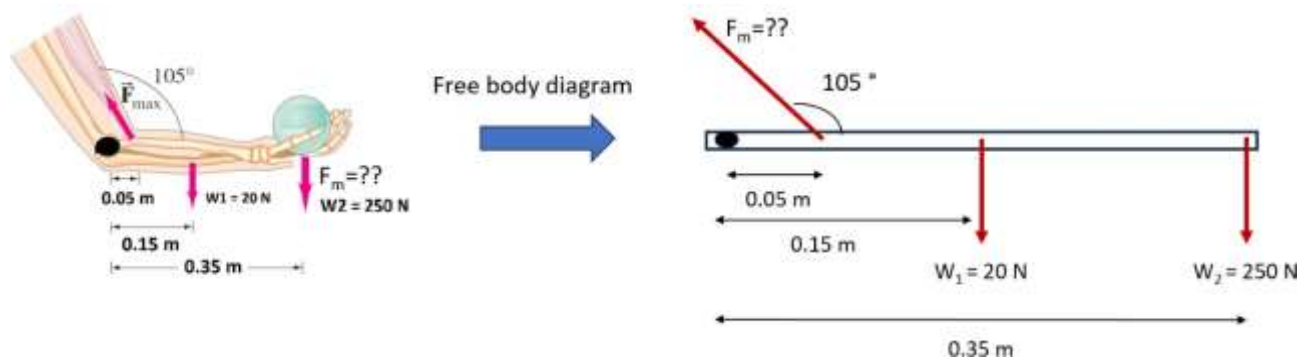


### Solution:

The arm is not rotating at this position, so the arm satisfies the rotational equilibrium. So

$$\sum \vec{\tau} = 0 \quad \text{or} \quad \sum \tau_{cw} = \sum \tau_{ccw}$$

We must determine the torque due to each force and then equalize all the torques in the c.w direction to all the torques in the c.c.w direction. Starting with the free-body diagram:

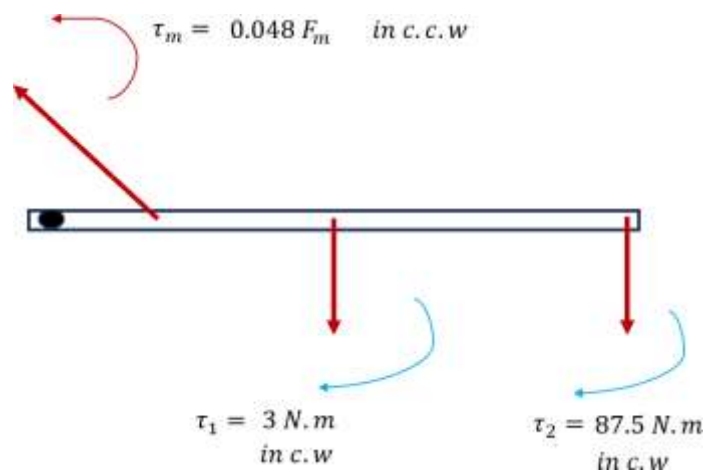


The torque due to each force:

$$\tau_m = r_m \cdot F_m \cdot \sin \theta = 0.05 \times F_m \times \sin 105 = 0.048 F_m \quad \text{in c.c.w}$$

$$\tau_1 = r_1 \cdot F_1 \cdot \sin \theta = 0.15 \times 20 \times \sin 90 = 3 \text{ N.m} \quad \text{in c.w}$$

$$\tau_2 = r_2 \cdot F_2 \cdot \sin \theta = 0.35 \times 250 \times \sin 90 = 87.5 \text{ N.m} \quad \text{in c.w}$$



Now, apply the rotational equilibrium

$$\sum \tau_{cw} = \sum \tau_{ccw}$$

$$\tau_1 + \tau_2 = \tau_m$$

$$3 + 87.5 = 0.048 F_m$$

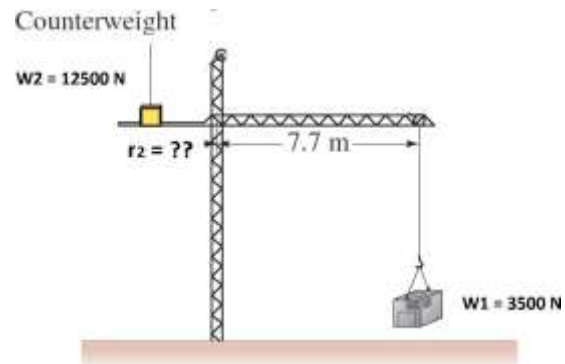
$$90.5 = 0.048 F_m$$

$$F_m = \frac{90.5}{0.048} = 1885 \text{ N}$$

□

### Exercise 3

A tower crane (Figure below) must always be carefully balanced so that there is no net torque tending to tip it. A particular crane at a building site is about to lift a 3500 N weight. Where must the 12500 N crane's counterweight be placed when the load is lifted from the ground?



The answer:  $r_2 = 2.2 \text{ m}$

### Exercise 4

If the system of forces applied to the object shown in the figure below satisfies the translational equilibrium, calculate the value of  $F_4$ .



The answer:  $F_4 = 450 \text{ N}$

## **Static equilibrium in the human body**

### **Levers**

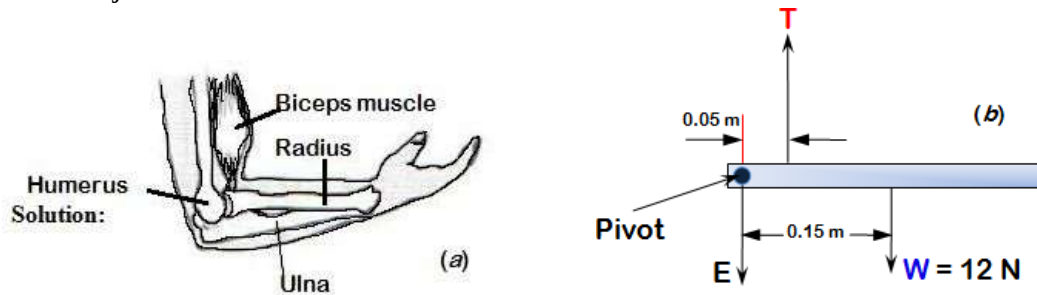
- Humans moves through a system of levers
- Levers rotate about an axis as a result of force being applied to cause its movement against a resistance or weight.

### **In the human body**

- bones represent the bars
- joints are the axes
- muscles contract to apply force

### Example 5

A model for a forearm satisfies the equilibrium conditions as shown in the figure below. If the weight  $w$  of the forearm is 12 N, find the tension  $T$  exerted by the biceps muscle and the force  $E$  exerted by the elbow joint.



### Solution:

The forearm satisfies the rotational equilibrium and the translational equilibrium.

**For the rotational equilibrium condition.** First, calculate the torque due to each force

$$\tau_T = r_T \cdot T \cdot \sin \theta = 0.05 \times T \times \sin 90 = 0.05 T \quad \text{in c.c.w}$$

$$\tau_W = r_W \cdot W \cdot \sin \theta = 0.15 \times 12 \times \sin 90 = 1.8 \text{ N.m} \quad \text{in c.w}$$

$$\tau_E = r_E \cdot E \cdot \sin \theta = 0 \times E \times \sin 90 = 0$$

**Now, apply the rotational equilibrium**

$$\sum \tau_{cw} = \sum \tau_{ccw}$$

$$1.8 = 0.05 \times T$$

$$T = \frac{1.8}{0.05} = 36 \text{ N}$$

**For the translational equilibrium condition**

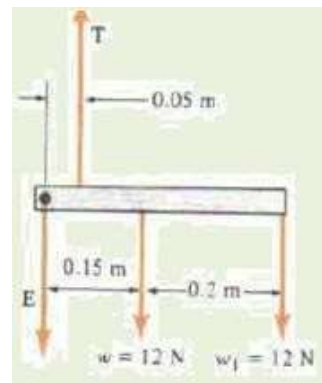
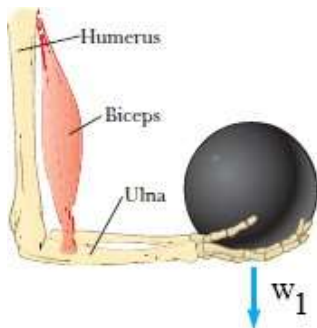
$$\sum \vec{F} \text{ in one direction} = \sum \vec{F} \text{ in opposite direction}$$

$$T = E + w$$

$$36 = E + 12 \quad \Rightarrow \quad E = 24 \text{ N}$$

### Exercise 5

A model for a forearm satisfies the equilibrium conditions as shown in the figure below. If the weight  $w$  of the forearm is 12 N and the weight of the ball  $w_1$  is 12 N. Find the tension  $T$  exerted by the biceps muscle and the force  $E$  exerted by the elbow joint.

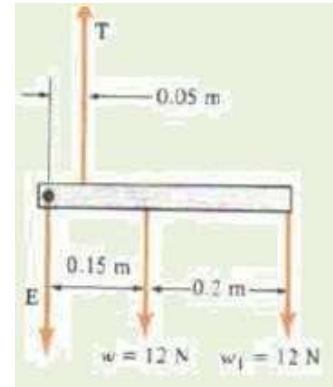
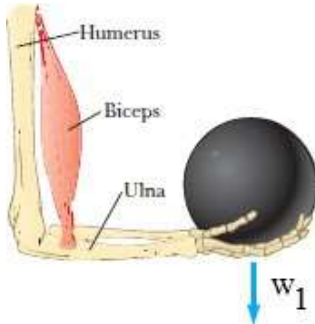


**See the solution of the question at the end of the chapter**

**The answer:**  $T = 120 \text{ N}$  and  $E = 96 \text{ N}$

### Exercise 5/ Solution

A model for a forearm satisfies the equilibrium conditions as shown in the figure below. If the weight  $w$  of the forearm is 12 N and the weight of the ball  $w_1$  is 12 N. Find the tension  $T$  exerted by the biceps muscle and the force  $E$  exerted by the elbow joint.



### Solution:

The forearm satisfies the rotational equilibrium and the translational equilibrium.

**For the rotational equilibrium condition.** First, calculate the torque due to each force

$$\tau_T = r_T \cdot T \cdot \sin \theta = 0.05 \times T \times \sin 90 = 0.05 T \quad \text{in c. c. w}$$

$$\tau_w = r_w \cdot w \cdot \sin \theta = 0.15 \times 12 \times \sin 90 = 1.8 \text{ N.m} \quad \text{in c. w}$$

$$\tau_{w1} = r_{w1} \cdot w_1 \cdot \sin \theta = 0.35 \times 12 \times \sin 90 = 4.2 \text{ N.m} \quad \text{in c. w}$$

$$\tau_E = r_E \cdot E \cdot \sin \theta = 0 \times E \times \sin 90 = 0$$

$$\sum \tau_{cw} = \sum \tau_{ccw}$$

$$1.8 + 4.2 = 0.05 \times T$$

$$6 = 0.05 \times T \Rightarrow T = \frac{6}{0.05} = 120 \text{ N}$$

**For the translational equilibrium condition**

$$\sum \vec{F} \text{ in one direction} = \sum \vec{F} \text{ in opposite direction}$$

$$T = E + w + w_1$$

$$120 = E + 12 + 12$$

$$E = 96 \text{ N}$$

# **Chapter Six:**

## **Fluid Dynamics**

## **Intended Learning Outcomes:**

### **At the end of this chapter, the students will be able to:**

- Solve mathematical problems related to the continuity equation, Bernoulli's equation, and Poiseuille's law for the dynamic fluid.

## The Equation of Continuity

- If a fluid enters one end of a pipe at a certain rate, it must also leave the pipe with the same rate.
- The volume flow rate (Q): it is the volume of fluid per second that flows through a tube.

$$\text{Volume flow rate (Q)} = \frac{\text{Volume}}{\text{Time}}$$

- For Incompressible fluid (where the density is constant) the volume flow rate is conserved. (Q = constant at any point)
- The volume flow rate Q has another formula

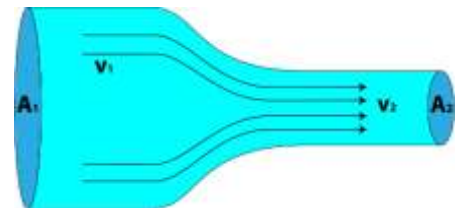
$$Q = \text{Area} \times \text{velocity} = \text{constant}$$

- Therefore,

$$Q_1 = Q_2$$

So,

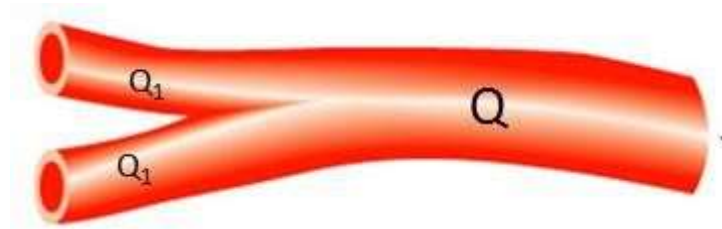
$$A_1 \times v_1 = A_2 \times v_2$$



- The SI unit of the volume flow rate (Q) is  $m^3/s$

## Parallel arteries

We can apply the continuity equation to the parallel arteries



The volume flow rate  $Q$

$$Q = Q_1 + Q_2 + \cdots Q_n$$

Or

$$Q = N \times Q_1$$

Or

$$A \times v = N \times A_1 \times v_1$$

Where  $N$  is the number of branches

### Example 2

A pipe with  $0.05 \text{ m}^2$  area cross section contains water with velocity of  $3 \text{ m/s}$ . what is the volume flow rate in the water pipe? **Solution:**

$$Q = A \times v$$

$$Q = 0.05 \times 3 = 0.15 \text{ m}^3/\text{s}$$

---

### Example 3

The area cross section of a water pipe decreases from  $0.2 \text{ m}^2$  to  $0.01 \text{ m}^2$ . If the velocity in the wider portion is  $4 \text{ m/s}$ . Find the velocity in the narrower region.

**Solution:**

$$Q_1 = Q_2$$

$$0.2 \times 4 = 0.01 \times v_2$$

$$v_2 = \frac{0.2 \times 4}{0.01} = 80 \text{ m/s}$$

---

### Example 4

A blood vessel of  $2 \times 10^{-4} \text{ m}^2$  area cross section splits into three vessels, each with  $1 \times 10^{-4} \text{ m}^2$  area cross section. If the velocity in the larger vessel is  $0.5 \text{ m/s}$ , find the velocity in each of the smaller vessels. **Solution:**

$$Q = Q_1 + Q_2 + Q_3$$

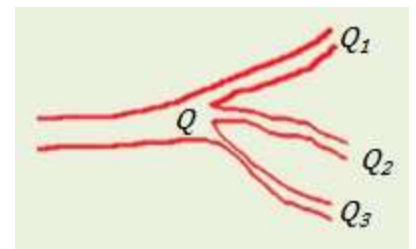
$$A \times v = A_1 \times v_1 + A_2 \times v_2 + A_3 \times v_3$$

Or,

$$A \times v = N \times A_1 \times v_1$$

$$2 \times 10^{-4} \times 0.5 = 3 \times 1 \times 10^{-4} \times v_1$$

$$v_1 = \frac{2 \times 10^{-4} \times 0.5}{3 \times 1 \times 10^{-4}} = 0.33 \frac{\text{m}}{\text{s}} = v_2 = v_3$$



**Example 5**

The aorta has an inside approximately  $0.005 \text{ m}^2$  area cross section compared to that of a capillary, which has about  $1.0 \times 10^{-4} \text{ m}^2$ . In addition, the average speed of flow is approximately  $1.0 \text{ m/s}$  in the aorta and  $0.01 \text{ m/s}$  in a capillary. Assuming that all the blood that flows through the aorta also flows through the capillaries, how many capillaries does the circulatory system have?

**Solution:**

From the continuity equation

$$A \times v = N \times A_1 \times v_1$$

$$0.005 \times 1 = N \times 1 \times 10^{-4} \times 0.01$$

$$N = \frac{0.005}{1 \times 10^{-4} \times 0.01}$$

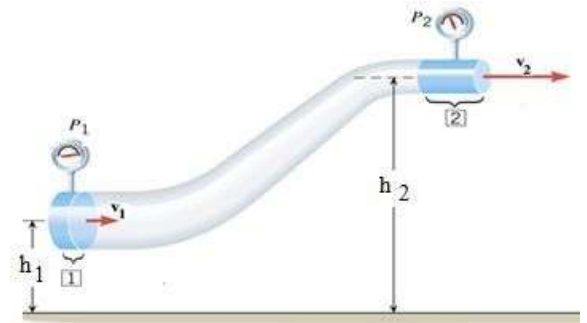
$$N = 5 \times 10^3$$

## Bernoulli's Equation

As a fluid moves through a pipe of varying cross section and elevation, the pressure changes along the pipe. In 1738 the Swiss physicist Daniel Bernoulli (1700–1782) derived an expression that relates the pressure of a fluid to its speed and elevation.

- **Ideal fluid:** is the non-viscose incompressible fluid.
- **Vescosity:** it is the amount of internal friction in the real fluids ( $\eta \rightarrow \text{Pa} \cdot \text{s}$  or  $\text{N} \cdot \text{s} / \text{m}^2$ ).
- Bernoulli's equation is a result of applying the conservation of energy to the ideal fluid.
- The conservation law of energy (Bernoulli's equation): the sum of the ideal fluid pressure and the mechanical energy density (the kinetic energy per unit volume plus the gravitational potential energy per unit volume) remain constant at any point through the pipe. Mathematically,

$$p + \rho gh + \frac{1}{2} \rho v^2 = \text{Constant}$$



Where,

p: is the fluid pressure (pa)

$\rho$ : is the fluid density ( $\text{kg}/\text{m}^3$ )

V: is the velocity of the fluid (m/s)

h: is the height of the fluid from the reference level (m)

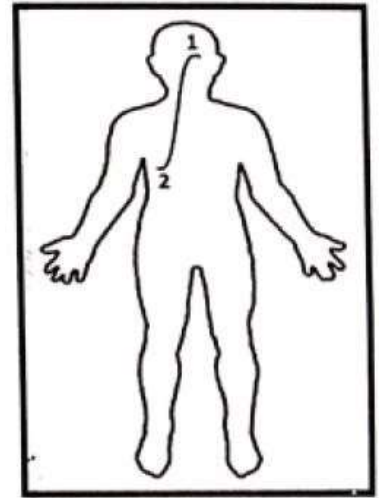
So,

$$p_1 + \rho gh_1 + \frac{1}{2} \rho v_1^2 = p_2 + \rho gh_2 + \frac{1}{2} \rho v_2^2$$

- **Bernoulli's principle:** where the fluid velocity is high, the pressure is low, and where the velocity is low, the pressure is high.

### Example 6

Blood flows through an S-shaped artery. At one end (position 1 in the figure) the blood in the artery has a pressure of  $3 \times 10^5$  Pa a speed of 9.5 m/s and a height of 1.7 m above the earth's surface. At the other end (position 2 in the figure) if the speed of the blood is 10 m/s and the height above the earth's surface is 0.6 m what is the pressure at the second point (at point 2 in the figure)? Consider  $g$  is  $10 \text{ m/s}^2$  and the density of the blood is  $1025 \text{ kg/m}^3$ . (Assume that the blood acts as an ideal fluid)



**Solution:**

$$p + \rho gh + \frac{1}{2} \rho v^2 = \text{Constant}$$

So,

$$p_1 + \rho gh_1 + \frac{1}{2} \rho v_1^2 = p_2 + \rho gh_2 + \frac{1}{2} \rho v_2^2$$

$$\begin{aligned} (3 \times 10^5) + (1025 \times 10 \times 1.7) + \left(\frac{1}{2} \times 1025 \times (9.5)^2\right) \\ = (p_2) + (1025 \times 10 \times 0.6) + \left(\frac{1}{2} \times 1025 \times (10)^2\right) \end{aligned}$$

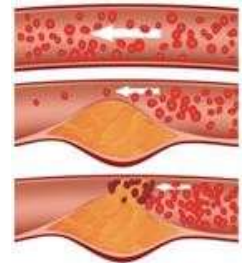
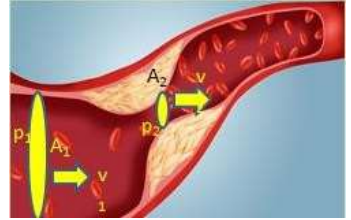
$$363678.1 = p_2 + 57400$$

$$p_2 = 306278.1 \text{ pa} \approx 3.1 \times 10^5 \text{ pa}$$

□

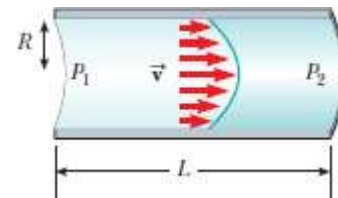
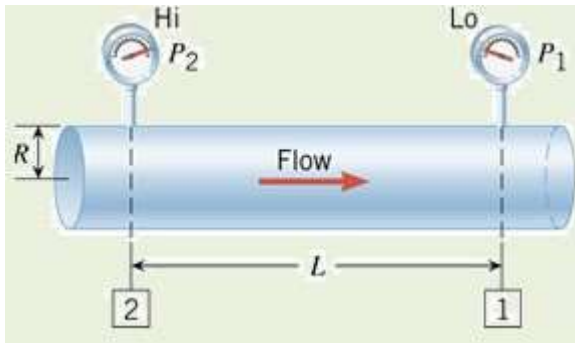
## Arteriosclerosis (Clogged Arteries)

- Atherosclerosis occurs when a deposit forms on an arterial wall.
- The smaller cross-sectional area at the clogging site, leads to a faster flow speed due to the continuity of flow ( $A_1 v_1 = A_2 v_2$ ) and a drop in pressure according to Bernoulli's equation.
- The pressure drop increases with blood speed and so it is expected to drop more with physical activity.



## Poiseuille's Law

We have very important principle in fluid mechanics called Poiseuille's principle. Poiseuille's principle state that the flow ( $Q \rightarrow \text{m}^3/\text{s}$ ) of fluid (blood) is related to a number of factors: the viscosity ( $\eta \rightarrow \text{N.s/m}^2$ ) of the fluid (blood), the pressure gradient across the tubing ( $P \rightarrow \text{pa}$ ) (or cross the blood vessel), and the length ( $L \rightarrow \text{m}$ ) and raduis ( $R \rightarrow \text{m}$ ) of the tubing (or for the blood vessel). Mathematically, poisuille's law is given by:



$$Q = \frac{\pi R^4 (P_2 - P_1)}{8 \eta L} = \frac{\pi R^4 \Delta P}{8 \eta L}$$

$$Q^{**} = \frac{\Delta P}{R_f}, \text{ where } R_f(\text{flow resistance}) = \frac{8 \eta L}{\pi R^4}$$

**Example 7**

A straight horizontal pipe with a radius (R) of  $1 \times 10^{-2}$  m and a length (L) of 50 m carries oil with a coefficient of viscosity ( $\eta$ ) of  $0.12 \text{ N.s/m}^2$ . At the output of the pipe, the volume flow rate (Q) is  $8.6 \times 10^{-5} \text{ m}^3/\text{s}$  and the pressure ( $p_1$ ) is  $1 \times 10^5$  pa. Find the pressure ( $p_2$ ) at the pipe input.

**Solution:**

$$Q = \frac{\pi R^4 (P_2 - P_1)}{8 \eta L}$$

$$8.6 \times 10^{-5} = \frac{\pi (1 \times 10^{-2})^4 \times (P_2 - 1 \times 10^5)}{8 \times 0.12 \times 50}$$

$$131.46 \times 10^{-5} \times 10^8 = (P_2 - 1 \times 10^5)$$

$$P_2 \cong 2.3 \times 10^5 \text{ pa}$$

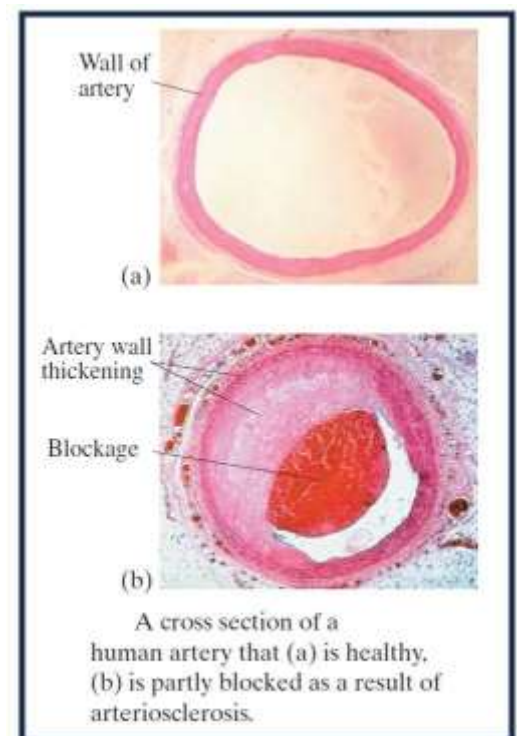
## Applications\*\*

Some diseases like Atherosclerosis, transient ischemic attack (TIA), Aneurysm, and brain strokes may be explained by the fluid mechanics laws.

An interesting example of this  $r^4$  dependence is blood flow in the human body. Because the radius of arteries is reduced as a result of arteriosclerosis (thickening and hardening of artery walls) and by cholesterol buildup, the pressure gradient must be increased to maintain the same blood flow rate. If the radius is reduced by half, the heart would have to increase the pressure by a factor of about  $2^4 = 16$  in order to maintain the same blood flow rate. The heart must work much harder under these conditions, but usually cannot maintain the original flow rate. Thus, high blood pressure is an indication both that the heart is working harder and that the blood flow rate is reduced.

**Atherosclerosis\*\*** ("hardening of the arteries") is a condition where fatty deposits occur on the inner lining of the arteries, which dramatically increases the risk for both TIA's and stroke.

Atherosclerotic plaque is formed when damage occurs to the lining of an artery. Platelets clump around the area of injury as a normal part of the clotting and healing process. Cholesterol and other fats also collect at this site, forming a mass within the lining of the artery. A clot (thrombus) may form at the site of the plaque, triggered by irregular blood flow in this location, and the thrombus may then block blood vessels in the brain. Pieces of plaque or clots can also break off and travel through the bloodstream from distant locations, forming an embolus that can block the small arteries, causing TIA's.



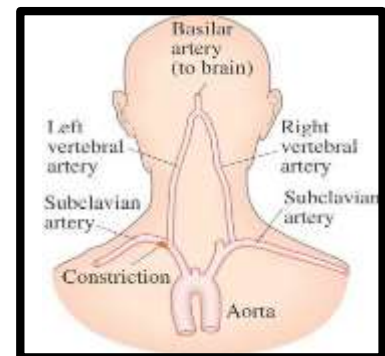
**An aneurysm\*\*** is a weak area in the wall of a blood vessel that causes the blood vessel to bulge or balloon out. When an aneurysm occurs in a blood vessel of the brain, it is called a cerebral aneurysm. [Aneurysms](#) in the brain occur when there is a weakened area in the wall of a blood vessel. An aneurysm may be present from birth (congenital). Or, it may develop later in life, such as after a blood vessel is injured.

**Transient ischemic attack (TIA) \*\*** is a “mini-stroke” caused by temporary interruption of blood supply to brain cells, resulting in a sudden, brief decrease in brain function. If the symptoms resolve completely in less than 24 hours, this is called a TIA or “mini stroke.” If the symptoms do not resolve, the event is called a stroke.

**Symptoms vary with the area of the brain affected and may include\*\*:**

- Changes in vision or trouble seeing in one or both eyes
- Confusion or difficulty in talking or understanding speech
- Numbness or weakness in the face, arm, or leg – especially on one side of the body
- Difficulty with walking, dizziness or loss of balance and coordination Vertigo
- Decreased movement or sensation in a part of the body
- Changes in the level of consciousness.

The constriction in the Subclavian artery causes the blood in the region to speed up and thus produces low pressure (according to Bernoulli's principle). The blood moving UP the LVA is then pushed DOWN instead of down causing a lack of blood flow to the brain. This condition is called **TIA (transient ischemic attack)** or Subclavian Steal Syndrome.



### Exercise 1

A hose delivers  $2 \text{ m}^3/\text{s}$  of water. The area cross-section of the hose is  $0.02 \text{ m}^2$ . What is the velocity of the water in the hose?

**The answer:**  $v = 100 \text{ m/s}$

### Exercise 2

The area cross-section of a water pipe decreases from  $0.4 \text{ m}^2$  to  $0.1 \text{ m}^2$ . If the velocity in the narrower portion is  $6 \text{ m/s}$ . Find the velocity in the wider region.

**The answer:**  $v_1 = 1.5 \text{ m/s}$

### Exercise 3

A blood vessel splits into four vessels, each with  $1 \times 10^{-4} \text{ m}^2$  area cross-section. If the velocity of the blood in the large vessel is  $1.5 \text{ m/s}$  and in each small vessel is  $1 \text{ m/s}$ , find the area cross-section of the large vessel.

**The answer:**  $A = 2.7 \times 10^{-4} \text{ m}^2$

#### Exercise 4

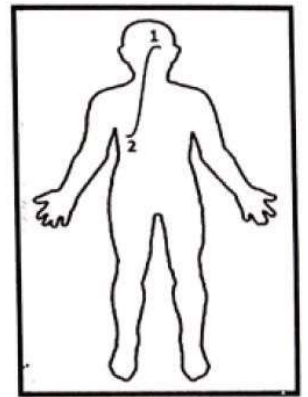
A blood vessel splits into three vessels. If the volume flow rate in each small vessel is  $2 \times 10^{-4} \text{ m}^3/\text{s}$ , what is the volume flow rate in the large vessel?

**The answer:**  $Q = 6 \times 10^{-4} \text{ m}^3/\text{s}$

#### Exercise 5

Blood flows through an S-shaped artery. At one end (position 1 in the figure) the blood in the artery has a pressure of  $4 \times 10^5 \text{ Pa}$  a speed of  $10 \text{ m/s}$  and a height of  $1.8 \text{ m}$  above the earth's surface. At the other end (position 2 in the figure) if the speed of the blood is  $10.2 \text{ m/s}$  and the pressure is  $4.1 \times 10^5 \text{ Pa}$ . What is the height of the second point? Consider  $g$  is  $10 \text{ m/s}^2$  and the density of the blood is  $1025 \text{ kg/m}^3$ . (Assume that the blood acts as an ideal fluid)

**The answer:**  $h_2 \approx 0.62 \text{ m}$



#### Exercise 6

What must be the pressure difference ( $\Delta P = ?$ ) between the two ends of a  $200 \text{ m}$  section of pipe,  $0.1 \text{ m}$  in radius, if it is to transport oil at a rate of  $0.05 \text{ m}^3/\text{s}$ ? Consider the oil viscosity ( $\eta$ ) is  $0.2 \text{ Pa}\cdot\text{s}$ .

**The answer:**  $\Delta P = 5.1 \times 10^4 \text{ Pa}$

## **Chapter Seven:**

### **Light and optics**

**Intended Learning Outcomes:**

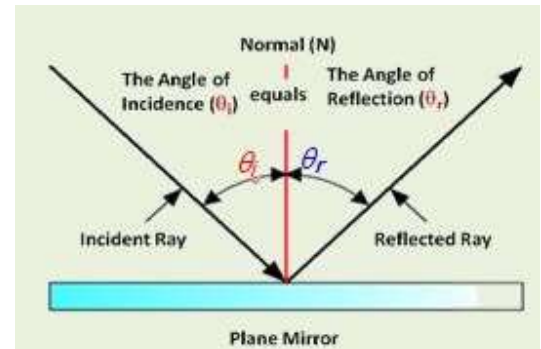
**At the end of this chapter, the students will be able to:**

- Recall the laws of reflection and the refractive index and the related relations.
- State the meaning of internal reflection and the critical angle.
- Recall the types of lens, lens parameters, lens equations, and Magnification.
- Recall the mechanism of vision in human eyes, its defects, and how to correct it.

## Reflection of Light\*\*

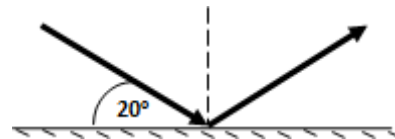
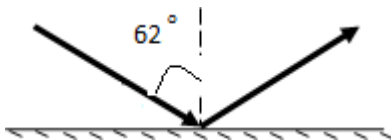
- Light reflected back when strike a reflected surface.
- The incident ray, the reflected ray, and the normal (N) all lie in the same plane.
- The angle of incidence ( $\theta_i$ ) equals to the angle of reflection ( $\theta_r$ )

$$\theta_i = \theta_r$$



### Example 1\*\*

For each figure shown below, find the angle of reflection.



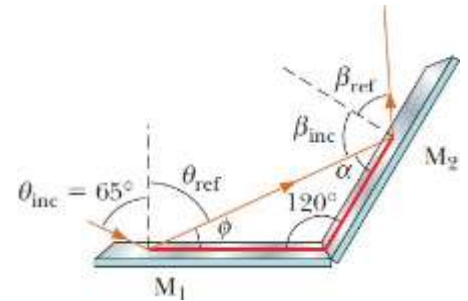
### Solution:

The angle of reflection for the first figure =  $62^\circ$

The angle of reflection for the second figure =  $70^\circ$   
(Because the angle of incidence is  $= 90^\circ - 20^\circ = 70^\circ$ )

### Example 2\*\*

Two mirrors make an angle of  $120^\circ$  with each other, as in shown in the Figure below. A ray is incident on mirror  $M_1$  at an angle of  $65^\circ$  to the normal. Find the angle the ray makes with the normal to  $M_2$  after it is reflected from both mirrors (find the angle of reflection from the second mirror  $M_2$ ).



### Solution strategy:

Apply the law of reflection twice. Given the incident ray at angle  $\theta_{inc}$ , find the final resultant angle,  $\beta_{ref}$ .

Apply the law of reflection to  $M_1$  to find the angle of reflection,  $\theta_{ref}$ :

$$\theta_{ref} = \theta_{inc} = 65^\circ$$

Find the angle  $\phi$  that is the complement of the angle  $\theta_{ref}$ :

$$\phi = 90^\circ - \theta_{ref} = 90^\circ - 65^\circ = 25^\circ$$

Find the unknown angle  $\alpha$  in the triangle of  $M_1$ ,  $M_2$ , and the ray traveling from  $M_1$  to  $M_2$ , using the fact that the three angles sum to  $180^\circ$ :

$$180^\circ = 25^\circ + 120^\circ + \alpha \rightarrow \alpha = 35^\circ$$

The angle  $\alpha$  is complementary to the angle of incidence,  $\beta_{inc}$ , for  $M_2$ :

$$\alpha + \beta_{inc} = 90^\circ \rightarrow \beta_{inc} = 90^\circ - 35^\circ = 55^\circ$$

Apply the law of reflection a second time, obtaining  $\beta_{ref}$ :

$$\beta_{ref} = \beta_{inc} = 55^\circ$$

### Exercise 1\*\*

Two plane mirrors meet at a  $135^\circ$  angle. If light rays strike one mirror at  $34^\circ$  as shown below, what is the angle of reflection from the second mirror  $M_2$ ?

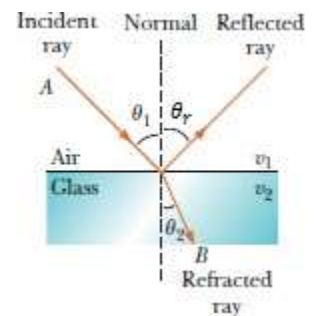
(The answer:  $\theta_{R2} = 79^\circ$ )



## Refraction of light

When a ray of light traveling through a transparent medium encounters a boundary leading into another transparent medium, as shown in the figure below, part of the ray is reflected and part enters the second medium. The ray that enters the second medium is bent at the boundary and is said to be refracted. The angle of refraction,  $\theta_2$  in the Figure below, depends on the properties of the two media and on the angle of incidence  $\theta_1$ .

- Light bends when entering a transparent medium.

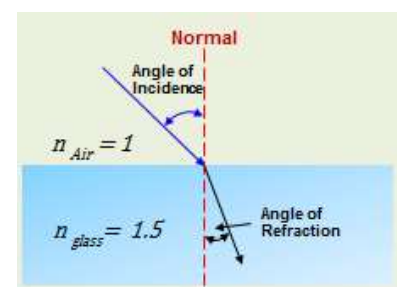


## Index of refraction

In general, the speed of light in any material is less than its speed in vacuum. In fact, light travels at its maximum speed in vacuum. It is convenient to define the index of refraction  $n$  of a medium to be the ratio

$$n = \frac{\text{speed of light in vacuum}}{\text{speed of light in a medium}} = \frac{C}{V}$$

- The index of refraction for a material is the ratio of the velocity of light in a vacuum to the velocity through the material.
- $C$  is the Velocity of light in vacuum or air  $\approx 3 \times 10^8$  m/s



### Example 3

What is the index of refraction for glass? Consider the velocity of light in the glass is  $2 \times 10^8$  m/s and the velocity of light in air or vacuum is  $3 \times 10^8$  m/s.

### Solution

$$n_{\text{glass}} = \frac{c}{V_{\text{in glass}}} = \frac{3 \times 10^8}{2 \times 10^8} = 1.5$$

### Exercise 2

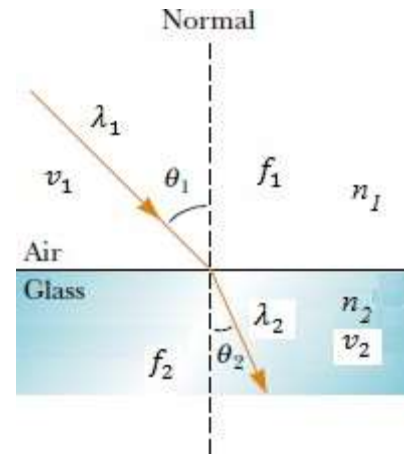
The speed of light in ice is  $2.29 \times 10^8$  m/s What is the index of refraction of ice? Consider the velocity of light in air or vacuum is  $3 \times 10^8$  m/s.

(The answer:  $n_{\text{ice}} = 1.31$  )

## Snell's Law

- The ratio of the sine of the angle of incidence  $\theta_1$  to the sine of the angle of refraction  $\theta_2$  is equal to the ratio of the index of refraction of the refractive medium  $n_2$  to the index of refraction of the incident medium  $n_1$

$$\frac{\sin \theta_1}{\sin \theta_2} = \frac{n_2}{n_1}$$



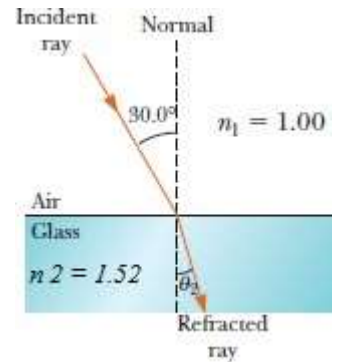
- As the light moves from medium 1 to medium 2, its wavelength ( $\lambda$ ) and velocity ( $v$ ) are changed, but its frequency ( $f$ ) remains constant due to the conservation of energy.
- As light travels from one medium to another, its frequency doesn't change ( $f_1 = f_2$ ).
- Since \*\*

$$v_1 = \lambda_1 \times f_1, \quad v_2 = \lambda_2 \times f_2, \quad \text{and} \quad n = \frac{c}{v}, \quad \text{Then}$$

$$\frac{\sin \theta_1}{\sin \theta_2} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2} = \frac{n_2}{n_1}$$

#### Example 4

A light ray traveling through air ( $n_1 = 1$ ) is incident on a glass ( $n_2 = 1.52$ ) at an angle of  $30.0^\circ$  to the normal, as sketched in Figure below. Find the angle of refraction,  $\theta_2$ .



#### Solution

$$n_1 = 1, \quad n_2 = 1.52, \quad \theta_1 = 30^\circ, \quad \theta_2 = ??$$

$$\frac{\sin \theta_1}{\sin \theta_2} = \frac{n_2}{n_1}$$

$$\frac{\sin 30}{\sin \theta_2} = \frac{1.52}{1} \Rightarrow 1.52 \times \sin \theta_2 = \sin 30 \Rightarrow \sin \theta_2 = \frac{\sin 30}{1.52}$$

$$\sin \theta_2 = 0.329 \Rightarrow \theta_2 = \sin^{-1} 0.329 = 19.2^\circ$$

#### Exercise 3

A diver shines a flashlight upward from beneath the water (medium 1) at a  $35.2^\circ$  angle to the vertical. At what angle does the light leave the water into the air (medium 2)?

Consider  $n_{\text{air}} = 1$  and  $n_{\text{water}} = 1.33$ .

(The answer:  $\theta_2 = 50.1^\circ$ )

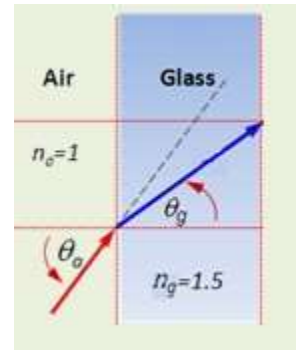
### Example 5 \*\*

A helium-neon laser emits a beam of wavelength 632nm in the air ( $n_a = 1$ ). This beam enters a slab of glass that has a 1.5 index of refraction ( $n_g = 1.5$ ). What is the wavelength of the beam inside the glass?

#### Solution

$$\frac{\lambda_1}{\lambda_2} = \frac{n_2}{n_1} \Rightarrow \frac{632}{\lambda_2} = \frac{1.5}{1}$$

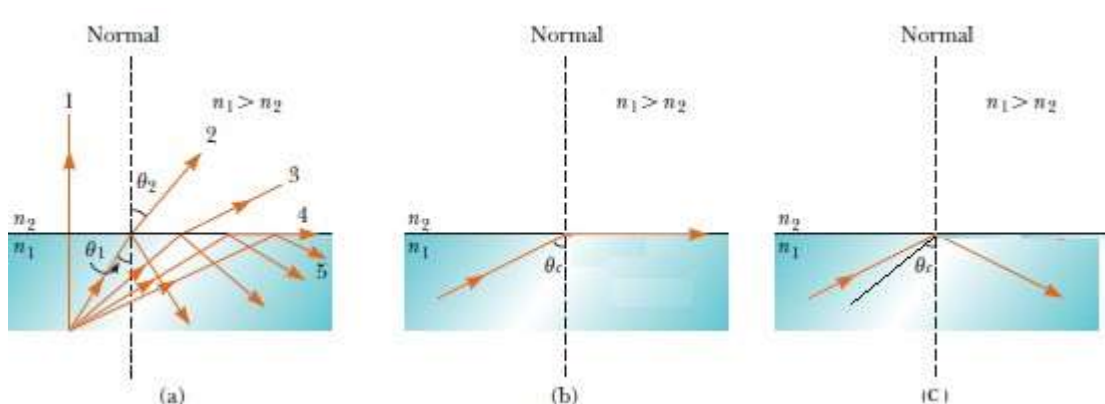
$$\lambda_2 = \frac{632}{1.5} = 421.3 \text{ nm}$$



### Total Internal Reflection

An interesting effect called total internal reflection can occur when light encounters the boundary between a medium with a higher index of refraction and one with a lower index of refraction.

Consider a light beam traveling in medium 1 and meeting the boundary between medium 1 and medium 2, where  $n_1$  is greater than  $n_2$  (Fig. a). Possible directions of the beam are indicated by rays 1 through 5. Note that the refracted rays are bent away from the normal because  $n_1$  is greater than  $n_2$ . At some particular angle of incidence  $\theta_c$ , called the critical angle, the refracted light ray moves parallel to the boundary so that  $\theta_2 = 90^\circ$  (Fig.b). For angles of incidence greater than  $\theta_c$ , the beam is entirely reflected at the boundary, as is ray 5 in Figures a and c. This ray is reflected as though it had struck a perfectly reflecting surface. It and all rays like it obey the law of reflection: the angle of incidence equals the angle of reflection.



- When a light beam is making a certain angle with the normal, passing through a medium of higher index of refraction to another medium of lower index of refraction, the emerging beam bends away from the normal.
- The critical angle  $\theta_c$  is the angle of incidence in a denser medium that makes an angle of refraction equal to  $90^\circ$  in the rare medium. According to Fig b,

$$\sin \theta_c = \frac{n_2}{n_1} \quad \text{or} \quad \theta_c = \sin^{-1} \left( \frac{n_2}{n_1} \right) \quad \text{where } n_2 < n_1$$

- With the increasing of the angle of incidence, the angle of refraction increases, until it reaches a certain value, at which the incident beam of light will be reflected internally.
- **The conditions of the total internal reflection:**
  - ❖ Total internal reflection occurs only when light attempts to move from a medium of higher index of refraction (dense) to a medium of lower index of refraction (rare).
  - ❖ The angle of incidence in the dense medium should be greater than the critical angle.

Scan this barcode to see the total internal reflection experiment.



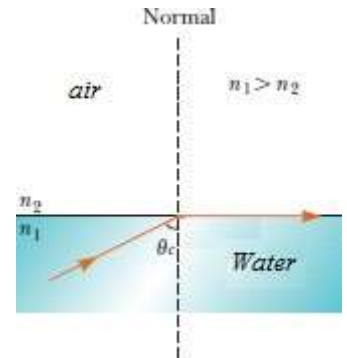
### Example 6

Find the critical angle between the water and air. Consider  $n_{air} = 1$ , and  $n_{water} = 1.33$ .

#### Solution

$$\theta_c = \sin^{-1} \left( \frac{n_2}{n_1} \right) \Rightarrow \theta_c = \sin^{-1} \left( \frac{1}{1.33} \right)$$

$$\theta_c = 48.75^\circ$$



### Exercise 4

What is the critical angle between water and glass? Consider  $n_{water} = 1.33$ , and  $n_{glass} = 1.5$ .

(The answer:  $\theta_c = 62.5^\circ$ )

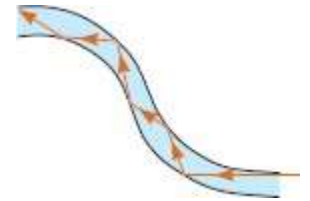
### Exercise 5

The critical angle for a certain liquid–air surface is  $47.2^\circ$ . What is the index of refraction of the liquid? Consider  $n_{air} = 1$ .

(The answer:  $n_{liquid} = 1.36$ )

## Fiber Optics\*\*

Another interesting application of total internal reflection is the use of solid glass or transparent plastic rods to “pipe” light from one place to another. As indicated in the figure shown, light is confined to traveling within the rods, even around gentle curves, as a result of successive internal reflections. Such a light pipe can be quite flexible if thin fibers are used rather than thick rods. If a bundle of parallel fibers is used to construct an optical transmission line, images can be transferred from one point to another.



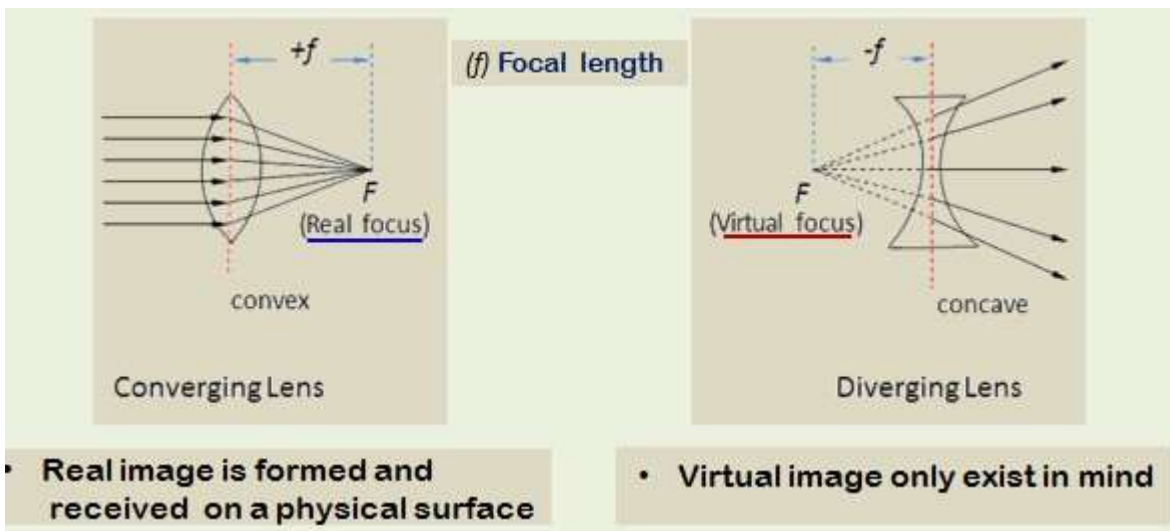
Very little light intensity is lost in these fibers as a result of reflections on the sides. Any loss of intensity is due essentially to reflections from the two ends and absorption by the fiber material. Fiber-optic devices are particularly useful for viewing images produced at inaccessible locations. Physicians often use fiber-optic cables to aid in the diagnosis and correction of certain medical problems without the intrusion of major surgery. For example, a fiber-optic cable can be threaded through the esophagus and into the stomach to look for ulcers. In this application the cable consists of two fiber-optic lines: one to transmit a beam of light into the stomach for illumination and the other to allow the light to be transmitted out of the stomach. The resulting image can, in some cases, be viewed directly by the physician, but more often is displayed on a television monitor or saved in digital form. In a similar way, fiber-optic cables can be used to examine the colon or to help physicians perform surgery without the need for large incisions. The field of fiber optics has revolutionized the entire communications industry. Billions of kilometers of optical fiber have been installed in the United States to carry high-speed Internet traffic, radio and television signals, and telephone calls. The fibers can carry much higher volumes of telephone calls and other forms of communication than electrical wires because of the higher frequency of the infrared light used to carry the information on optical fibers. Optical fibers are also preferable to copper wires because they are insulators and don't pick up stray electric and magnetic fields or electronic “noise.”



## Lenses

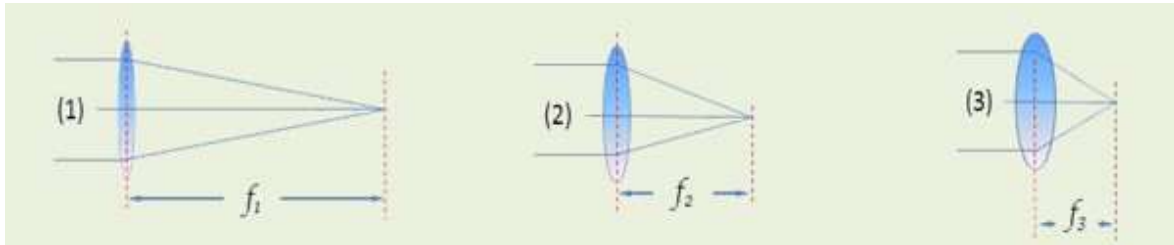
Lens is a piece of glass or other transparent material with curved sides for concentrating or dispersing light rays, used singly (as in a magnifying glass) or with other lenses (as in a telescope).

- In geometrical optics, images are formed where light rays intersect.



- The focal length  $f$  is positive for the convex (converging) lens.
- The focal length is negative for the concave (diverging) lens.

## The Refractive Power of the Lenses



- The more a lens bends light ray, the greater is its “*refractive power*”.
- The refractive power is measured in terms of unit called “*diopter*” ( $\frac{1}{m}$  or  $m^{-1}$ ) and has the symbol “*D*”.
- The refractive power (*P*) in diopters of convex lens is equal to 1 divided by its focal length (*f*) in meter.

•

$$P = \frac{1}{f}$$

The effect of focal length on the refractive power of lenses

$$f_1 > f_2 > f_3 \quad \Rightarrow \quad P_1 < P_2 < P_3$$

### Example 7

Calculate the refractive power (P) in diopter for a:

a) Convex lens has a focal length ( $f$ ) of 25 cm.

b) Concave lens has a focal length ( $f$ ) of 2 m.

### Solution

*Remember that:*  $cm = \frac{1}{100} m$

$$a) f = 25 \text{ cm} = \frac{25}{100} m = 0.25 m \quad \text{So,} \quad P = \frac{1}{f} = \frac{1}{0.25} = 4 \text{ D}$$

$$b) f = -2m \text{ (negative for the concave lens)} \quad \text{so,} \quad P = \frac{1}{f} = \frac{1}{-2} = -0.5 \text{ D}$$

### Exercise 6

Calculate the refractive power (P) in a diopter for a convex lens that has a focal length of 30 cm.

(The answer:  $p = 3.33 \text{ D}$ )

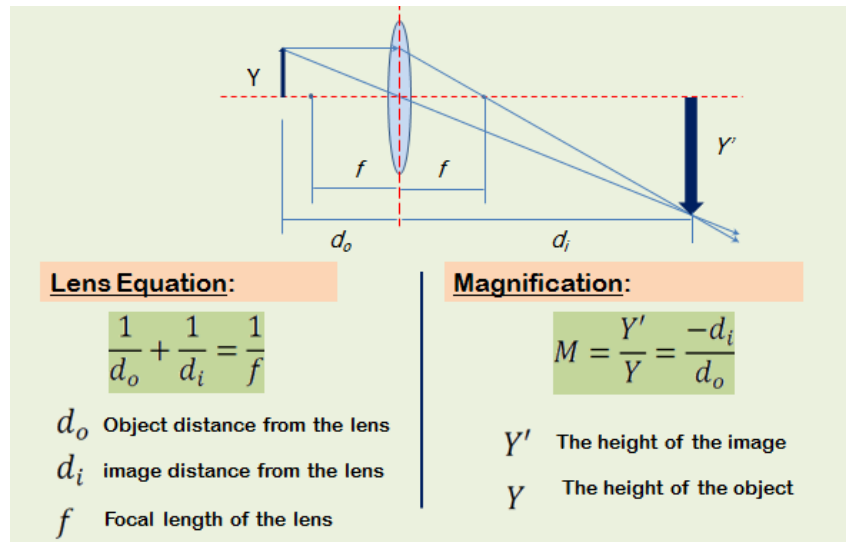
### Exercise 7

Calculate the refractive power (P) in a diopter for a concave lens that has a focal length of 4 m.

(The answer:  $p = -0.25 \text{ D}$ )

## Lens Equation and Magnification

### Analytical Approach to Imaging



### Signs convention

- The focal length ( $f$ ) is positive for the convex lens and negative for the concave lens.
- The object distance ( $d_o$ ) is positive for a real object and negative for a virtual object.
- The image distance ( $d_i$ ) is positive for a real image and negative for a virtual image.
- The magnification ( $M$ ) is positive for an upright image and negative for an inverted image.
- The image height ( $y'$ ) is positive for an upright image and negative for an inverted image.
- The magnification of the image:
  - The image is magnified if  $|M| > 1$
  - The image is minimized if  $|M| < 1$
  - The size of the image equals the size of the object if  $|M| = 1$

### Example 8

A converging lens has a focal length of 30 cm. If a 20 cm object height is located 90 cm from the lens, find:

- 1) The image distance ( $d_i = ??$ ).
- 2) The image is real or virtual?
- 3) The image is upright or inverted?
- 4) The image magnification?
- 5) The image height ( $y' = ??$ ).

### Solution

1)  $f = +30 \text{ cm}, \quad d_o = 90 \text{ cm}, \quad d_i = ??$

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \rightarrow \frac{1}{30} = \frac{1}{90} + \frac{1}{d_i} \rightarrow \frac{1}{d_i} = \frac{1}{30} - \frac{1}{90} \rightarrow \frac{1}{d_i} = \frac{2}{90} \rightarrow$$

$$d_i = +45 \text{ cm}$$

2) The image is real because  $d_i$  is positive.

3)  $M = \frac{-d_i}{d_o} = \frac{-(45)}{90} = -0.5$  so, the image is inverted because  $M$  is negative.

4) The image is minimized because  $|M| < 1$ . ( $0.5 < 1$ )

5) The image height  $y' = ??$   $M = \frac{y'}{y} \rightarrow y' = M \times y = -0.5 \times 20 = -10 \text{ cm}.$

### Exercise 8

A diverging lens has a focal length of 30 cm. If a 20 cm object height is located 90 cm from the lens, find:

- 1) The image distance ( $d_i = ??$ ).
- 2) The image is real or virtual?
- 3) The image is upright or inverted?
- 4) The image magnification?
- 5) The image height ( $y' = ??$ ).

**The answer:**

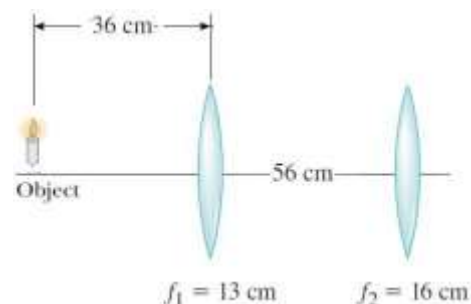
1)  $d_i = -22.5 \text{ cm},$  2) Virtual, 3)  $M = 0.25$ , upright, 4) Minimized, 5)  $y' = 5 \text{ cm}$

## Combinations of Lenses

Many optical instruments use lenses in combination. When light passes through more than one lens, we find the image formed by the first lens as if it were alone. Then this image becomes the object for the second lens. Next, we find the image formed by this second lens using the first image as an object. This second image is the final image if there are only two lenses. The total magnification will be the product of the separate magnifications of each lens. Even if the second lens intercepts the light from the first lens before it forms an image, this technique still works.

### Example 9

Two thin converging lenses of focal lengths  $f_1 = 13 \text{ cm}$  and  $f_2 = 16 \text{ cm}$  are separated by  $56 \text{ cm}$ , as illustrated in the Figure below. An object is placed  $36 \text{ cm}$  to the left of lens 1. Find the position and the magnification of the final image



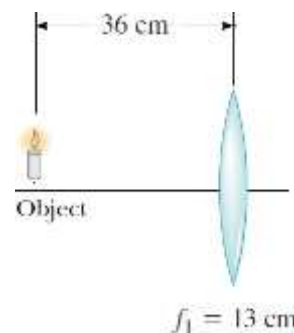
### Solution

First, we find the image formed by the first lens (ignore the second lens)

$$f_1 = +13 \text{ cm}, \quad d_{o1} = 36 \text{ cm}, \quad d_{i1} = ??$$

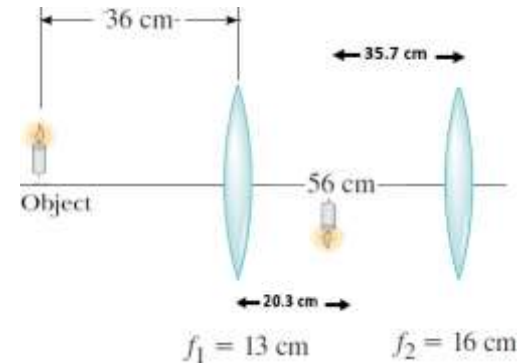
$$\frac{1}{f_1} = \frac{1}{d_{o1}} + \frac{1}{d_{i1}} \rightarrow \frac{1}{13} = \frac{1}{36} + \frac{1}{d_{i1}} \rightarrow \frac{1}{d_{i1}} = \frac{1}{13} - \frac{1}{36} \rightarrow$$

$$\frac{1}{d_{i1}} = 0.049 \rightarrow d_{i1} = +20.3 \text{ cm (the image is real)}$$



The magnification through the first lens  $M_1 = \frac{-d_{i1}}{d_{o1}} = \frac{-(20.3)}{36} = -0.57$  (inverted)

Now, the image formed by this lens (the first lens) acts as the object for the second lens (the **real** image formed by the first lens acts as a **real** object for the second lens, and **vice versa**). Thus, the object distance for the second lens is  $56 \text{ cm} - 20.3 \text{ cm} = 35.7 \text{ cm}$ . We again apply the thin lens equation to find the location of the final image:



$$f_2 = +16 \text{ cm}, \quad d_{o2} = 35.7 \text{ cm}, \quad d_{i2} = ??$$

$$\frac{1}{f_2} = \frac{1}{d_{o2}} + \frac{1}{d_{i2}} \rightarrow \frac{1}{16} = \frac{1}{35.7} + \frac{1}{d_{i2}} \rightarrow \frac{1}{d_{i2}} = \frac{1}{16} - \frac{1}{35.7} \rightarrow$$

$$\frac{1}{d_{i2}} = 0.0345 \rightarrow d_{i2} = +29 \text{ cm} \text{ (The position of the final image)}$$

$$\text{The magnification through the first lens } M_2 = \frac{-d_{i2}}{d_{o2}} = \frac{-(29)}{35.7} = -0.8$$

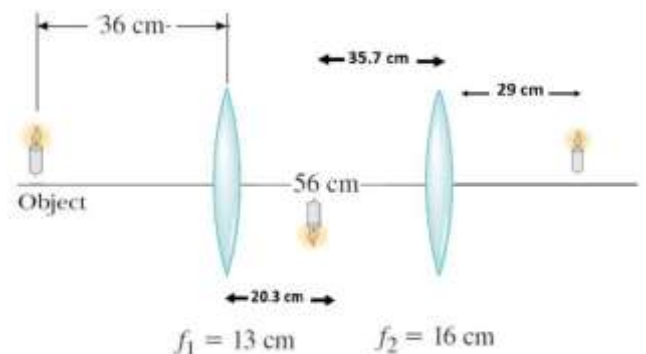
Thus, the final magnification of the system is

$$M_{\text{final}} = M_1 \cdot M_2 = (-0.57) \cdot (-0.8) = +0.46$$

The final image is minimized (with respect to the original object) because

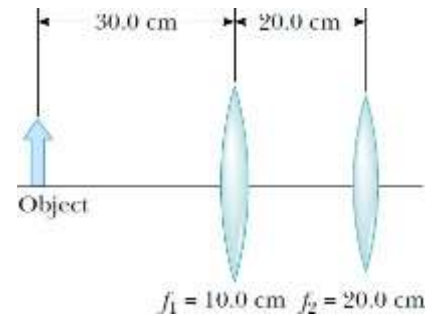
$$|M| < 1. \quad (0.46 < 1)$$

To finalize the problem, note that the positive sign of the final magnification indicates that the final image is upright with respect to the initial (original) object. The fact that the absolute value of the magnification is less than one tells us that the final image is smaller than the object. The fact that  $d_{i2}$  is positive tells us that the final image is behind, or right, side of lens 2. All of these conclusions are shown in the Figure below



### Exercise 9

Two thin converging lenses of focal lengths  $f_1 = 10\text{ cm}$  and  $f_2 = 20\text{ cm}$  are separated by 20 cm, as illustrated in the Figure below. An object is placed 30 cm to the left of lens 1. Find the position and the magnification of the final image.



(See the solution at the end of the chapter)

**The answer:**

The final position of the image is  $d_{i2} = -6.67\text{ cm}$

the final magnification of the system is  $M_{final} = -0.67$

### Exercise 10

Two lenses, one converging with focal length 20.0 cm and one diverging with focal length 10.0 cm, are placed 25.0 cm apart. An object is placed 60.0 cm in front of the converging lens. Determine (a) the position and (b) the magnification of the final image formed.

**The answer:**

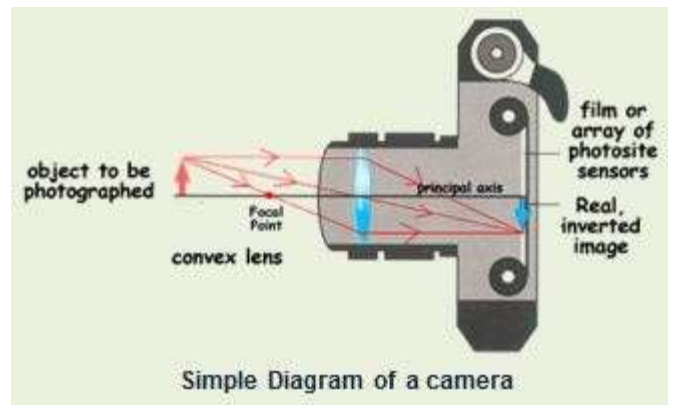
The final position of the image is  $d_{i2} = -5\text{ cm}$

The final magnification of the system is  $M_{final} = -0.25$

## Vision in human eyes

### The camera\*\*

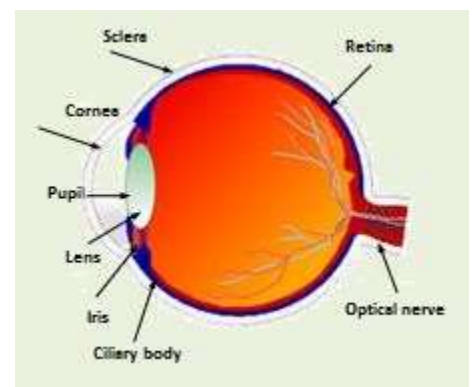
- A typical camera uses a converging lens to focus a real inverted image onto a receiving element (photographic film or sensor).
- The camera lens cannot change its shape; it moves closer to or farther away from the sensor (acts as the traditional camera film) in order to focus the image.



### The human eye\*\*

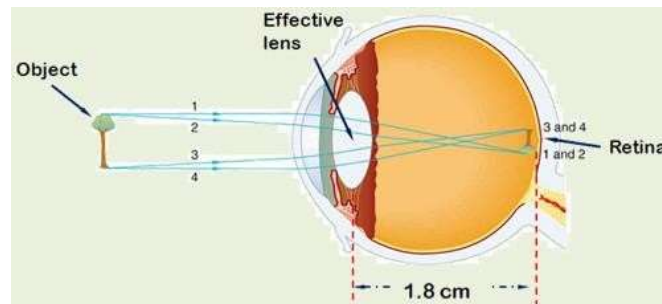
#### The eyeball is just like a camera.

- Sclera: Outer walls, hard, like a light-tight box.
- Cornea and crystalline lens (eye lens system): The two lens system.
- Ciliary body: Controlling the curvature of the eye lens.
- Retina : at the back of eyeball ,like the film.
- Iris: Like diaphragms or stop in a camera. The iris controls the size of the pupil fully open at low light level and small opening at high light level.
- Pupil: camera aperture.
- Optical nerve: delivering the generated signal to the brain



## Image formation

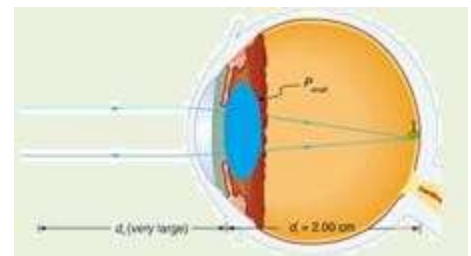
- The cornea and eye lens form a compound lens system, producing a real inverted image on the retina.



- Accommodation is the process of adjusting the eye's focal length to produce the picture on the retina, by contracting or relaxing the ciliary muscles around the lens.

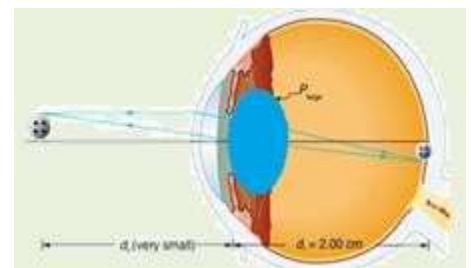
### 1) Viewing Distant Objects

Light rays from the same point on a distant object must be nearly parallel while entering the eye and more easily converge to produce an image on the retina. By the relaxing of ciliary muscle the curvature of the lens decreases resulting in low refractive power eye lens.



### 2) Viewing Nearby Objects

Light rays from a nearby object can diverge more and still enter the eye. A more powerful lens is needed to converge them on the retina by the contracting of ciliary muscle the curvature of the lens increases resulting in a high refractive power eye lens.

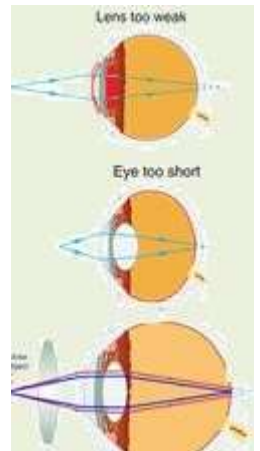


## Visual Defects and Correction

- **Visual Defects:** When an eye cannot focus an object's image on the retina, an Image formed in front of or behind the retina, results in blurred vision
- **Typical causes:**
  - Abnormal length of the eyeball
  - Abnormal curvature of the cornea
  - Abnormal accommodation
- **Correction**  
Glasses or Contact lenses

## Hyperopia (Farsightedness)

- The inability of the eye to focus on nearby objects. Images are formed behind the retina. The near point is located farther away from the eye.
- No difficulty focusing on distant objects "Can see Far object clearly".
- **Causes**
  - Cornea is too flat
  - Lens cannot perform a highly convex shape
  - Shortened eyeball (retina is closer than normal to the cornea lens system)
- **Correction**  
Need to refocus the image on the retina by adding a converging lens (positive power +D) to decrease the focal length of the cornea-lens system.



## Myopia (Nearsightedness)

- Inability of the eye to focus on distant objects. Images are formed in front of the retina. Far point is formed closer than normal.

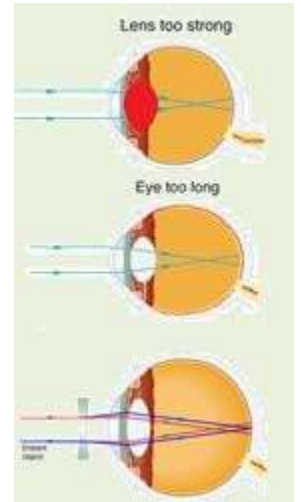
- No difficulty focusing on nearby objects; “Can see near object clearly”.

- **Causes**

- Bulging cornea (greater curvature)
- Lens cannot perform a less curved shape
- Elongated eyeball (retina is farther away than normal from the cornea lens system)

- **Correction**

Need to refocus the image on the retina by adding a diverging lens (negative power -D) to increase the focal length of the cornea-lens system.



## Astigmatism

- Astigmatism is a condition in which an abnormal curvature of the cornea can cause two focal points to fall in two different locations, making close and far objects appear blurry (different focus at different planes).



- **Causes**

Irregularly shaped cornea or lens (Refractive power differs between regions)

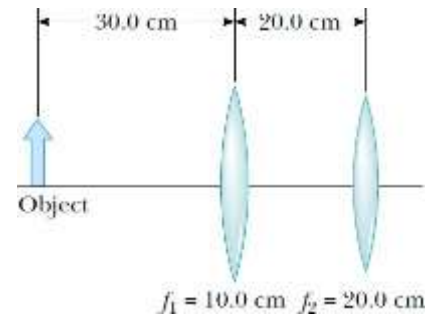
- **Correction**

- Glasses of Cylindrical lens
- Lenses with different radii of curvature in different planes



### Exercise 9 / solution

Two thin converging lenses of focal lengths  $f_1 = 10 \text{ cm}$  and  $f_2 = 20 \text{ cm}$  are separated by  $20 \text{ cm}$ , as illustrated in the Figure below. An object is placed  $30 \text{ cm}$  to the left of lens 1. Find the position and the magnification of the final image.



### Solution

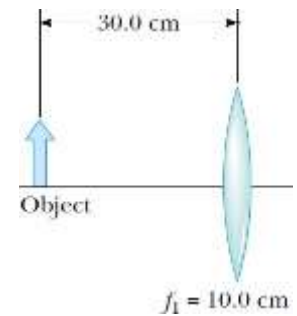
First, we find the image formed by the first lens (ignore the second lens)

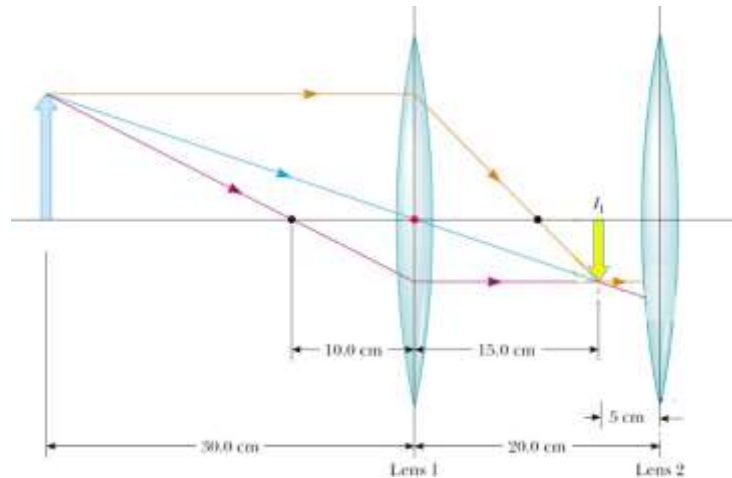
$$f_1 = +10 \text{ cm}, \quad d_{o1} = 30 \text{ cm}, \quad d_{i1} = ??$$

$$\frac{1}{f_1} = \frac{1}{d_{o1}} + \frac{1}{d_{i1}} \rightarrow \frac{1}{10} = \frac{1}{30} + \frac{1}{d_{i1}} \rightarrow \frac{1}{d_{i1}} = \frac{1}{10} - \frac{1}{30} \rightarrow \frac{1}{d_{i1}} = \frac{2}{30} \rightarrow$$

$$d_{i1} = +15 \text{ cm}$$

$$\text{The magnification through the first lens } M_1 = \frac{-d_{i1}}{d_{o1}} = \frac{-(15)}{30} = -0.5$$





Now, the image formed by this lens (the first lens) acts as the object for the second lens. Thus, the object distance for the second lens is  $20.0 \text{ cm} - 15.0 \text{ cm} = 5.00 \text{ cm}$ . We again apply the thin lens equation to find the location of the final image:

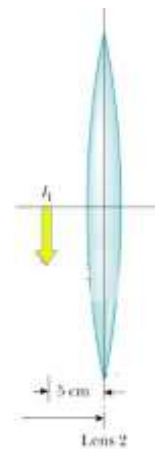
So,

$$f_2 = +20 \text{ cm}, \quad d_{o2} = 5 \text{ cm}, \quad d_{i2} = ??$$

$$\frac{1}{f_2} = \frac{1}{d_{o2}} + \frac{1}{d_{i2}} \rightarrow \frac{1}{20} = \frac{1}{5} + \frac{1}{d_{i2}} \rightarrow \frac{1}{d_{i2}} = \frac{1}{20} - \frac{1}{5} \rightarrow$$

$$\frac{1}{d_{i2}} = \frac{-3}{20} \rightarrow d_{i2} = -6.67 \text{ cm}$$

The magnification through the second lens  $M_2 = \frac{-d_{i2}}{d_{o2}} = \frac{-(-6.67)}{5} = +1.33$



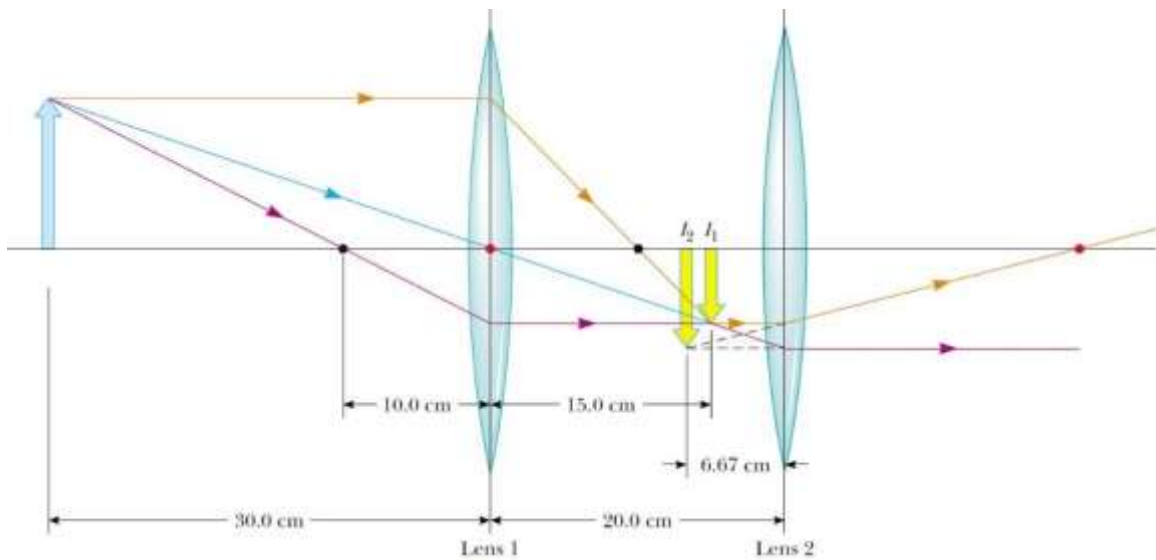
Thus, the overall magnification of the system is

$$M_{final} = M_1 \cdot M_2 = (-0.5) \cdot (1.33) = -0.67$$

The final image is minimized (with respect to the original object) because

$$|M| < 1. \quad (0.67 < 1)$$

To finalize the problem, note that the negative sign on the overall magnification indicates that the final image is inverted with respect to the initial object. The fact that the absolute value of the magnification is less than one tells us that the final image is smaller than the object. The fact that  $d_{i2}$  is negative tells us that the final image is on the front, or left, side of lens 2. All of these conclusions are shown in the Figure below.



## **Chapter Eight:**

## **Nuclear physics**

**Intended Learning Outcomes:**

**At the end of this chapter, the students will be able to:**

- Define Radioactive decay and radiation.
- Count the different types of radiation.
- Discuss the radioactive decay law.
- Define the half-life time constant.

## Radioactive decay and radiation

### What is radioactive decay?

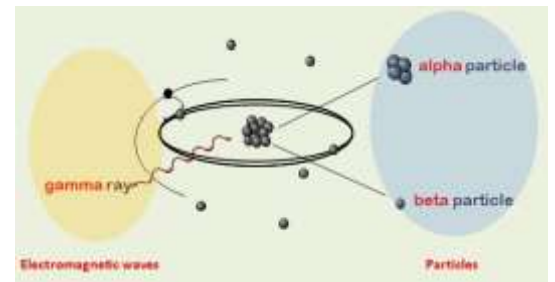
It is the spontaneous disintegration of the unstable nucleus to more stable nucleus by emitting radiation (particles or/and electromagnetic waves).

### What is Radiation?

It is the energy emitted from the unstable nuclei in form of emitted particles or emitted electromagnetic waves (or both).

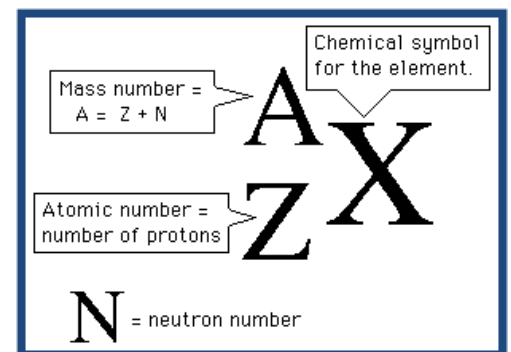
### The most common types of radiation are

1. Alpha particle ( $\alpha$ )
2. Beta particle ( $\beta$ )
3. Gamma ray ( $\gamma$ )



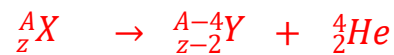
### Nuclear Notation

The standard nuclear notation shows the chemical symbol, the mass number, and the atomic number of the element.

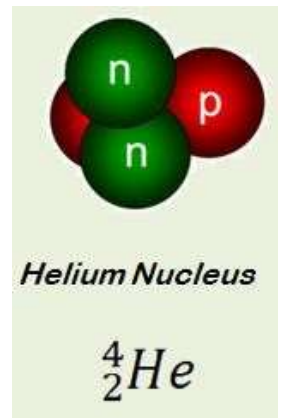
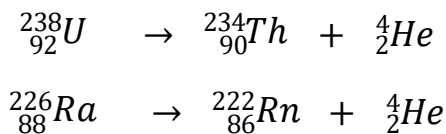


## Alpha radiation ( $\alpha$ )

If a nucleus emits an alpha particle (  ${}^4_2\text{He}$  ), it loses two protons and two neutrons. Therefore, the mass number A decreases by 4 and the atomic number Z decreases by 2. The decay can be written symbolically as



Where X is called the parent nucleus and Y is known as the daughter nucleus. As examples,  ${}^{238}\text{U}$  and  ${}^{226}\text{Ra}$  are both alpha emitters and decay according to the schemes



**Description:** 2 neutrons, 2 protons (Helium nucleus)

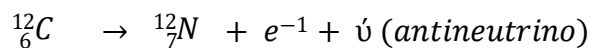
**Electric Charge:** +2

**Penetrating power:** very Low penetrating power, stopped by paper or a few cm of air

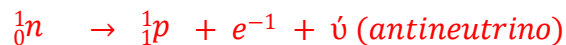
**Effects of Magnetic/Electric Field:** weakly deflected

## Beta radiation ( $\beta^{-1}$ )

When a radioactive nucleus undergoes beta decay, the daughter nucleus has the same mass number A as the parent nucleus, but the atomic number Z increases by 1.



The emission of electrons from a nucleus is surprising because as we know the nucleus is composed of protons and neutrons only. This apparent discrepancy can be explained by noting that the emitted electron is created in the nucleus by a process in which a neutron is transformed into a proton. This process can be represented by

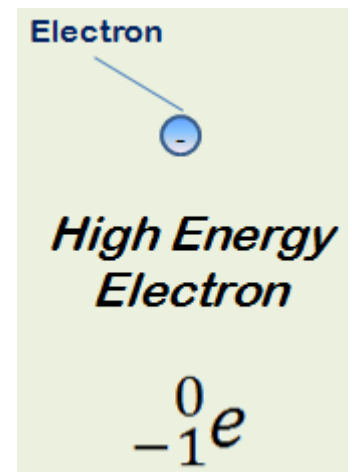


**Description:** high energy electron

**Electric Charge:** -1

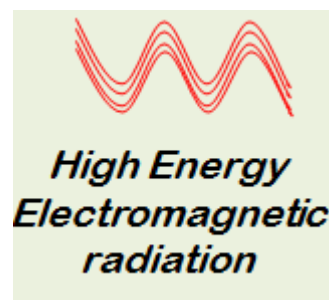
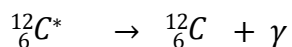
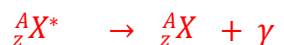
**Penetrating power:** intermediate penetrating power, stopped by few mm of aluminium

**Effects of Magnetic/Electric Field:** strongly deflected



## Gamma ray ( $\gamma$ )

Very often a nucleus that undergoes radioactive decay is left in an excited energy state. The nucleus can then undergo a second decay to a lower energy state-perhaps even to the ground state-by emitting one or more high-energy photons. The process is similar to the emission of light by an atom. An atom emits radiation to release some extra energy when an electron “jumps” from a state of high energy to a state of lower energy. Likewise, the nucleus uses essentially the same method to release any extra energy it may have following a decay or some other nuclear event. In nuclear de-excitation, the “jumps” that release energy are made by protons or neutrons in the nucleus as they move from a higher energy level to a lower level. The photons emitted in the process are called **gamma rays**, which have very high energy relative to the energy of visible light.



**Description:** High energy electromagnetic radiation

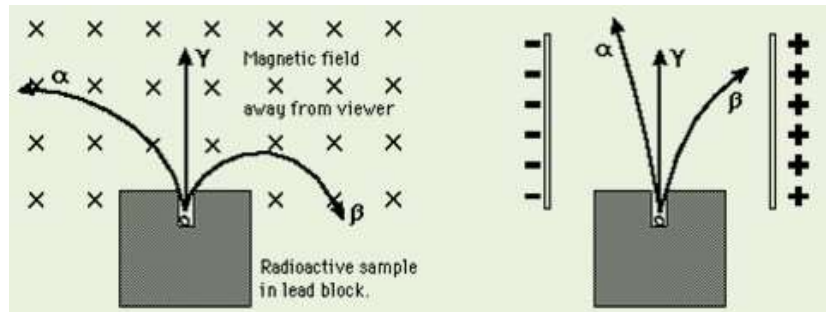
**Electric Charge:** 0

**Penetrating power:** very high penetrating power, reduced by several cm's of lead or several metres of concrete

**Effects of Magnetic/Electric Field:** no deflection

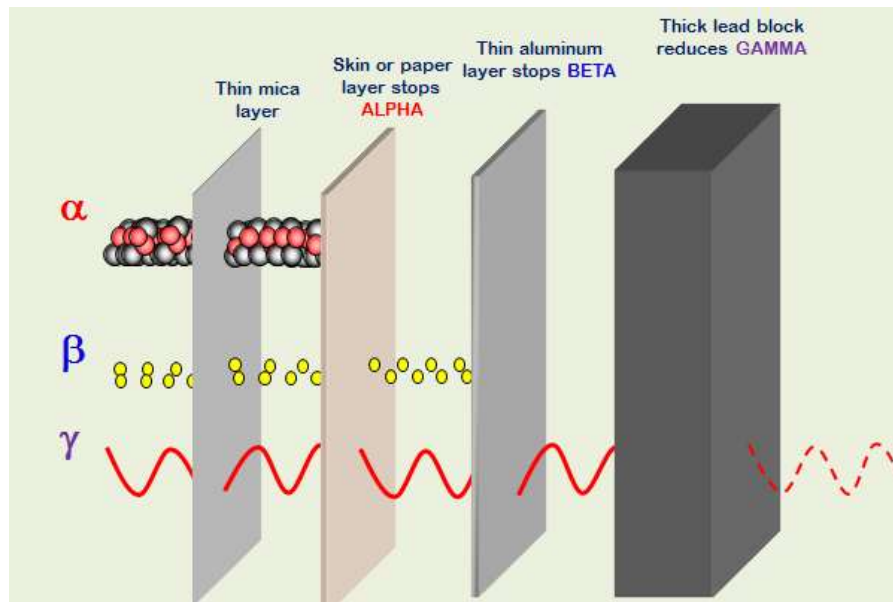
## The effect of electric and magnetic fields on radiation

- Beta radiation has a  $-1$  charge and a small mass so is strongly deflected
- Alpha radiation has a  $+2$  charge and large mass, so is only weakly deflected
- Gamma radiation has no mass or charge so it is not deflected



## The Penetration Power of Radiation

The penetration power of different radiations is shown in the figure below.



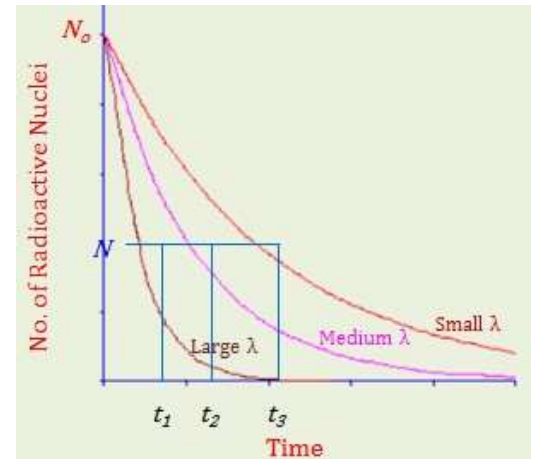
## The radioactive decay law

The number of atoms after time  $t$  can be calculated by using the following equation:-

$$N = N_0 \times e^{-\lambda \times t}$$

Where,

- $N$  is the number of atoms after time  $t$ .
- $N_0$  is the number of atoms at  $t = 0$
- $\lambda$  is the decay constant, which represents the probability of disintegration per unit time.
- SI unit for  $\lambda$  is  $1/s$ .
- The greater  $\lambda$  is, the greater the rate of decay.

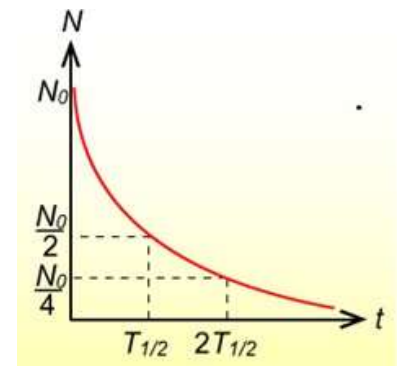


## The half-life time ( $\tau_{\frac{1}{2}}$ )

- The radioactive half-life for a given radioisotope is a measure of the tendency of the nucleus to "decay" or "disintegrate"
- The "half-life" ( $\tau_{\frac{1}{2}}$ ) is the time it takes for half of the atoms of a radioactive substance to decay.

$$\tau_{\frac{1}{2}} = \frac{0.693}{\lambda}$$

Where  $\lambda$  is the decay constant



### **Example 1**

Assume that we had 20,000 atoms of a radioactive substance. If the half-life for this radioactive substance is 1 hour, how many atoms of that substance would be left after 1 hour, 2 hours, and 3 hours?

**The answer:**

| Time                        | #<br>atoms<br>remaining | %<br>of atoms<br>remaining |
|-----------------------------|-------------------------|----------------------------|
| 1 hour (one lifetime) ?     | 10,000                  | (50%)                      |
| 2 hours (two lifetimes) ?   | 5,000                   | (25%)                      |
| 3 hours (three lifetimes) ? | 2,500                   | (12.5%)                    |

### Example 2

If the half-life of radium (Ra) is  $1.5 \times 10^3$  years, what is the decay constant?

### Solution

$$\tau_{\frac{1}{2}} = \frac{0.693}{\lambda} \quad \Rightarrow \quad 1.5 \times 10^3 = \frac{0.693}{\lambda}$$

$$1.5 \times 10^3 \times \lambda = 0.693 \quad \Rightarrow \quad \lambda = \frac{0.693}{1.5 \times 10^3} \quad \Rightarrow \quad \lambda = 0.462 \times 10^{-3} \text{ year}^{-1}$$

### Example 3

The decay constant of Cesium (Cs) is  $0.0231 \text{ year}^{-1}$ . If the sample contains  $6 \times 10^{20}$  nuclei. Find the number of nuclei after  $10^3$  years.

### Solution

$$N = N_0 \times e^{-\lambda \times t}$$

$$N = 6 \times 10^{20} \times e^{-0.0231 \times 10^3}$$

$$N = 6 \times 10^{20} \times e^{-23.1}$$

$$N = 6 \times 10^{20} \times 5.57 \times 10^{-10}$$

$$N \approx 5.6 \times 10^{10} \text{ nuclei}$$

#### Example 4

The decay constant of radium (Ra) is  $1.4 \times 10^{-11} \text{ s}^{-1}$ . If the sample contains  $2 \times 10^{16}$  nuclei. Find the number of nuclei after  $5 \times 10^3$  years.

#### Solution

$$\lambda = 1.4 \times 10^{-11} \text{ s}^{-1}$$

$$N_0 = 2 \times 10^{16}$$

$$N = ??$$

$$t = 5 \times 10^3 \text{ years}$$

$$t = 5 \times 10^3 \text{ years} \times 365 \text{ days} \times 24 \text{ hours} \times 60 \text{ min} \times 60 \text{ s} = 15768 \times 10^7 \text{ s}$$

$$N = N_0 \times e^{-\lambda \times t}$$

$$N = 2 \times 10^{16} \times e^{-1.4 \times 10^{-11} \text{ s}^{-1} \times 15768 \times 10^7 \text{ s}}$$

$$N \approx 22 \times 10^{14} \text{ nuclei}$$

### Exercise 1

What is the decay constant of  ${}^{238}_{92}\text{U}$  whose half-life is  $4.5 \times 10^9 \text{ year}$ ?

**The answer:**  $0.154 \times 10^{-9} \text{ year}^{-1}$

### Exercise 2

The decay constant of  ${}^{238}_{92}\text{U}$  is  $3.2 \times 10^{-5} \text{ s}^{-1}$ . What is its half-life?

**The answer:**  $0.22 \times 10^5 \text{ s}$

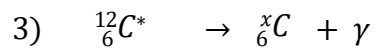
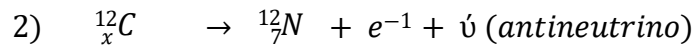
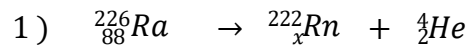
### Exercise 3

The radioactive isotope  ${}^{198}_{79}\text{Au}$  has a decay constant of  $0.01 \text{ h}^{-1}$ . A sample containing this isotope has  $9 \times 10^{12} \text{ nuclei}$ . Calculate the number of nuclei after 120 h.

**The answer:**  $2.7 \times 10^9 \text{ nuclei}$

#### Exercise 4

Find the value of x in each of the following equations.



**The answer:** 1)  $x = 86$                       2)  $x = 6$                       3)  $x = 12$

#### Exercise 5

The radioactive isotope  ${}_{79}^{198}\text{Au}$  has a decay constant of  $0.01 \text{ h}^{-1}$ . A sample containing this isotope has  $8 \times 10^{15} \text{ nuclei}$ . Calculate the number of nuclei after 3600 s.

**The answer:**  $7.92 \times 10^{15} \text{ nuclei}$

## **Chapter Nine:**

### **Electricity**

## **Intended Learning Outcomes:**

### **At the end of this chapter, the students will be able to:**

- Explore Coulomb's Law (Electrical force)
- Define electric current, resistance, electrical force, electrical field, and electrical potential
- Solve mathematical problems on electrical current, resistance, electrical force, electrical field, and electrical potential
- Explore the meaning of the capacitance
- Solve mathematical problems related to the capacitance

## Introduction\*\*

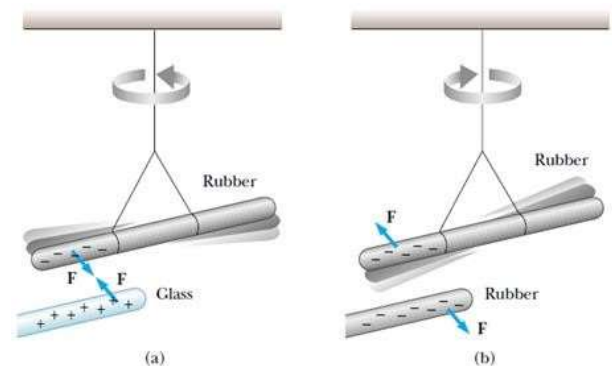
- The electromagnetic force between charged particles is one of the fundamental forces of nature.
- Charge is an intrinsic property of matter.
- There are two kinds of electric charges: Positive and negative.
- Charges of the same sign repel one another and charges with opposite signs attracted one another.

To verify that there are two types of charge:

- Hard rubber rod is rubbed with fur. ( negative charge )
- Glass rod is rubbed with silk. ( positive charge )
- When a glass rod that has been rubbed with silk is brought near the suspended rubber rod, the two attract each other.
- If two charged rubber rods (or two charged glass rods) are brought each other, the two repel each other.

( a ) A negatively charged rubber rod suspended by a thread is attracted to a positively charged glass rod.

( b ) A negatively charged rubber rod is repelled by another negatively charged rubber rod.



|                                                                                               |                   |
|-----------------------------------------------------------------------------------------------|-------------------|
| <div>Tend to lose electrons</div> <div>↑</div> <div>↓</div> <div>Tend to gain electrons</div> | (+)               |
|                                                                                               | human hands (dry) |
|                                                                                               | glass             |
|                                                                                               | human hair        |
|                                                                                               | nylon             |
|                                                                                               | cat fur           |
|                                                                                               | silk              |
|                                                                                               | cotton            |
|                                                                                               | steel             |
|                                                                                               | wood              |
|                                                                                               | amber             |
|                                                                                               | ebonite           |
|                                                                                               | plastic wrap      |
|                                                                                               | Teflon®           |
| (-)                                                                                           |                   |

## Properties of Electric Charges\*\*

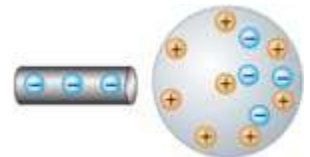
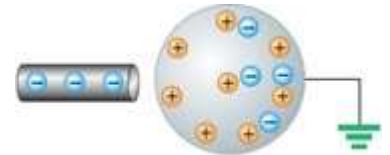
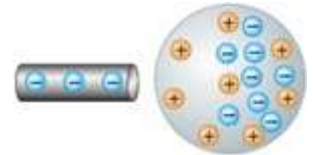
- There are two kinds of charges in nature positive and negative.
- Charges of the same sign repel one another and charges with opposite signs attract one another.
- Total charge in an isolated system is conserved.
- Charge is quantized.

## Materials classified to\*\*

- **Conductors** are materials in which some of the electrons are free, not bound to atom and move relatively freely through the material. Like Cu, Al, Ag,...
- **Insulators** are materials in which all electrons are bound to atoms and cannot move freely through the material. Like glass, rubber and wood...
- **Semiconductors** are materials with electrical properties are somewhere between those of insulators and conductors. Like Si, Ge, ....

## Charging conductors by induction\*\*

- Equal number of  $-Ve$  and  $+Ve$  charge (charge on the sphere = zero)
- When a negatively charged rubber rod is brought near the sphere, electrons in the region nearest the rod experience a repulsive force and migrate to the other side of the sphere.
- Some of the electrons in the conductors are so strongly repelled and move out of the sphere through the wire and into the Earth.
- If the wire to ground is then removed, the conducting sphere contains an excess of induced positive charge because it has fewer electrons than it needs to cancel out the positive charge of the protons.
- When the rubber rod is removed from the vicinity of the sphere, this induced positive charge remains on the ungrounded sphere.
- Note that the rubber rod loses none of its negative charge during this process.
- Charging an object by induction requires no contact with the object inducing the charge. This is in contrast to charging an object by rubbing (that is, by conduction) which does require contact between the two objects.



## Prefixes

The prefixes are used to help in writing the units, those prefixes are just abbreviations placed in front of a physical quantity unit.

Important prefixes in this chapter:

$c = \text{centi} = 10^{-2}$ , Example:  $5 \text{ cm} = 5 \times 10^{-2} \text{ m}$

$m = \text{milli} = 10^{-3}$ , Example:  $7 \text{ mg} = 7 \times 10^{-3} \text{ g}$

$\mu = \text{micro} = 10^{-6}$ , Example:  $9 \mu\text{C} = 9 \times 10^{-6} \text{ C}$

$n = \text{nano} = 10^{-9}$ , Example:  $13 \text{ ns} = 13 \times 10^{-9} \text{ s}$

$p = \text{pico} = 10^{-12}$ , Example:  $2 \text{ pHz} = 2 \times 10^{-12} \text{ Hz}$

| Prefixes for Powers of Ten |        |              |
|----------------------------|--------|--------------|
| Power                      | Prefix | Abbreviation |
| $10^{-24}$                 | yocto  | y            |
| $10^{-21}$                 | zepto  | z            |
| $10^{-18}$                 | atto   | a            |
| $10^{-15}$                 | femto  | f            |
| $10^{-12}$                 | pico   | p            |
| $10^{-9}$                  | nano   | n            |
| $10^{-6}$                  | micro  | $\mu$        |
| $10^{-3}$                  | milli  | m            |
| $10^{-2}$                  | centi  | c            |
| $10^{-1}$                  | deci   | d            |
| $10^3$                     | kilo   | k            |
| $10^6$                     | mega   | M            |
| $10^9$                     | giga   | G            |
| $10^{12}$                  | tera   | T            |
| $10^{15}$                  | peta   | P            |
| $10^{18}$                  | exa    | E            |
| $10^{21}$                  | zetta  | Z            |
| $10^{24}$                  | yotta  | Y            |

## Coulomb's Law

- Charles Coulomb measured the magnitudes of the electric forces between charged objects.
- From Coulomb's experiments, we can generalize the following properties of the electric force (  $F$  ) between two stationary charged particles. The electric force

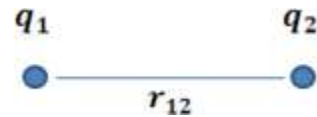
a) is inversely proportional to the square of the separation  $r$  between the particles and directed along the line joining them.

b) is proportional to the product of the charges  $q_1$  and  $q_2$  of the two particles.

c) is attractive if the charges are of opposite signs and repulsive if the charges have the same sign.

- Coulomb's law mathematically takes the following form:

$$\vec{F} = K_e \times \frac{q_1 \times q_2}{r^2}$$



Where:

$q_1$  is the charge of the first particle ( coulomb [ C ] ).

$q_2$  is the charge of the second particle ( coulomb [ C ] ).

$r$  is the separation distance between the charges ( m ).

$K_e$  is a constant called a coulomb constant. Where

$$K_e = \frac{1}{4 \times \pi \times \epsilon}$$

$\epsilon$  : is known as the permittivity constant.

For free space,  $\epsilon = \epsilon_0$  (for vacuum), where

$$\epsilon_0 = 8.8542 \times 10^{-12} \frac{\text{C}^2}{\text{N} \cdot \text{m}^2}$$

So, in a vacuum (or in the air) Coulomb's constant can be approximated to

$$K_e = \frac{1}{4 \times \pi \times \epsilon_0} = \frac{1}{4 \times \pi \times 8.8542 \times 10^{-12}} = 9 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}$$

Therefore,

In the air (or vacuum) Coulomb's law takes the following form:

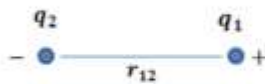
$$\vec{F} = 9 \times 10^9 \times \frac{q_1 \times q_2}{r^2}$$

Notes:

- All quantities used in Coulomb's equation should be in SI units.
- Negative signs (for the charges) are not substituted in Coulomb's law.
- The direction of the force should be considered.

- The charge that the question asks to calculate the electrical force on it can move, while the other charges are fixed.

In the Figure below, what is the direction of the electrical force on  $q_1$  due to  $q_2$ ? Direction of  $F_{21}$ ?

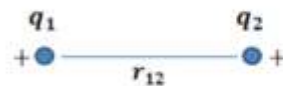


The electrical force on  $q_1$  due to  $q_2$  is denoted as  $F_{21}$ . So  $q_2$  is fixed and  $q_1$  moves according to the signs of the charges.

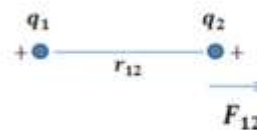


Here,  $F_{21}$  is to the left

In the Figure below, what is the direction of the electrical force on  $q_2$  due to  $q_1$ ? Direction of  $F_{12}$ ?



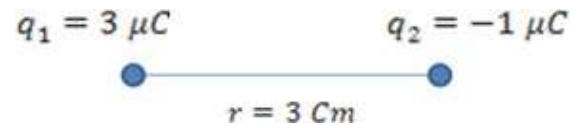
The electrical force on  $q_2$  due to  $q_1$  is denoted as  $F_{12}$ . So  $q_1$  is fixed and  $q_2$  moves according to the signs of the charges.



Here,  $F_{12}$  is to the right

### Example 1

Consider two point charges  $q_1$  and  $q_2$  separated by a distance of 3 cm as shown in the figure below. Calculate the electrical force on  $q_1$



### Solution:

The quantities should be in SI unit

$$q_1 = 3 \mu C = 3 \times 10^{-6} C$$

$$q_2 = -1 \mu C = -1 \times 10^{-6} C$$

$$r = 3 \text{ cm} = 3 \times 10^{-2} m$$

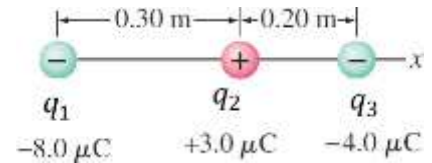
The electrical force on  $q_1$  due to  $q_2$  ( $F_{21} = ??$ ) is

$$F_{21} = 9 \times 10^9 \times \frac{q_1 \times q_2}{r^2} = 9 \times 10^9 \times \frac{3 \times 10^{-6} \times 1 \times 10^{-6}}{(3 \times 10^{-2})^2}$$

$$F_{21} = 30 \text{ N to the right}$$

### Example 2

Three charged particles are arranged in a line, as shown in the Figure below. Calculate the net electrostatic force on particle 3 ( $q_3$ ) due to the other two charges.



#### Solution:

The quantities should be in SI unit

$$q_1 = -8 \mu\text{C} = -8 \times 10^{-6} \text{ C}$$

$$q_2 = 3 \mu\text{C} = 3 \times 10^{-6} \text{ C}$$

$$q_3 = -4 \mu\text{C} = -4 \times 10^{-6} \text{ C}$$

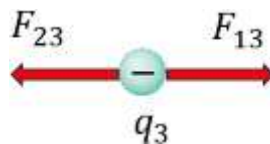
$$r_{13} = 0.3 + 0.2 = 0.5 \text{ m}$$

$$r_{23} = 0.2 \text{ m}$$

The electrical forces on  $q_3$  due to  $q_1$  and  $q_2$  ( $F_{13} = ??$  and  $F_{23} = ??$ ) are

$$F_{13} = 9 \times 10^9 \times \frac{q_1 \times q_3}{r_{13}^2} = 9 \times 10^9 \times \frac{8 \times 10^{-6} \times 4 \times 10^{-6}}{(0.5)^2} = 1.2 \text{ N (to the right)}$$

$$F_{23} = 9 \times 10^9 \times \frac{q_2 \times q_3}{r_{23}^2} = 9 \times 10^9 \times \frac{3 \times 10^{-6} \times 4 \times 10^{-6}}{(0.2)^2} = 2.7 \text{ N (to the left)}$$



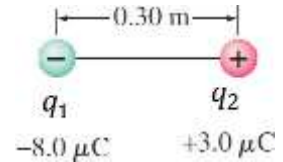
The net force on  $q_3$  is

$$\sum F_x = F_{23} + F_{13} = -2.7 + 1.2 = -1.5 \text{ N to the left}$$

### Exercise 1

Two point charges  $q_1$  and  $q_2$  are 0.3 m apart as shown in the Figure below. Calculate the electrical force on  $q_2$  due to  $q_1$ .

The answer:  $F_{12} = 2.4 \text{ N to the left}$



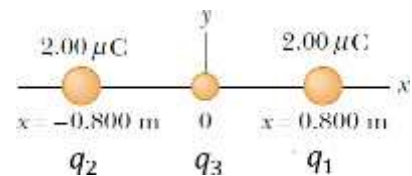
### Exercise 2

Two identical positive charges exert a repulsive force of  $36 \times 10^{19} \text{ N}$  when separated by a distance of 2 m. Calculate the charge of each.

The answer:  $q_1 = q_2 = 4 \times 10^5 \text{ C}$

### Exercise 3

Given two charges  $q_1$  and  $q_2$ , as shown in the Figure below, and a positive charge  $q_3 = 1.8 \times 10^{-18} \text{ C}$  at the origin, what is the net force exerted on the charge  $q_3$  by the two charges ( $q_1$  and  $q_2$ )?



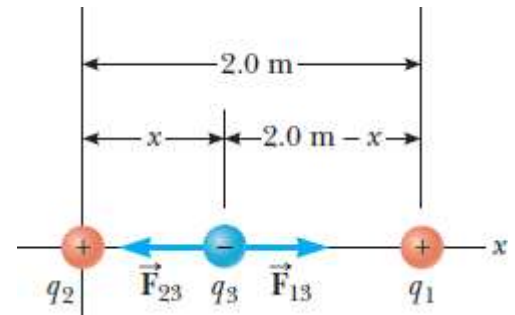
The answer: The net force on  $q_3$  is zero

#### Exercise 4

Three charges lie along the x-axis as in the Figure below. The positive charge  $q_1 = 4 \mu\text{C}$  is at 2.0 m to the right of the origin point, and the positive charge  $q_2 = 9 \mu\text{C}$  is at the origin. Where must a negative charge  $q_3$  be placed on the x-axis so that the resultant electric force on it is zero?

See the solution at the end of the chapter

The answer:  $x = 0.12 \text{ m}$



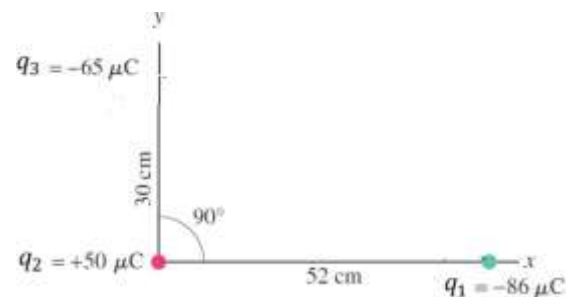
#### Exercise 5\*\*

Calculate the net electrical force exerted on the charge  $q_2$  by the two charges ( $q_1$  and  $q_3$ ) in the Figure shown below.

The answer:

$$F = 355.1 \text{ N}$$

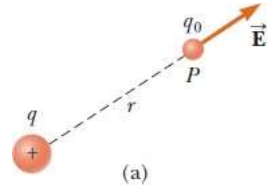
$$\theta_x = 66.3^\circ$$



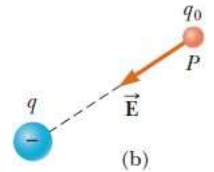
## Electric Field \*\*

- The electric field vector ( $\vec{E}$ ) at a point in space is defined as the electric force  $\vec{F}_e$  acting on a positive test charge  $q_0$  placed at that point divided by the test charge:

$$\vec{E} = \frac{\vec{F}_e}{q_0} \quad (N/C)$$



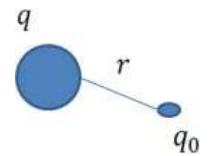
- A test charge  $q_0$  at P is a distance  $r$  from a point charge  $q$ . (a) If  $q$  is positive, the electric field at P points radially outwards from  $q$ . (b) If  $q$  is negative, the electric field at P points radially inwards toward  $q$ .



- The electric field is a vector quantity.
- Electric field exists at a point if a test charge at that point experiences an electric force.

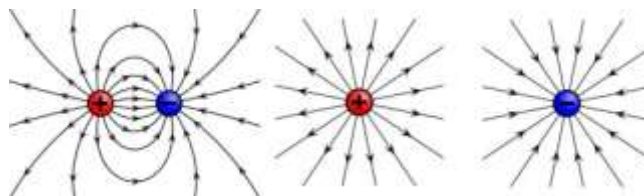
$$\vec{E} = \frac{\vec{F}_e}{q_0} \quad \text{Where} \quad \vec{F} = 9 \times 10^9 \times \frac{q_1 \times q_2}{r^2} \quad \text{Then}$$

$$\vec{E} = 9 \times 10^9 \times \frac{q}{r^2}$$



## The Rule for Drawing Electric Field Lines \*\*

- Electric field lines start at a positive charge and terminate at a negative charge.
- For one type of charge, lines start or end at infinity.
- The number of electric field lines proportional to the magnitude of the charge.
- Electric field lines never cross each other.



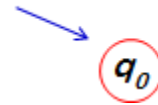
### Example 3\*\*

Point Charge (test charge) ( $q_0$ ) with  $2 \times 10^{-6} \text{ C}$  affected by  $20 \text{ N}$  electric forces, find the electric field on the charge point.

**Solution:**

$$\vec{E} = \frac{\vec{F_e}}{q_0} = \frac{20}{2 \times 10^{-6}} = 1 \times 10^7 \text{ N/C}$$

Point charge



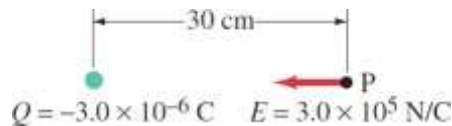
### Example 4\*\*

Calculate the magnitude and direction of the electric field at point P which is 30 cm to the right of a point charge  $Q = -3 \times 10^{-6} \text{ C}$

**Solution**

$$\vec{E} = 9 \times 10^9 \times \frac{q}{r^2} = 9 \times 10^9 \times \frac{3 \times 10^{-6}}{(0.3)^2} = 3 \times 10^5 \text{ N/C}$$

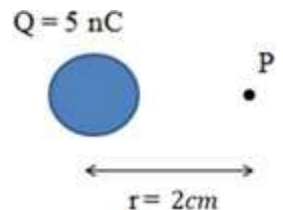
The direction of the electric field is toward the charge Q, to the left as shown in the Figure below.



### Exercise 6\*\*

In the following figure, calculate the electric field due to the charge Q at point p.

**The answer:**  $E = 11.25 \times 10^4 \text{ N/C}$  away from the charge Q.

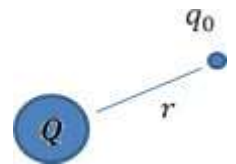


## Electric Potential

A gravitational analogy was relied upon to explain the reasoning behind the relationship between location and potential energy. Moving a positive test charge against the direction of an electric field is like moving a mass upward within Earth's gravitational field. Both movements would be like going against nature and would require work by an external force. This work would in turn increase the potential energy of the object. On the other hand, the movement of a positive test charge in the direction of an electric field would be like a mass falling downward within Earth's gravitational field. This motion would result in the loss of potential energy. Potential energy is the stored energy of the position of an object and it is related to the location of the object within a field.

- Electric potential,  $V$ , is the energy (electric potential energy) per unit charge.

$$V = \frac{U}{q_0} \quad \text{or} \quad V = 9 \times 10^9 \times \frac{q}{r}$$



- The SI unit of the electric potential is the volt V. (volt = Joule/Coulomb).
- The electric field is a scalar quantity.
- The potential difference between the two points is:  $V_{BA} = V_B - V_A$

### Example 5

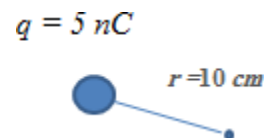
Find the electric potential 10cm away from 5 nC charge.

**Solution:**

$$q = 5 \text{ nC} = 5 \times 10^{-9} \text{ C}$$

$$r = 10 \text{ cm} = 10 \times 10^{-2} \text{ m} = 10^{-1} \text{ m} = 0.1 \text{ m}$$

$$V = 9 \times 10^9 \times \frac{5 \times 10^{-9}}{0.1} = 450 \text{ volt}$$



### Exercise 7

Determine the potential at a point 0.50 m (a) from a  $20 \mu C$  point charge, (b) from a  $-20 \mu C$  point charge.

**The answer:** a)  $V = 3.6 \times 10^5 V$ , b)  $V = -3.6 \times 10^5 V$

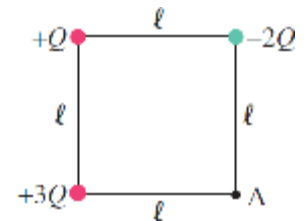
### Exercise 8

What is the electric potential 15.0 cm from a  $20 \mu C$  point charge?

**The answer:**  $V = 12 \times 10^5 V$

### Exercise 9\*\*

Three-point charges are arranged at the corners of a square of side as shown in Figure below. What is the potential at the fourth corner (point A)?

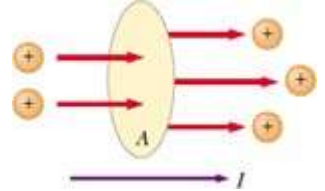


**The answer:**  $V = 9 \times 10^9 \frac{Q(2+\sqrt{2})}{2l}$

## Electrical Current

- The current is the rate at which charge flows through a specific surface.
- The current  $I$  is given by

$$I = \frac{\Delta Q}{\Delta t}$$



- The unit of current is the ampere (  $A$  ):  $1A = \frac{1C}{1s}$
- It is conventional to assign to the current the same direction as the flow of positive charges.
- The direction of the current flow is opposite to the direction of the flow of electrons.

### Example 6

What is the current in amperes if  $1\text{ Na}^+$  ion flow across a cell membrane in  $3\text{ }\mu\text{s}$ . The charge on the sodium ion is  $1.6 \times 10^{-19}\text{ C}$ .

**Solution:**

$$\Delta Q = 1.6 \times 10^{-19}\text{ C}$$

$$\Delta t = 3\text{ }\mu\text{s} = 3 \times 10^{-6}\text{ s}$$

$$I = ??$$

$$I = \frac{\Delta Q}{\Delta t}$$

$$I = \frac{1.6 \times 10^{-19}}{3 \times 10^{-6}} = 0.5 \times 10^{-13}\text{ C}$$

**Example 7**

A steady current of 2.5 A exists in a wire for 4.0 min. How much total charge passes by a given point in the circuit during those 4.0 minutes?

**Solution:**

$$I = 2.5 \text{ A}$$

$$\Delta t = 4 \times 60 = 240 \text{ s}$$

$$\Delta Q = ??$$

$$I = \frac{\Delta Q}{\Delta t}$$

$$2.5 = \frac{\Delta Q}{240} \Rightarrow \Delta Q = 2.5 \times 240 = 600 \text{ C}$$

**Exercise 10**

10 coulombs of charge flow through the bulb in 50 seconds. Calculate the current flowing through the bulb.

**The answer:**  $I = 0.2 \text{ A}$

**Exercise 11**

A service station charges a battery using a current of 6.7 A for 5.0 h. How much charge passes through the battery?

**The answer:**  $\Delta Q = 120600 \text{ C}$

## Resistance ( $R$ )

- In a conductor, the potential difference through the conductor is directly proportional to the current through the conductor.
- The resistance  $R$  of a conductor is defined as  $R = \frac{\Delta V}{I}$

Where  $\Delta V$  is the potential difference across it, and  $I$  is the current it carries.

- The SI unit of resistance is volts per ampere, which is defined to be 1 ohm ( $\Omega$ ); that is,  $1\Omega = 1 \text{ V/A}$ .

## Resistance of a wire

- The resistance of wire is given by  $R = \frac{\rho \times L}{A}$

Where:

$L$  is the length of the wire (m),  $A$  is the area cross section of the wire ( $\text{m}^2$ ), and  $\rho$  is the resistivity constant ( $\Omega \cdot \text{m}$ )

- Resistivity ( $\rho$ ) is defined as a constant which is the resistance of 1m of the material with a specific cross-section with a specific temperature.

## Example 8

Calculate the resistance of an aluminum wire that has a length of  $10 \text{ cm}$  and a cross-sectional area of  $2 \times 10^{-4} \text{ m}^2$  and the resistivity for  $Al$  is  $2.82 \times 10^{-8} \text{ m} \cdot \Omega$

**Solution:**

$$R = \rho \frac{L}{A}$$

$$R = 2.82 \times 10^{-8} \frac{10 \times 10^{-2}}{2 \times 10^{-4}}$$

$$R = 1.41 \times 10^{-5} \Omega$$

### Example 9

What is the resistance of a toaster if 120 V produces a current of 4.6 A?

**Solution:**

$$\Delta V = 120 \text{ V}$$

$$I = 4.6 \text{ A}$$

$$R = ??$$

$$R = \frac{\Delta V}{I}$$

$$R = \frac{120}{4.6} \Rightarrow R = 26.1 \text{ } \Omega$$

### Exercise 12

A hair dryer draws 13.5 A when plugged into a 120-V. What is its resistance?

**The answer:**  $R = 8.9 \text{ } \Omega$

### Exercise 13

What is the area cross-section of a 1.00-m length of tungsten wire whose resistance is  $0.32 \text{ } \Omega$ ? Consider the resistivity of the tungsten is  $5.6 \times 10^{-8} \text{ } \Omega \cdot m$ .

**The answer:**  $A = 17.5 \times 10^{-8} \text{ } m^2$

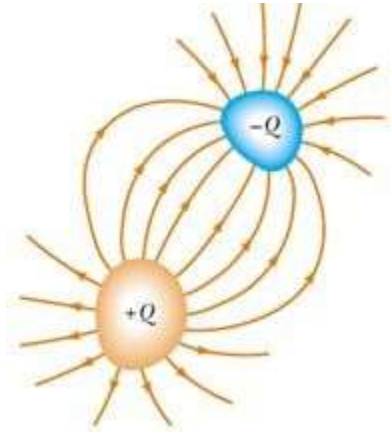
## Capacitance

- Consider two conductors carrying charges of equal magnitude and opposite sign, as in the Figure. Such a combination of two conductors is called a capacitor.
- The capacitance  $C$  of a capacitor is defined as the ratio of the magnitude of the charge on either conductor to the magnitude of the potential difference between the conductors

$$C = \frac{|Q|}{\Delta V}$$

- Capacitance is always a positive quantity.
- The SI unit of capacitance is the farad ( F), which was named in honor of Michal Faraday:

$$1F = \frac{1C}{1V}$$



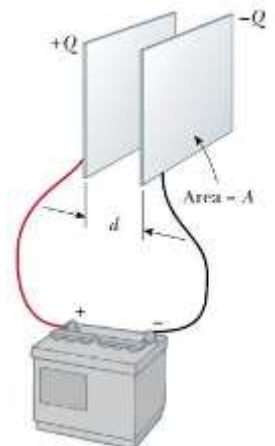
## Parallel Plate Capacitors

Two parallel metallic plates of equal area  $A$  are separated by a distance  $d$ , as shown in Figure. One plate carries a charge  $Q$ , and the other carries a charge  $-Q$ . What is the capacitance here?

The capacitance of a parallel-plate capacitor is proportional to the area  $A$  of its plates and inversely proportional to the plate separation  $d$ . It's given by:

$$C = \frac{A \epsilon_0}{d}$$

Where  $\epsilon_0$  is the permittivity constant.



## Capacitor with Dielectrics

If the dielectric completely fills the space between the plates, the capacitance increases by a dimensionless factor  $\kappa$ , which is called the dielectric constant of the material.

- The dielectric constant varies from one material to another.
- For a parallel plate capacitor, the capacitance of a capacitor filled by air (or vacuum):

$$C_0 = \frac{A \epsilon_0}{d}$$

- We can express the capacitance when the capacitor is filled with dielectric as:

$$C = \kappa \times C_0 \quad \text{or} \quad C = \kappa \times \frac{A \epsilon_0}{d}$$

### Example 10

A parallel Plate Capacitor has a plate area of  $0.2 \text{ m}^2$  separated by a  $0.01 \text{ m}$  thickness of paper. Find its capacitance.

Consider:  $\kappa = 3.7$  for paper, and  $\epsilon_0 = 8.8542 \times 10^{-12} \frac{\text{C}^2}{\text{N.m}^2}$

**Solution:**

$$A = 0.2 \text{ m}^2, d = 0.01 \text{ m}, k = 3.7$$

$$C = k \frac{A \epsilon_0}{d}$$

$$C = 3.7 \times \frac{0.2 \times 8.85 \times 10^{-12}}{0.01}$$

$$C = 654.9 \times 10^{-12} \text{ F}$$

| Dielectric - constants (at 20°C) |                       |                           |
|----------------------------------|-----------------------|---------------------------|
| Material                         | Dielectric constant K | Dielectric strength (V/m) |
| Vacuum                           | 1.0000                |                           |
| Air (1 atm)                      | 1.0006                | $3 \times 10^6$           |
| Paraffin                         | 2.2                   | $10 \times 10^6$          |
| Polystyrene                      | 2.6                   | $24 \times 10^6$          |
| Vinyl (plastic)                  | 2-4                   | $50 \times 10^6$          |
| Paper                            | 3.7                   | $15 \times 10^6$          |
| Quartz                           | 4.3                   | $8 \times 10^6$           |
| Oil                              | 4                     | $12 \times 10^6$          |

**Example 11**

The two plates of a capacitor hold  $+ 2500 \mu C$  and  $- 2500 \mu C$  of charge, respectively, when the potential difference is 960 V. What is the capacitance?

**Solution:**

$$Q = 2500 \mu C = 2500 \times 10^{-6} C$$

$$\Delta V = 960 V$$

$$C = \frac{|Q|}{\Delta V} = \frac{2500 \times 10^{-6}}{960} = 2.6 \times 10^{-6} F$$

**Exercise 14**

An 8500-pF capacitor holds plus and minus charges of  $16.5 \times 10^{-8} C$ . What is the voltage across the capacitor?

**The answer:**  $\Delta V = 19.4 V$

**Exercise 15**

A parallel Plate Capacitor has a plate area of  $0.04 m^2$  separated by a 1.8 mm thickness of paraffin. Find its capacitance. Consider:  $\kappa = 2.2$  for paraffin, and  $\epsilon_0 = 8.8542 \times 10^{-12} \frac{C^2}{N.m^2}$

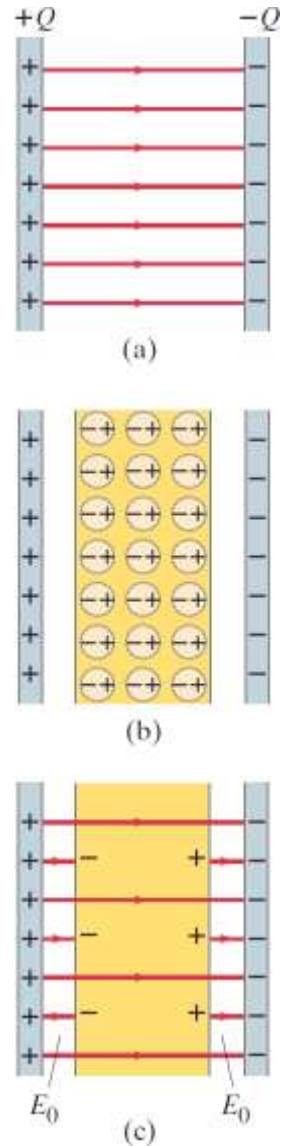
**The answer:**  $C = 43.3 \times 10^{-11} F$

## Molecular Description of Dielectrics\*\*

Let us examine, from the molecular point of view, why the capacitance of a capacitor should be larger when a dielectric is between the plates. A capacitor  $C_0$  whose plates are separated by an air gap has a charge  $+Q$  on one plate and  $-Q$  on the other (Fig. a). Assume it is isolated (not connected to a battery) so charge cannot flow to or from the plates. The potential difference between the plates  $V_0$  is given by

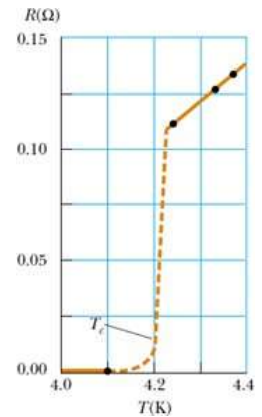
$$C_0 = \frac{Q}{V_0} \Rightarrow Q = C_0 V_0$$

where the subscripts refer to air between the plates. Now we insert a dielectric between the plates (Fig. b). Because of the electric field between the capacitor plates, the dielectric molecules will tend to become oriented as shown in Fig. b. If the dielectric molecules are polar, the positive end is attracted to the negative plate and vice versa. Even if the dielectric molecules are not polar, electrons within them will tend to move slightly toward the positive capacitor plate, so the effect is the same. The net effect of the aligned dipoles is a net negative charge on the outer edge of the dielectric facing the positive plate, and a net positive charge on the opposite side, as shown in Fig. c. Some of the electric field lines, then, do not pass through the dielectric but instead end on charges induced on the surface as shown in Fig. c. Hence the electric field within the dielectric is less than in air. That is, the electric field in the space between the capacitor plates, assumed filled by the dielectric, has been reduced by some factor  $\kappa$ . The voltage across the capacitor is reduced by the same factor  $\kappa$  because  $V = E \cdot d$  and hence  $Q = C V$ , the capacitance  $C$  must increase by that same factor  $\kappa$  to keep  $Q$  constant.



## Superconductors\*\*

- There a class of metals and compounds whose resistance decreases to zero when they are below a certain temperature  $T_c$  known as the critical temperature. These materials are known as superconductors.
- The resistance – temperature graph for a superconductor follows that of a normal metal at temperature above  $T_c$  shown in the Figure.
- When the temperature is at or below  $T_c$ , the resistivity drops suddenly to zero.
- Recent measurements have shown that the resistivities of superconductors below their  $T_c$  values are less than  $4 \times 10^{-25} \Omega \cdot m$  – around  $10^{17}$  times smaller than the resistivity of copper and in practice considered to be zero.
- Today thousands of superconductors are known, see the Table, the critical temperature of recently discovered superconductors are substantially higher than initially thought possible.
- The value of  $T_c$  is sensitive to chemical composition, pressure, and molecular structure. It is interesting to note that copper, silver, and gold, which are excellent conductors, do not exhibit superconductivity.
- One of the truly remarkable features of superconductors is that once a current is set up in them, it persists without applied potential difference (because  $R = 0$ ). Steady currents have been observed to persist in superconducting loops for several years with no apparent decay!
- An important and useful application of superconductivity is in the development of superconducting magnets, in which the magnitudes of the magnetic field are about ten times greater than those produced by the best normal electromagnets.
- A small permanent magnet levitated above a disk of the superconductor  $YBa_2Cu_3O_7$ , which is at 77 K.



| Material                                                         | $T_c(K)$ |
|------------------------------------------------------------------|----------|
| HgBa <sub>2</sub> Ca <sub>2</sub> Cu <sub>3</sub> O <sub>8</sub> | 134      |
| Tl-Ba-Ca-Cu-O                                                    | 125      |
| Bi-Sr-Ca-Cu-O                                                    | 105      |
| YBa <sub>2</sub> Cu <sub>3</sub> O <sub>7</sub>                  | 92       |
| Nb <sub>3</sub> Ge                                               | 23.2     |
| Nb <sub>3</sub> Sn                                               | 18.05    |
| Nb                                                               | 9.46     |
| Pb                                                               | 7.18     |
| Hg                                                               | 4.15     |
| Sn                                                               | 3.72     |
| Al                                                               | 1.19     |
| Zn                                                               | 0.88     |



## **The action potential in the nerve \*\***

### Introduction: NERVES

The nervous system coordinates the activity of the muscles, monitors the organs, constructs and also stops input from the senses, and initiates actions. Prominent participants in a nervous system include neurons and nerves, which play roles in such coordination. Our nervous tissue only consists of two types of cells. These cells are neurons and neuroglia cells. The neurons are responsible for transmitting nerve impulses. Neuroglia cells are responsible for supporting and nourishing the neuron cells.

### **Types of Neurons**

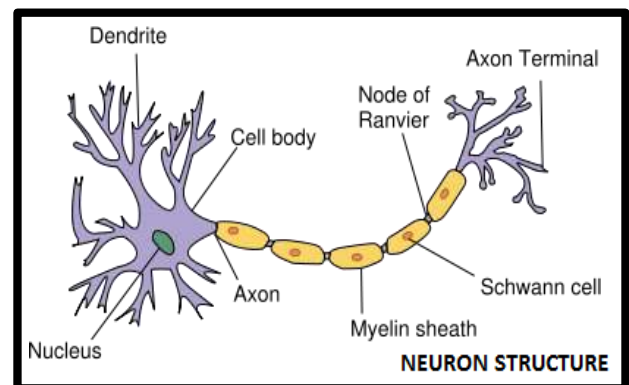
There are three types of neurons in the body. We have sensory neurons, interneurons, and motor neurons. Neurons are a major class of cells in the nervous system. Neurons are sometimes called nerve cells, though this term is technically imprecise, as many neurons do not form nerves. In vertebrates, neurons are found in the brain, the spinal cord and in the nerves and ganglia of the peripheral nervous system. Their main role is to process and transmit information. Neurons have excitable membranes, which allow them to generate and propagate electrical impulses. Sensory neuron takes nerve impulses or messages right from the sensory receptor and delivers it to the central nervous system. A sensory receptor is a structure that can find any kind of change in its surroundings or environment.

### **Structure of a neuron**

Neurons have three different parts to them. They all have an axon, a cell body and dendrites. The axon is the part of the neuron that conducts nerve impulses. Axons can get to be quite long. When an axon is present in nerves, it is called a nerve fiber. A cell body has a nucleus and it also has other organelles. The dendrites are the short pieces that come off of the cell body that receive the signals from sensory receptors and other neurons.

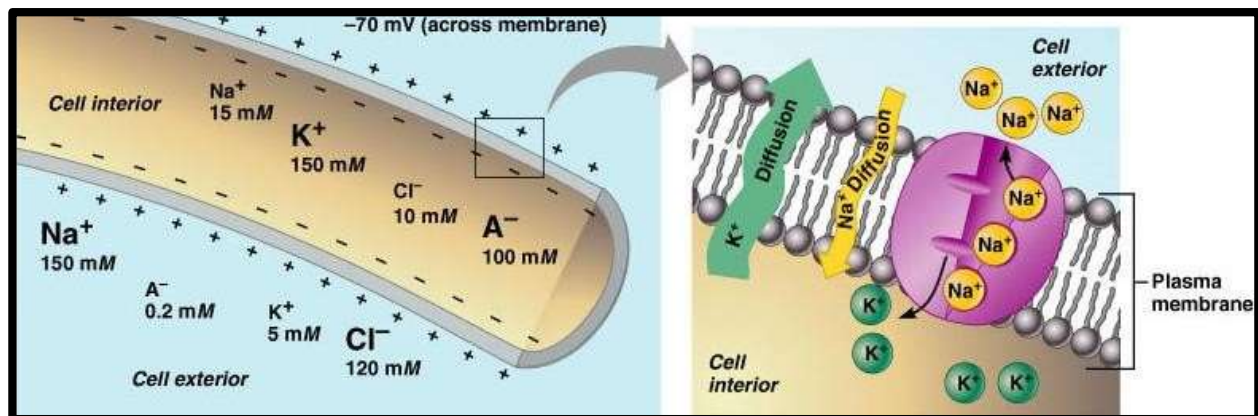
### **Myelin Sheath**

Schwann cells contain a lipid substance called myelin in their plasma membranes. When schwann cells wrap around axons, a myelin sheath forms. There are gaps that have no myelin sheath around them; these gaps are called nodes of Ranvier. Myelin sheathes make excellent insulators. Axons that are longer have a myelin sheath, while shorter axons do not.



## The resting potential

**membrane potential** - An electrical voltage across the cell membrane, due to ionic disequilibria (unequal distribution of ions), primarily the differences in sodium and potassium ion concentrations inside and outside the cell leading to a more negative cell cytoplasm and a more positive intercellular fluid outside the cell membrane when the cell is in the resting state; this ionic disequilibrium is maintained by the action of the sodium-potassium pump and consequently, by the expenditure of much ATP energy.



**Resting membrane potential**- The membrane potential of a nonconducting neuron, due to ionic disequilibria, primarily the differences in sodium and potassium concentrations inside and outside the cell because of the action of the  $\text{Na}^+/\text{K}^+$  ATPase pump; a nonconducting neuron is charged positively on the outside and negatively on the inside with a typical value of  $-70 \text{ mV}$ .

**$\text{Na}^+/\text{K}^+$  ATPase** - An integral membrane protein active transport molecule which has the capacity to bind and hydrolyze ATP and use the energy from the ATP hydrolysis to move  $3 \text{ Na}^+$  ions from the cytoplasm to the exterior of the cell while simultaneously moving  $2 \text{ K}^+$  ions from the exterior of the cell into the cytoplasm; the action of this ion pump is the main factor in establishing a resting membrane potential for the cell.

## Nernst equation

We can determine whether an ion is in equilibrium by calculating a theoretical resting potential for which there would be no net flow of that ion across the cell membrane. At this equilibrium potential difference across the cell membrane, the flows due to the concentration and potential differences are exactly balanced.

The equilibrium potential difference for an ion can be found from the Nernst equation. A potential difference across a membrane will produce a concentration imbalance on the two sides of the membrane. Equilibrium occurs when the potential energy of one ion with charge  $q$ ,  $q(V_i - V_o)$ , is equal to the work necessary to transfer it to the region of higher concentration. Hence the ion is in equilibrium if the concentrations satisfy the Nernst equation,

$$q(V_i - V_o) = k_B T \ln \frac{c_o}{c_i}$$

Where,

$q$ : is the ion charge

$V_i$ : The potential inside the axon

$V_o$ : The potential outside the axon

$k_B$ : Is the Boltzmann constant.

$T$ : Is the temperature in Kelvin

$c_o$ : Is the concentration outside the axon

$c_i$ : Is the concentration inside the axon

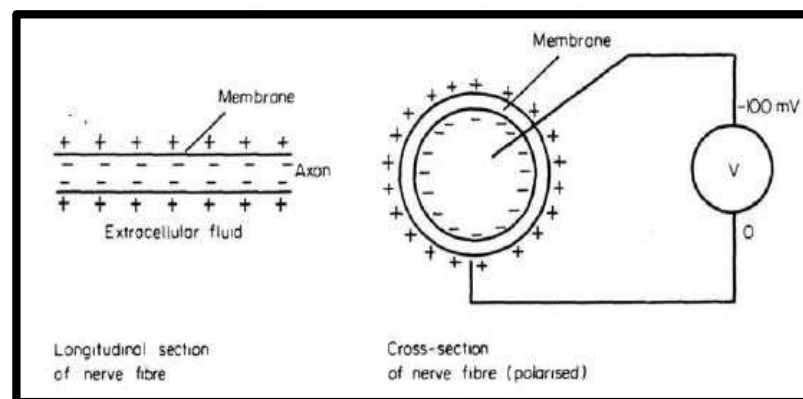
## The Capacitance and Resistance of the membrane

### Capacitance

The plasma membrane is made of a molecular lipid bilayer. Inserted in this bilayer, there are membrane proteins that have the important function of transporting materials across the membrane. The lipid bilayer acts like an insulator separating two conducting media: the external medium of the axon and the internal medium or axoplasm. This geometry constitutes an electric capacitor where the two conducting plates are the ionic media and the membrane is the dielectric. The capacitance  $c$  of a capacitor increases with the area of the plates and decreases with the separation between the plates according to the relation

$$C = k \frac{\epsilon_o A}{d}, \text{ where } A = 2\pi r \cdot L$$

where  $A$  is the membrane area,  $d$  is the separation between the plates or the membrane thickness,  $\epsilon_o$  is the permittivity constant,  $L$  is the length of the axon,  $r$  is the axon radius and  $k$  is the dielectric constant (for the membrane).



## Resistance

In the neuron, there are two substances that exhibit electrical resistance: the axoplasm itself and the cell membrane plus myelin sheath, if present.

The electrical resistance  $R$  along the length of the axon follows the same principles as a wire:

$$R = \frac{\rho \cdot L}{A} = \frac{\rho \cdot L}{\pi r^2}$$

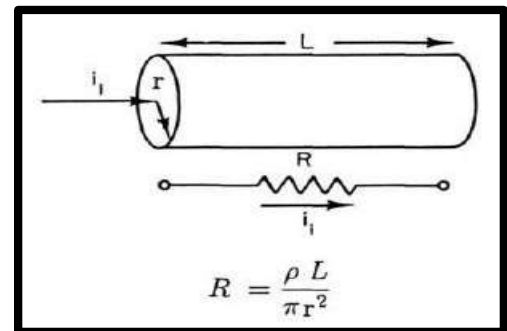
Where

$\rho$ : is the resistivity

$A$ : is the area cross section

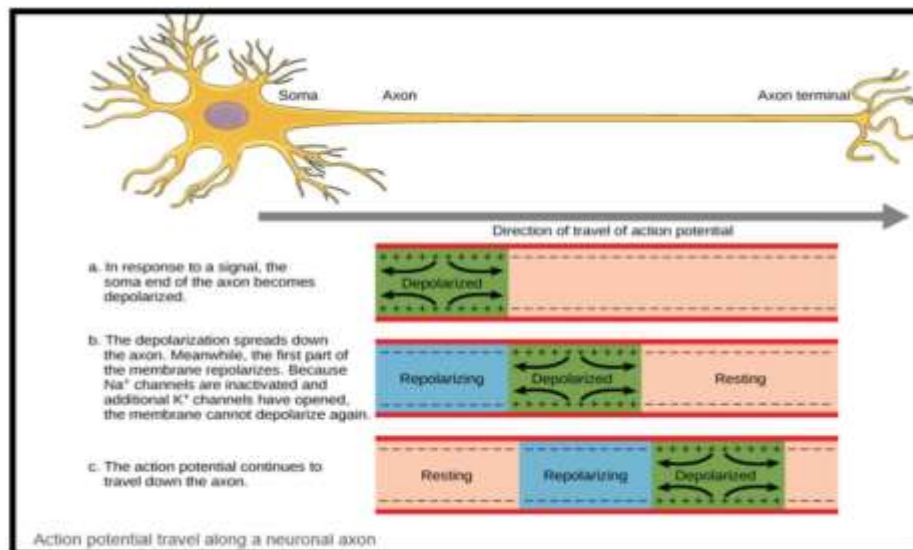
$L$ : is the length of the axon

$r$ : is the axon radius



## The Nerve Impulse

When a nerve is stimulated the resting potential changes. Examples of such stimuli are pressure, electricity, chemicals, etc. Different neurons are sensitive to different stimuli (although most can register pain). The stimulus causes sodium ion channels to open. The rapid change in polarity that moves along the nerve fiber is called the "ACTION POTENTIAL." This moving change in polarity has several stages:



## Depolarization

The upswing is caused when positively charged sodium ions ( $\text{Na}^+$ ) suddenly rush through open sodium gates into a nerve cell. The membrane potential of the stimulated cell undergoes a localized change from -65 millivolts to 0 in a limited area. As additional sodium rushes in, the membrane potential actually reverses its polarity so that the outside of the membrane is negative relative to the inside. During this change of polarity the membrane actually develops a positive value for a moment (+40 millivolts). The change in voltage stimulates the opening of additional sodium channels (called a voltage-gated ion channel).

## Repolarization

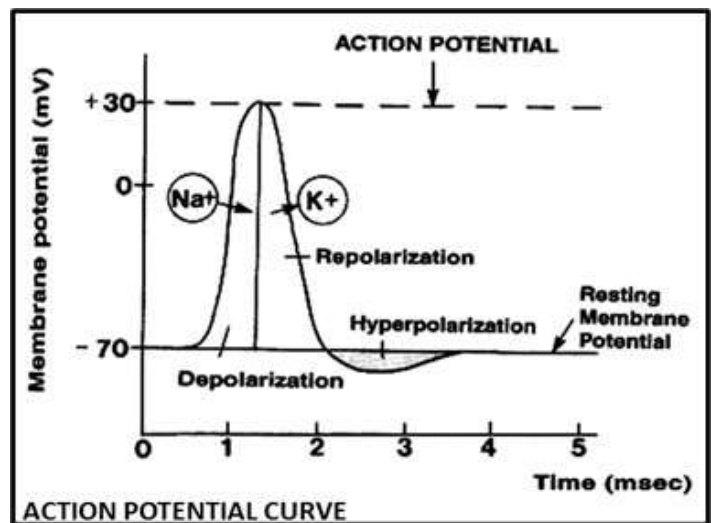
(The downswing) is caused by the closing of sodium ion channels and the opening of potassium ion channels; this will release the positively charged potassium ions ( $\text{K}^+$ ) from the nerve cell when potassium gates open. This acts to restore the localized negative membrane potential of the cell (about -65 or -70 mV is typical for nerves).

## Hyperpolarization

When stimulation causes the inside of the cell to become more negative than the rmp (rest membrane potential) the line will deflect down. This is called hyperpolarization. This can be caused by positive charges leaving the cell or negative charges entering.

## Refractory phase

Is a short period of time after the depolarization stage. Shortly after the sodium gates open they close and go into an inactive conformation. The sodium gates cannot be opened again until the membrane is repolarized to its normal resting potential. The sodium-potassium pump returns sodium ions to the outside and potassium ions to the inside. During the refractory phase this particular area of the nerve cell membrane cannot be depolarized.



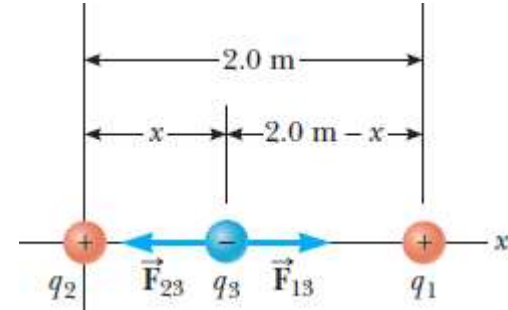
#### Exercise 4 /solution

Three charges lie along the x-axis as in the Figure below. The positive charge  $q_1 = 4 \mu\text{C}$  is at 2.0 m to the right of the origin point, and the positive charge  $q_2 = 9 \mu\text{C}$  is at the origin. Where must a negative charge  $q_3$  be placed on the x-axis so that the resultant electric force on it is zero?

#### Solution

If  $q_3$  is to the right or left of the other two charges, the net force on  $q_3$  can't be zero because then  $F_{13}$  and  $F_{23}$  act in the same direction. Consequently,  $q_3$  must lie between the two other charges. Write  $F_{13}$  and  $F_{23}$  in terms of the unknown coordinate position  $x$ , then sum them and set them equal to zero, solving for the unknown. The solution can be obtained with the quadratic formula.

The resultant force on  $q_3$  is zero when



$$F_{23} = F_{13}$$

$$9 \times 10^9 \times \frac{q_2 \times q_3}{r_{23}^2} = 9 \times 10^9 \times \frac{q_1 \times q_3}{r_{13}^2}$$

$$\frac{9 \times 10^9 \times 9 \times 10^{-6} \times q_3}{(x)^2} = \frac{9 \times 10^9 \times 4 \times 10^{-6} \times q_3}{(0.2 - x)^2}$$

$$\frac{9}{(x)^2} = \frac{4}{(0.2 - x)^2}$$

Taking the square root for both sides

$$\frac{3}{x} = \frac{2}{0.2 - x}$$

$$\text{So, } 2x = 0.6 - 3x \Rightarrow x = 0.12 \text{ m}$$

## References:

- D. C. GIANCOLI, Physics: Principles with Applications, Addison-Wesley (2014).
- R. A. SERWAY and J. W. JEWETT, Physics for Scientists and Engineers, with Modern Physics, Brooks/Cole Publishing Company (2004).

## **Solutions to Exercises**

**Chapter One:**  
**Dimensional Analysis**

### Exercise 1

Find the dimension and the SI unit of:

4. Force?
5. Work?
6. Pressure?

### Solution:

1. *Force = mass x acceleration* →

$$\text{Dimension of } (F) = M \times \frac{L}{T^2} = \frac{M \cdot L}{T^2}, \quad \text{SI unit} = \frac{Kg \cdot m}{s^2}$$

2. *Work = Force x distance* →

$$\text{Dimension of } (W) = \frac{M \cdot L}{T^2} \times L = \frac{M \cdot L^2}{T^2}, \quad \text{SI unit} = \frac{Kg \cdot m^2}{s^2}$$

3. *Pressure =  $\frac{\text{Force}}{\text{Area}}$*  →

$$\text{Dimension of } (P) = \frac{\frac{M \cdot L}{T^2}}{L^2} = \frac{M \cdot L}{T^2} \div L^2 = \frac{M \cdot L}{T^2} \times \frac{1}{L^2} = \frac{M}{L \cdot T^2},$$

$$\text{SI unit} = \frac{Kg}{m \cdot s^2}$$

### Exercise 2

Find the dimension and the SI unit of the mathematical constant  $\pi$ . Consider

$$\textit{Area of the circle} = \pi \times (\textit{radius})^2$$

**Solution:**

$$\textit{Area of the circle} = \pi \times (\textit{radius})^2$$

$$\pi = \frac{\textit{Area}}{(\textit{radius})^2} = \frac{L^2}{L^2} = 1 \quad (\text{Dimensionless quantity})$$

### Exercise 3

Find the dimension and the SI unit of the variable  $k$  in the following equation

$$\textit{Force} = \frac{k \times (\textit{velocity})^2}{\textit{radius}}$$

**Solution:**

$$\frac{\textit{Force}}{1} = \frac{k \times (\textit{velocity})^2}{\textit{radius}} \quad \rightarrow \quad k \times (\textit{velocity})^2 = \textit{Force} \times \textit{radius}$$

$$k = \frac{\textit{Force} \times \textit{radius}}{(\textit{velocity})^2} = \frac{\frac{M \cdot L}{T^2} \times L}{\left(\frac{L}{T}\right)^2} = \frac{\frac{M \cdot L^2}{T^2}}{\frac{L^2}{T^2}} = \frac{M \cdot L^2}{T^2} \times \frac{T^2}{L^2} = M$$

$$k \rightarrow \textit{dimension} = M, \quad \textit{SI unit} = Kg$$

#### Exercise 4

Suppose we are told that the hydrostatic pressure  $P$  is proportional to some power of depth  $h$ , say  $h^n$ , some power of density  $\rho$ , say  $\rho^m$ , and the gravity acceleration  $g$ . Using the dimensional analysis rules, determine the values of  $n$  and  $m$ .

(Hint: find the values of  $n$  and  $m$  in the following equation:  $P = h^n \cdot \rho^m \cdot g$ )

#### Solution:

$$P = h^n \cdot \rho^m \cdot g$$

$$\frac{M}{L \cdot T^2} = L^n \cdot \left(\frac{M}{L^3}\right)^m \cdot \frac{L}{T^2}$$

$$\frac{M}{L \cdot T^2} = L^n \cdot \frac{M^m}{L^{3m}} \cdot \frac{L}{T^2}$$

$$\frac{M}{L \cdot \cancel{T^2}} = \frac{L^n \cdot M^m \cdot L}{L^{3m} \cdot \cancel{T^2}}$$

$$M^1 \cdot L^{-1} = L^{n+1-3m} \cdot M^m$$

After equating the powers of  $L$  and  $M$

$$m = 1$$

$$n + 1 - 3m = -1 \quad \rightarrow \quad n + 1 - 3 \cdot (1) = -1 \quad \rightarrow \quad n + 1 - 3 = -1$$

$$n - 2 = -1 \quad \rightarrow \quad n = -1 + 2 \quad \rightarrow \quad n = 1$$

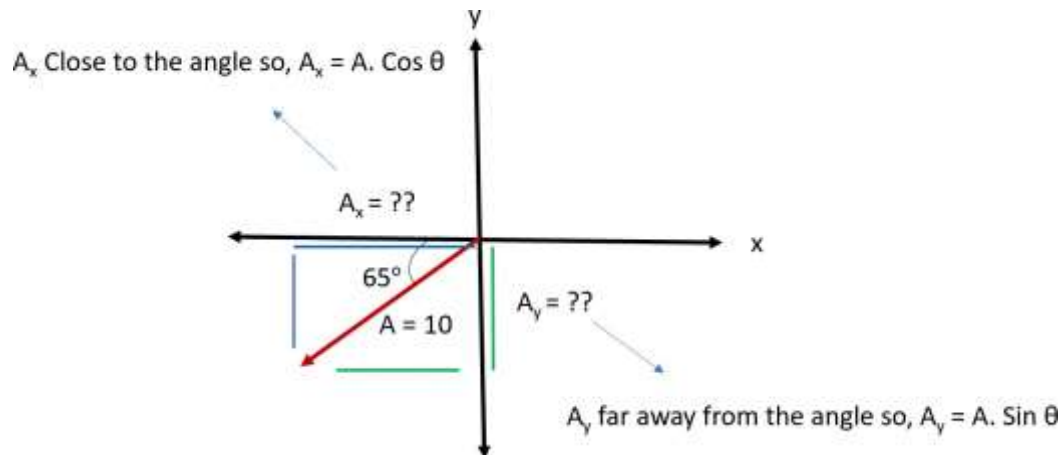
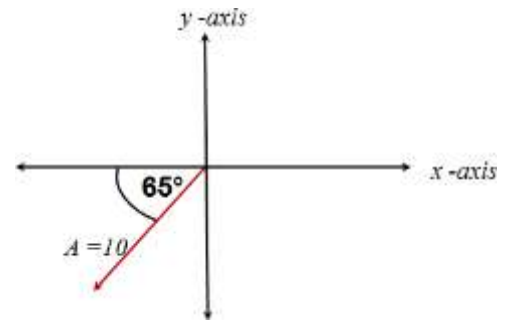
## **Chapter Two:**

### **Vectors**

### Exercise 1

Find the x and y components for vector A shown in the following figures.

**Solution:**



$$A_x = A \cdot \cos \theta = 10 \cos 65^\circ = 10 (0.423) = 4.23$$

$\rightarrow -4.2$  Because it is in the negative axis side.

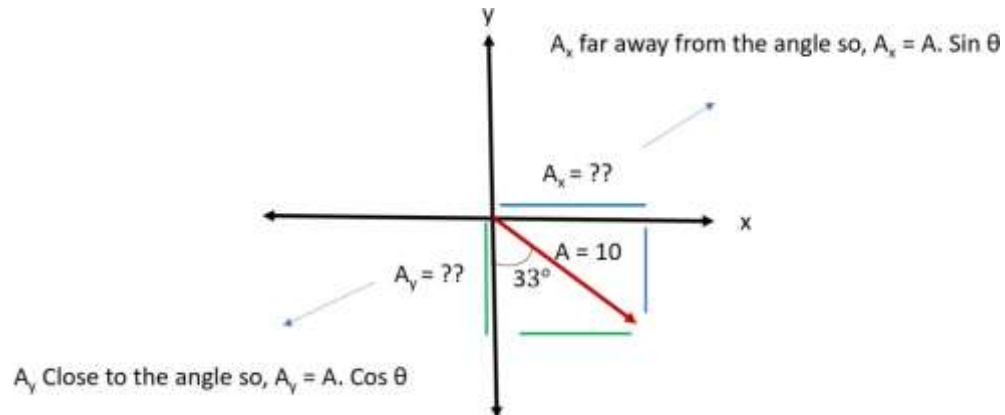
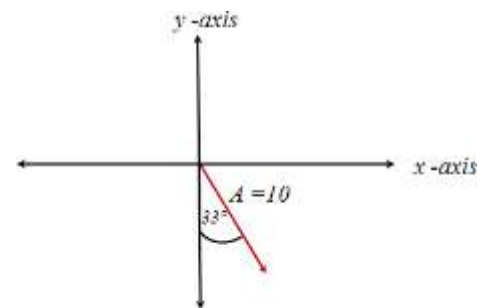
$$A_y = A \cdot \sin \theta = 10 \sin 60^\circ = 10 (0.906) = 9.06$$

$\rightarrow -9.1$  Because it is in the negative axis side.

## Exercise 2

Find the  $x$  and  $y$  components for vector  $A$  shown in the following figures.

**Solution:**



$$A_x = A \cdot \sin \theta = 10 \sin 33^\circ = 10 (0.545) = 5.45$$

$\rightarrow + 5.5$  Because it is in the positive axis side.

$$A_y = A \cdot \cos \theta = 10 \cos 33^\circ = 10 (0.839) = 8.39$$

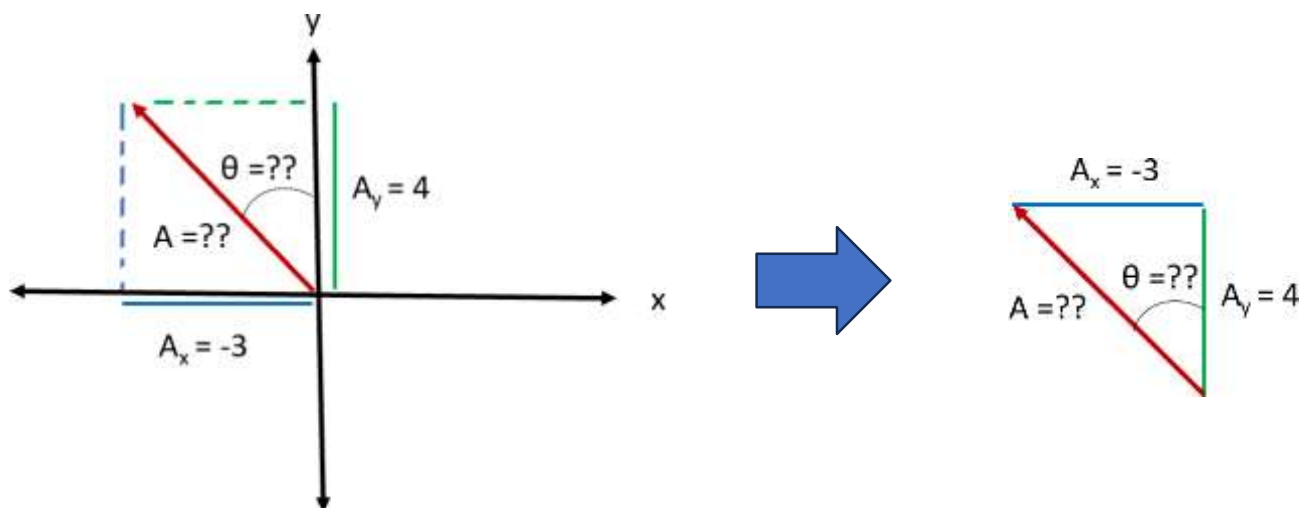
$\rightarrow - 8.4$  Because it is in the negative axis side.

### Exercise 3

Find the magnitude and the direction (the angle with respect to +y axis) for vector A, if the components of this vector are:  $A_x = -3$  and  $A_y = 4$

#### Solution:

We have  $A_x = -3$  and  $A_y = 4$ , let us find 1. The magnitude of A, and 2. The direction of A (the angle  $\theta$  with respect to x-axis)



#### 1. ( $A = ??$ )

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(-3)^2 + 4^2} = 5$$

#### 2. ( $\theta = ??$ )

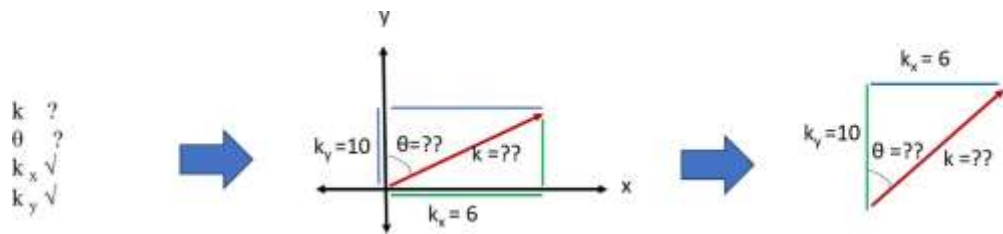
$$\theta_y = \tan^{-1} \left( \frac{|A_x|}{|A_y|} \right) = \tan^{-1} \left( \frac{3}{4} \right) = 36.9^\circ$$

#### Exercise 4\*\*

Find the magnitude and the direction (the angle with respect to  $y$ -axis) for vector  $K$ , where  $K$  is given by:

$$\vec{K} = 6\hat{i} + 10\hat{j}$$

**Solution:**



1. The magnitude ( $k = ??$ )

$$k = \sqrt{k_x^2 + k_y^2} = \sqrt{6^2 + 10^2} = 11.7$$

2. The direction of  $k$  ( the angle with respect to  $y$  axis) ( $\theta = ??$ )

$$\tan \theta = \frac{k_x}{k_y}$$

So,

$$\theta = \tan^{-1} \frac{k_x}{k_y}$$

$$\theta_y = \tan^{-1} \left( \frac{|k_x|}{|k_y|} \right) = \tan^{-1} \left( \frac{6}{10} \right) = 31^\circ$$

**Chapter Three:**  
**Kinematics in One and Two Dimensions**

### Exercise 1

In an electron gun, an electron is accelerated uniformly from rest to a velocity of  $4 \times 10^7$  m/s in a distance of 0.10 m. Calculate the acceleration.

#### Solution:

$$v_0 = 0, \quad v = 4 \times 10^7 \text{ m/s}, \quad a = ??, \quad x = 0.1 \text{ m}$$

$$v^2 = v_0^2 + 2 a x$$

$$(4 \times 10^7)^2 = (0)^2 + 2 \times a \times 0.1$$

$$(4 \times 10^7)^2 = a \times 0.2$$

$$16 \times 10^{14} = a \times 0.2$$

$$a = \frac{16 \times 10^{14}}{0.2} = 8 \times 10^{15} \text{ m/s}^2$$

### Exercise 2

A car slows down from 28 m/s to rest in a distance of 88 m. What was its acceleration, assumed constant?

#### Solution

$$v_0 = 28 \text{ m/s}, \quad v = 0, \quad a = ??, \quad x = 88 \text{ m}$$

$$v^2 = v_0^2 + 2 a x$$

$$0 = (28)^2 + 2 \times a \times 88$$

$$0 = 784 + a \times 176$$

$$-784 = a \times 176$$

$$a = \frac{-784}{176} = -4.45 = -4.5 \text{ m/s}^2$$

### Exercise 3

A car accelerates from 14 m/s to 21 m/s in 6 s. What was its acceleration? How far did it travel in this time? Assume constant acceleration.

#### Solution:

$$v_0 = 14 \text{ m/s}, \quad v = 21 \text{ m/s}, \quad t = 6 \text{ s}, \quad a = ??, \quad x = ??$$

$$a = ??$$

$$v = v_0 + a t$$

$$21 = 14 + a \cdot (6)$$

$$7 = 6 a$$

$$a = \frac{7}{6} = 1.166 = 1.2 \text{ m/s}^2$$

$$x = ??$$

$$v^2 = v_0^2 + 2 a x$$

$$21^2 = 14^2 + 2 \cdot (1.2) \cdot x$$

$$441 = 196 + 2.4 \cdot x$$

$$245 = 2.4 \cdot x$$

$$x = \frac{245}{2.4} = 102.1 = 102 \text{ m}$$

#### Exercise 4

A ball is dropped from a cliff top and takes 3s to reach the ground below. Calculate (a) the height of the cliff and (b) the ball's velocity before it reaches the ground. *Consider  $g = 9.8 \text{ m/s}^2$ .*

#### Solution:

We have  $v_0 = 0$  (the object dropped),

$$t = 3\text{s}$$

$$g = 9.8 \text{ m/s}^2$$

$$(a) \ y = ?$$

$$y = v_0 t - \frac{1}{2} g t^2$$

$$y = 0 \cdot (t) - \frac{1}{2} (9.8) \cdot 3^2$$

$$y = -44.1 \text{ m (below the reference line to negative direction)}$$

$$(b) \ v = ?$$

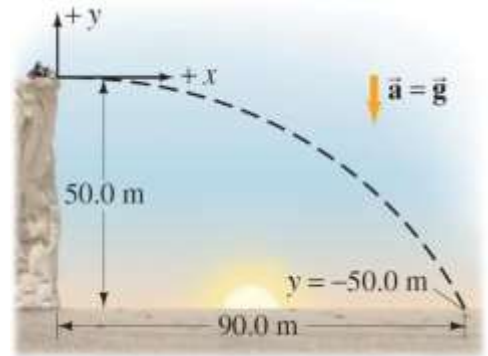
$$v = v_0 - g t$$

$$v = 0 - (9.8) 3$$

$$v = -29.4 \text{ m/s (negative sign indicates the direction to negative y axis)}$$

### Exercise 5 (Projectile Motion)\*\*

A movie stunt driver on a motorcycle speeds horizontally off a 50 m-high cliff. How fast must the motorcycle leave the cliff top to land on level ground below, 90 m from the base of the cliff where the cameras are? Consider  $g = 9.8 \text{ m/s}^2$



**Solution:**

We have

| Known                                | Unknown      |
|--------------------------------------|--------------|
| $v_{y0} = 0$ ( <i>horizontally</i> ) | time (t)     |
| $a_x = 0$                            | $v_{x0} = ?$ |
| $y = -50 \text{ m}$                  |              |
| $x = 90 \text{ m}$                   |              |
| $a_y = -g$                           |              |
| $g = 9.8 \text{ m/s}^2$              |              |

| Horizontal Motion<br>( $a_x = 0$ , $v_x = \text{constant}$ ) | Vertical Motion<br>( $a_y = -g = \text{constant}$ ) |
|--------------------------------------------------------------|-----------------------------------------------------|
| $v_x = v_{x0}$                                               | $v_y = v_{y0} - gt$                                 |
| $x = v_{x0}t$                                                | $y = v_{y0}t - \frac{1}{2}gt^2$                     |
|                                                              | $v_y^2 = v_{y0}^2 - 2gy$                            |

$$x = v_{x0} t$$

$x = 90 \text{ m}$ . So, to find  $v_{x0}$  we must calculate  $t$ .

From this equation, we can calculate the time

$$y = v_{y0} t - \frac{1}{2} g t^2$$

$$-50 = 0.(t)t - \frac{1}{2} \cdot (9.8) \cdot t^2$$

$$-50 = -4.9.t^2$$

$$t^2 = \frac{-50}{-4.9} = 10.2$$

$$t = \sqrt{10.2} = 3.19 \text{ s}$$

Then,

$$x = v_{x0} t$$

$$90 = v_{x0} \times 3.2$$

$$v_{x0} = \frac{90}{3.19} = 28.2 \text{ m/s}$$

### Exercise 6 (Projectile Motion)\*\*

A shell is fired from a gun with a velocity of 600 m/s at an angle of  $40^\circ$  to the ground (to the horizontal). Calculate (a) the time of flight, (b) the range, (c) the maximum height reached.

Consider  $g = 9.8 \text{ m/s}^2$

#### Solution:

The initial velocity components

$$v_{x0} = v_0 \cos \theta = 600 \times \cos 40 = 460 \text{ m/s}$$

$$v_{y0} = v_0 \sin \theta = 600 \times \sin 40 = 386 \text{ m/s}$$

We have

| Known                                        | Unknown   |
|----------------------------------------------|-----------|
| $v_{x0} = 460 \text{ m/s} = \text{constant}$ | Time (t)  |
| $v_{y0} = 386 \text{ m/s}$                   | x (range) |
| $a_x = 0$                                    | y         |
| $a_y = -g$                                   |           |
| $g = 9.8 \text{ m/s}^2$                      |           |

| Horizontal Motion<br>( $a_x = 0$ , $v_x = \text{constant}$ ) | Vertical Motion<br>( $a_y = -g = \text{constant}$ ) |
|--------------------------------------------------------------|-----------------------------------------------------|
| $v_x = v_{x0}$                                               | $v_y = v_{y0} - gt$                                 |
| $x = v_{x0}t$                                                | $y = v_{y0}t - \frac{1}{2}gt^2$                     |
|                                                              | $v_y^2 = v_{y0}^2 - 2gy$                            |

(a) The time of flight (finding t when  $y = 0$ )

$$y = v_{y0}t - \frac{1}{2}gt^2$$

$$0 = 386t - \frac{1}{2} \times 9.8 \times t^2$$

$$386t - 4.9 \times t^2 = 0$$

$$(386 - 4.9 t) t = 0$$

$$t = 0 \text{ ( at the beginning )}$$

$$386 - 4.9 t = 0$$

$$386 = 4.9 t$$

$$t = \frac{386}{4.9} = 78.8 \text{ s}$$

(b) The total distance traveled in the x direction (the range)

$$x = v_{x0} t = 460 \times 78.8 = 36248 \text{ m}$$

(c) The maximum height

$$y = ?? \text{ ( the maximum y is when } v_y = 0 \text{ )}$$

$$v_y^2 = v_{y0}^2 - 2 g y$$

$$0 = 386^2 - 2 \times 9.8 \times y$$

$$-148996 = -19.6 \times y$$

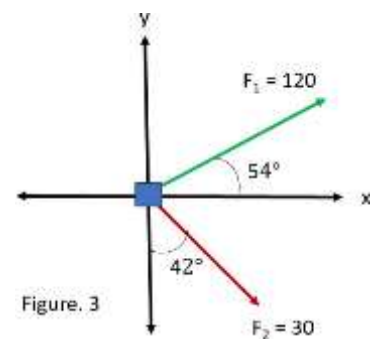
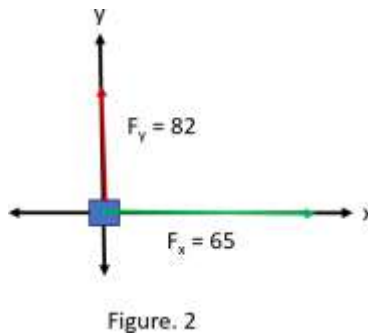
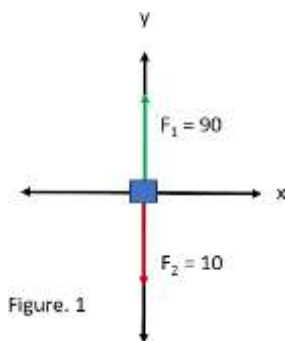
$$y = \frac{-148996}{-19.6} = 7602 \text{ m}$$

**Chapter Four:**  
**Dynamics: Newton's Laws of Motion**

### Exercise 1

For the following system of forces shown in each figure below, find

- (a) The resultant (net) force on the x-axis  $\sum F_x = ??$ ,
- (b) The resultant (net) force on the y-axis  $\sum F_y = ??$ , and
- (c) The magnitude and the direction of the resultant (net) force  $\sum \vec{F} = ??$



### Solution

#### Figure 1

$$\sum F_x = 0$$

$$\sum F_y = F_1 - F_2 = 90 - 10 = 80 \text{ N to the } + y \text{ axis direction ( to the } + y \text{ axis )}$$

$$\sum \vec{F} = 80 \text{ N ( to the } + y \text{ axis )}$$

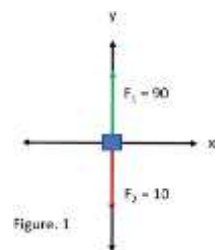
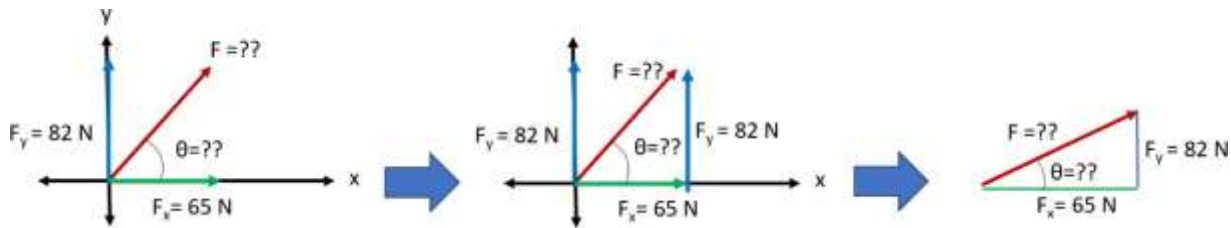


**Figure 2**

The resultant (net) force of these perpendicular forces is a force lying between them, making an angle  $\theta$  with respect to the x-axis (or y-axis).

(like examples 3 and 4 in chapter 2; vectors)



$$\sum F_x = 65 \text{ N}$$

$$\sum F_y = 82 \text{ N}$$

$$\sum \vec{F} = \sqrt{(F_x)^2 + (F_y)^2} = \sqrt{(65)^2 + (82)^2} = 104.6 \text{ N}$$

The direction of  $F$  ( the angle with respect to x-axis) ( $\theta = ??$ )

$$\tan \theta = \frac{F_y}{F_x}$$

So,

$$\theta = \tan^{-1} \frac{F_y}{F_x}$$

$$\theta_x = \tan^{-1} \left( \frac{82}{65} \right) = 51.6^\circ$$

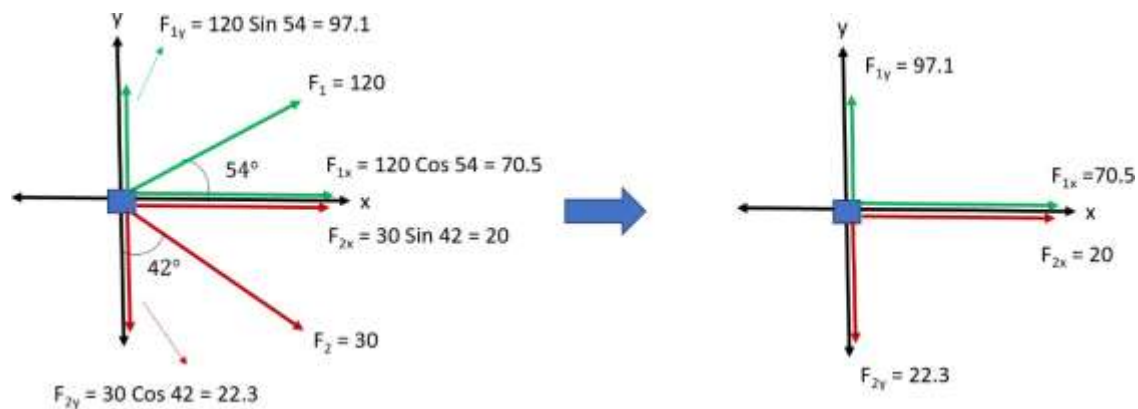
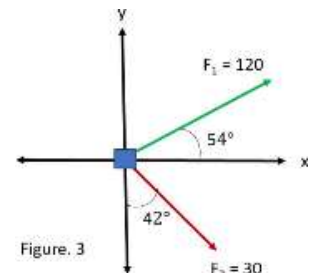
**Figure 3**

For such a system, we must

1. Analyze each vector (find the x and the y components for each vector.
2. Find the resultant force on each axis

( $\sum F_x = ??$ , and  $\sum F_y = ??$ )

3. Find the resultant (net) force of the perpendicular forces.  $\sum \vec{F} = ??$



The x and the y components for each vector are

$$F_{1x} = F_1 \cos 54 = 120 \cdot \cos 54 = 70.5 \text{ (to the } + x \text{ axis)}$$

$$F_{1y} = F_1 \sin 54 = 120 \cdot \sin 54 = 97.1 \text{ (to the } + y \text{ axis)}$$

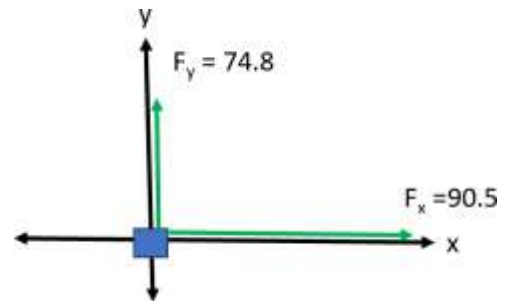
$$F_{2x} = F_2 \sin 42 = 30 \cdot \sin 42 = 20 \text{ (to the } + x \text{ axis)}$$

$$F_{2y} = F_2 \cos 42 = 30 \cdot \cos 42 = 22.3 \text{ (to the } - y \text{ axis)}$$

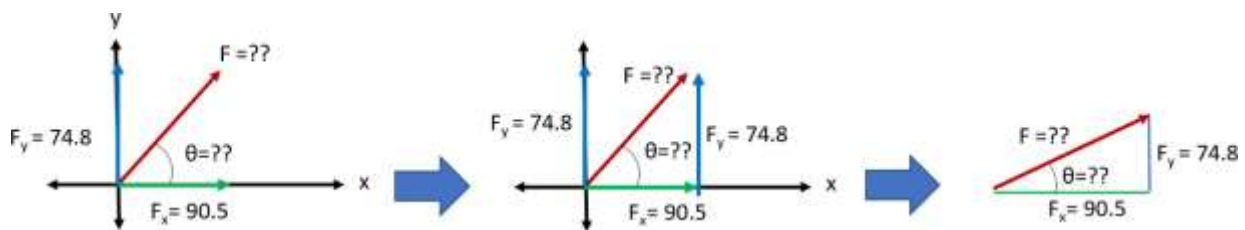
Now; the net forces on the x and y axis

$$\sum F_x = 70.5 + 20 = 90.5 \text{ N to the } +x \text{ axis}$$

$$\sum F_y = 97.1 - 22.3 = 74.8 \text{ N to the } +y \text{ axis}$$



The resultant (net) force  $\sum \vec{F}$  of these perpendicular forces is a force lying between them, making an angle  $\theta$  with respect to the x-axis (or y-axis).



$$\sum \vec{F} = \sqrt{(F_x)^2 + (F_y)^2} = \sqrt{(90.5)^2 + (74.8)^2} = 117.4 \text{ N}$$

The direction of  $F$  ( the angle with respect to x-axis) ( $\theta = ??$ )

$$\tan \theta = \frac{F_y}{F_x}$$

So,

$$\theta = \tan^{-1} \frac{F_y}{F_x}$$

$$\theta_x = \tan^{-1} \left( \frac{74.8}{90.5} \right) = 39.6^\circ$$

### Exercise 2

What net force is needed to give a 1000 kg car an acceleration of  $3 \text{ m/s}^2$ ?

**Solution:**

$$\sum F_x = m \cdot a_x$$

$$\sum F_x = 1000 \times 3 = 3000 \text{ N}$$

### Exercise 3

An elevator has a mass of 800 kg. It accelerates upward at  $5 \text{ m/s}^2$ . What is the force tension  $T$  exerted by the cable on the elevator? Consider  $g = 9.8 \text{ m/s}^2$

**Solution:**

For the vertical motion

$$\sum F_y = m \cdot a_y$$

The net force on the y-axis  $\sum F_y = T - W$

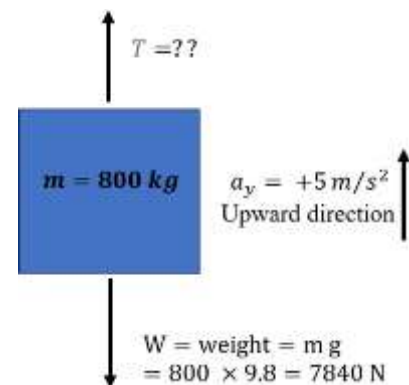
So

$$\sum F_y = m \cdot a_y$$

$$T - W = m \cdot a_y$$

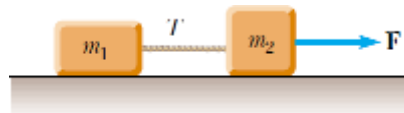
$$T - 7840 = 800 \times 5$$

$$T = 4000 + 7840 = 11840 \text{ N}$$



#### Exercise 4

Two blocks connected by a rope of negligible mass are being dragged by a horizontal force  $F$  as shown in the Figure below. Suppose that  $F = 68\text{ N}$ ,  $m_1 = 12\text{ kg}$ , and  $m_2 = 18\text{ kg}$ . (a) Draw a free-body diagram for each block. (b) Determine the tension  $T$  and the magnitude of the acceleration of the system.

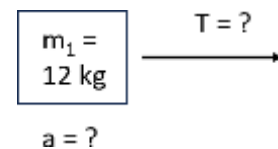


#### Solution:

For the first mass  $m_1$

$$\sum F_x = m \cdot a_x$$

$$T = 12 \cdot a_x \dots\dots\dots (1)$$



For  $m_2$

$$\sum F_x = m \cdot a_x$$

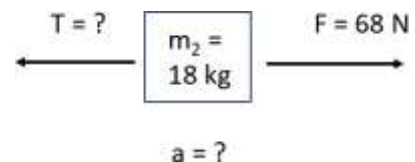
$$F - T = 18 \cdot a_x$$

$$68 - T = 18 \cdot a_x \dots\dots\dots (2)$$

Now, substituting equation 1 in 2

$$68 - (12 a_x) = 18 \cdot a_x$$

$$68 = 18 \cdot a_x + 12 a_x$$



$$68 = 30 \cdot a_x$$

$$a_x = \frac{68}{30} = 2.27 \text{ m/s}^2$$

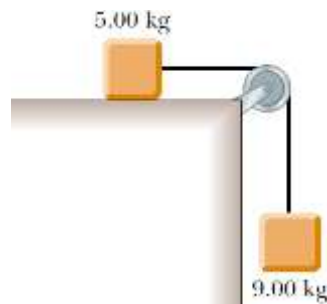
To find  $T$ , we can use equation 1 or equation 2. Using equation 1

$$T = 12 \cdot a_x$$

$$T = 12 \times 2.27 = 27.24 \text{ N}$$

### Exercise 5

A 5 kg object placed on a frictionless, horizontal table is connected to a string that passes over a pulley and then is fastened to a hanging 9 kg object, as shown in the Figure below. Draw free-body diagrams of both objects. Find the acceleration of the two objects and the tension in the string. Consider  $g = 9.8 \text{ m/s}^2$

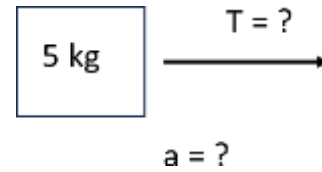


**Solution:**

For the first mass ( $5\text{kg}$ )

$$\sum F = m \cdot a$$

$$T = 5 a \quad \dots\dots\dots(1)$$

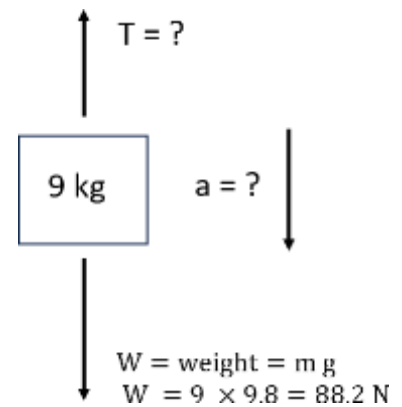


For the second mass ( $9\text{kg}$ )

$$\sum F = m \cdot a$$

$$T - W = m \cdot a$$

$$T - 88.2 = -9 \cdot a \quad \dots\dots\dots(2)$$



Now, substituting equation 1 in 2

$$T - 88.2 = -9 \cdot a$$

$$5 a - 88.2 = -9 \cdot a$$

$$-88.2 = -14 a$$

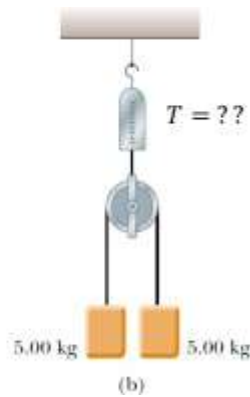
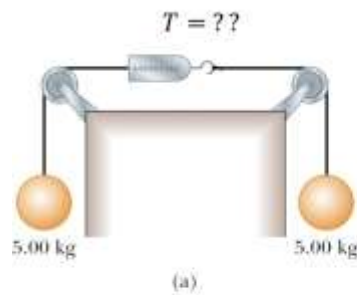
$$a = \frac{-88.2}{-14} = 6.3 \text{ m/s}^2$$

To find  $T$ , we can use equation 1 or equation 2. Using equation 1

$$T = 5 a = 5 \times 6.3 = 31.5 \text{ N}$$

### Exercise 6

The systems shown in the Figure below are in equilibrium. If the spring scales are calibrated in newtons, what do they read? (Find the tension  $T$  in each Figure). Consider  $g = 9.8 \text{ m/s}^2$ .



**Solution:**

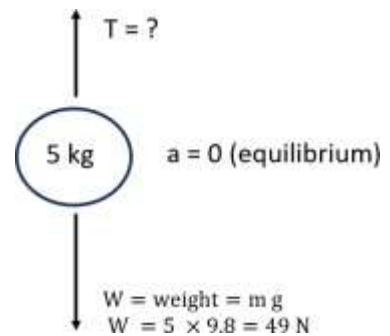
*a. Isolate either mass*

$$\sum F = m \cdot a$$

$$T - W = m \cdot a$$

$$T - 49 = 0$$

$$T = 49 \text{ N}$$



*b. Isolate the pulley*

$$T = 2 T_1$$

But, to find  $T_1$

$$\sum F = m \cdot a$$

$$T_1 - W = m a$$

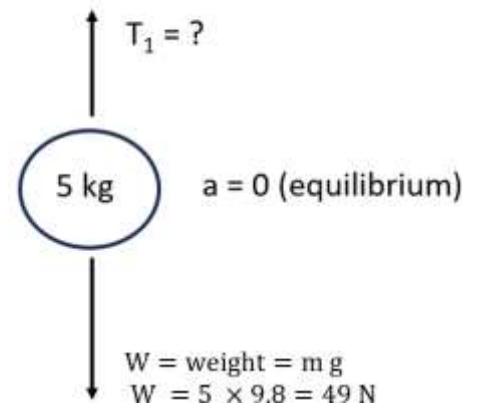
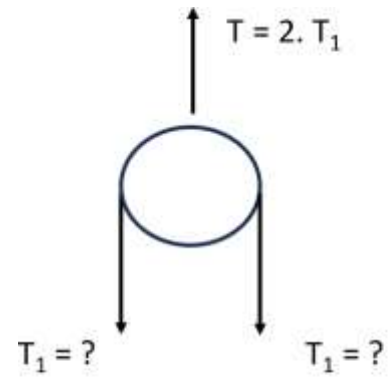
$$T_1 - 49 = 0$$

$$T_1 = 49 \text{ N}$$

So,

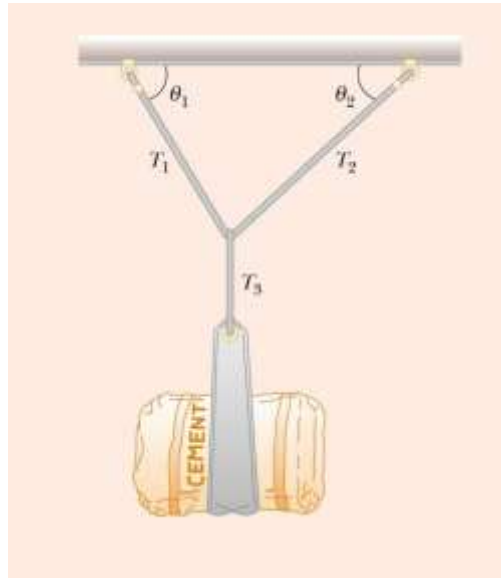
$$T = 2 T_1$$

$$T = 98 \text{ N}$$



### Exercise 7

A bag of cement of weight 325 N hangs from three wires as suggested in the Figure below. Two of the wires make angles  $\theta_1 = 60^\circ$  and  $\theta_2 = 25^\circ$  with the horizontal. If the system is in equilibrium, find the tensions  $T_1, T_2$ , and  $T_3$  in the wires.



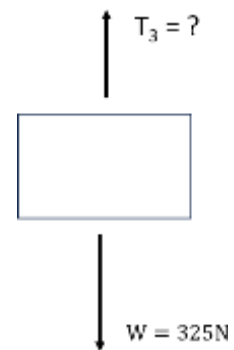
### Solution:

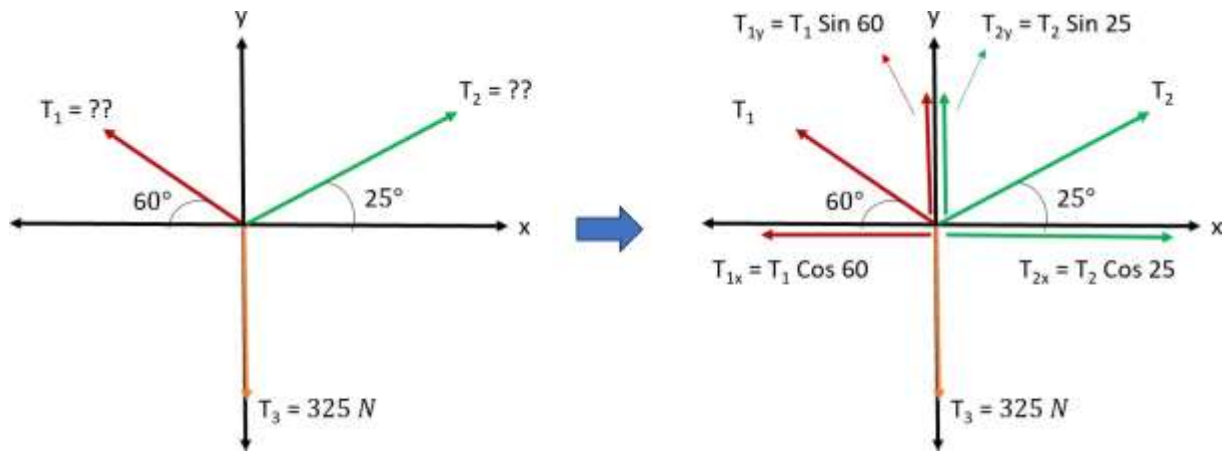
$$\sum F = m \cdot a$$

$$T_3 - W = 0$$

$$T_3 = W$$

$$T_3 = 325\text{N}$$





Apply the conditions for equilibrium to the knot, using the components in the table:

$$\sum F_x = 0 \quad \text{and} \quad \sum F_y = 0$$

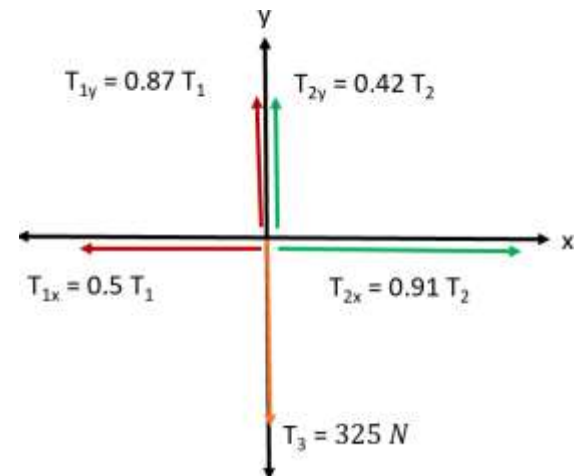
The first equilibrium condition  $\sum F_x = 0$

$$\sum F_x = T_2 \cos 25^\circ - T_1 \cos 60^\circ = 0$$

$$T_2 \cos 25^\circ = T_1 \cos 60^\circ$$

$$0.91 T_2 = 0.5 T_1$$

$$T_2 = \left( \frac{0.5}{0.91} \right) T_1 = 0.55 T_1 \quad \dots \dots \dots (1)$$



The second equilibrium condition  $\sum F_y = 0$

$$\sum F_y = T_1 \sin 60^\circ + T_2 \sin 25^\circ - 325 = 0$$

$$0.87 T_1 + 0.42 T_2 - 325 = 0 \dots\dots\dots(2)$$

Substitute the result for  $T_2$  (equation 1) into Equation (2):

$$0.87 T_1 + 0.42 T_2 - 325 = 0. \text{ So}$$

$$0.87 T_1 + 0.42 \cdot (0.55 T_1) - 325 = 0$$

$$0.87 T_1 + 0.23 T_1 = 325, \quad \text{so} \quad 1.1 T_1 = 325, \quad \text{so} \quad T_1 = \frac{325}{1.1}$$

$$T_1 = 295 \text{ N}$$

Using equation 1 to find  $T_2$

$$T_2 = 0.55 T_1 = 0.55 \times 295 = 162 \text{ N}$$

## **Chapter Five:**

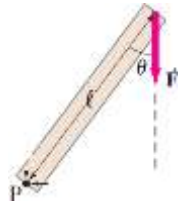
## **Biomechanics**

### Exercise 1

Consider a force  $F = 80\text{ N}$  applied to a beam as shown in the figure below. The length of the beam is  $l = 5\text{ m}$ , and  $\theta = 37^\circ$ . Calculate the torque due to the force  $F$  on the beam.

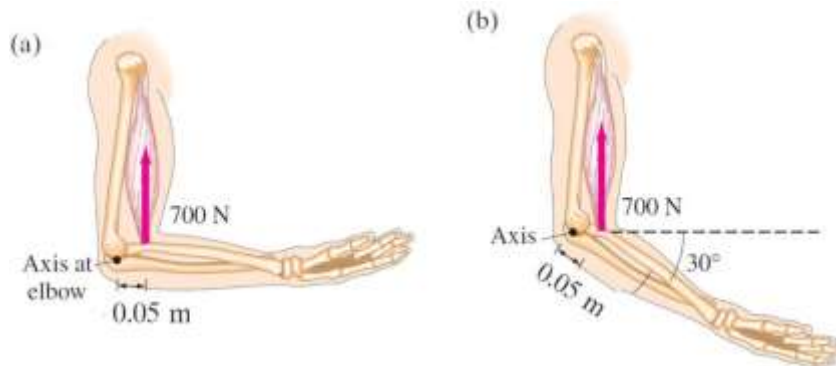
### Solution

$$\tau = r \cdot F \cdot \sin \theta = 80 \times 5 \times \sin 37 = 241\text{ N.m in c.c.w}$$



### Exercise 2

The biceps muscle exerts a vertical force on the lower arm, bent as shown in the Figure below. For each case, calculate the torque about the axis of rotation through the elbow joint, assuming the muscle is attached  $0.05\text{ m}$  from the elbow as shown.



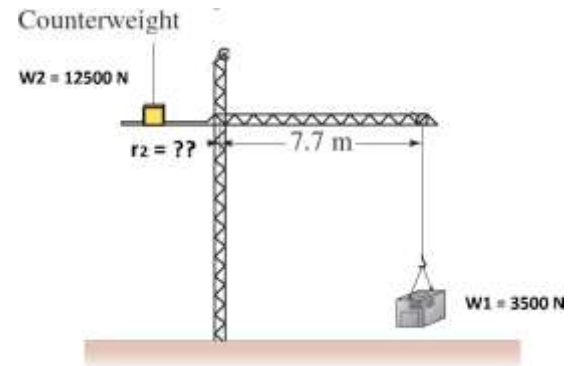
### Solution

$$a) \tau = r \cdot F \cdot \sin \theta = 700 \times 0.05 \times \sin 90 = 35\text{ N.m in c.c.w}$$

$$b) \tau = r \cdot F \cdot \sin \theta = 700 \times 0.05 \times \sin 120 = 30.3\text{ N.m in c.c.w}$$

### Exercise 3

A tower crane (Figure below) must always be carefully balanced so that there is no net torque tending to tip it. A particular crane at a building site is about to lift a 3500 N weight. Where must the 12500 N crane's counterweight be placed when the load is lifted from the ground?



### Solution

Because the crane satisfies the rotational equilibrium, then

$$\tau_1 = r_1 \cdot w_1 \cdot \sin \theta = 7.7 \times 3500 \times \sin 90 = 26950 \text{ N.m} \quad \text{in c. w}$$

$$\tau_2 = r_2 \cdot w_2 \cdot \sin \theta = r_2 \times 12500 \times \sin 90 = r_2 \cdot 12500 \text{ N.m} \quad \text{in c. c. w}$$

$$\tau_{cw} = \tau_{ccw}$$

$$26950 = r_2 \times 12500$$

$$r_2 = \frac{26950}{12500} = 2.2 \text{ m}$$

#### Exercise 4

If the system of forces applied to the object shown in the figure below satisfies the translational equilibrium, calculate the value of  $F_4$ .

#### Solution



$$\sum \vec{F} = 0, \text{ or}$$

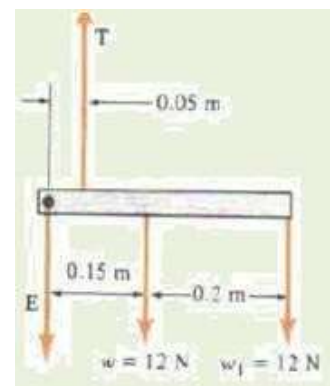
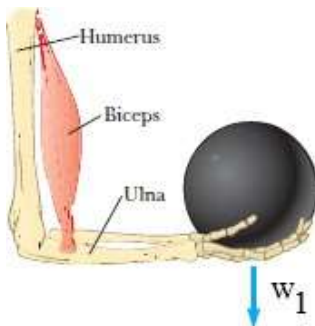
$$\sum \vec{F} \text{ in one direction} = \sum \vec{F} \text{ in opposite direction}$$

$$F_1 + F_4 = F_2 + F_3$$

$$50 + F_4 = 150 + 350 \quad \Rightarrow \quad F_4 = 450 \text{ N}$$

#### Exercise 5

A model for a forearm satisfies the equilibrium conditions as shown in the figure below. If the weight  $w$  of the forearm is 12 N and the weight of the ball  $w_1$  is 12 N. Find the tension  $T$  exerted by the biceps muscle and the force  $E$  exerted by the elbow joint.



### Solution

The forearm satisfies the rotational equilibrium and the translational equilibrium.

**For the rotational equilibrium condition.** First, calculate the torque due to each force

$$\tau_T = r_T \cdot T \cdot \sin \theta = 0.05 \times T \times \sin 90 = 0.05 T \quad \text{in c. c. w}$$

$$\tau_w = r_w \cdot w \cdot \sin \theta = 0.15 \times 12 \times \sin 90 = 1.8 \text{ N.m} \quad \text{in c. w}$$

$$\tau_{w1} = r_{w1} \cdot w1 \cdot \sin \theta = 0.35 \times 12 \times \sin 90 = 4.2 \text{ N.m} \quad \text{in c. w}$$

$$\tau_E = r_E \cdot E \cdot \sin \theta = 0 \times E \times \sin 90 = 0$$

$$\sum \tau_{cw} = \sum \tau_{ccw}$$

$$1.8 + 4.2 = 0.05 \times T$$

$$6 = 0.05 \times T \Rightarrow T = \frac{6}{0.05} = 120 \text{ N}$$

**For the translational equilibrium condition**

$$\sum \vec{F} \text{ in one direction} = \sum \vec{F} \text{ in opposite direction}$$

$$T = E + w + w_1$$

$$120 = E + 12 + 12$$

$$E = 96 \text{ N}$$

**Chapter Six:**  
**Fluid Dynamics**

**Exercise 1**

A hose delivers  $2 \text{ m}^3/\text{s}$  of water. The area cross-section of the hose is  $0.02 \text{ m}^2$ . What is the velocity of the water in the hose?

**Solution**

$$Q = A \times v$$

$$2 = 0.02 \times v$$

$$v = \frac{2}{0.02} = 100 \text{ m/s}$$

**Exercise 2**

The area cross-section of a water pipe decreases from  $0.4 \text{ m}^2$  to  $0.1 \text{ m}^2$ . If the velocity in the narrower portion is  $6 \text{ m/s}$ . Find the velocity in the wider region.

**The answer:**

$$Q_1 = Q_2$$

$$0.4 \times v_1 = 0.1 \times 6$$

$$v_1 = \frac{0.1 \times 6}{0.4} = 1.5 \text{ m/s}$$

### Exercise 3

A blood vessel splits into four vessels, each with  $1 \times 10^{-4} \text{ m}^2$  area cross-section. If the velocity of the blood in the large vessel is  $1.5 \text{ m/s}$  and in each small vessel is  $1 \text{ m/s}$ , find the area cross-section of the large vessel.

**Solution:**

$$Q = N \times Q_1$$

$$A \times v = N \times A_1 \times v_1$$

$$A \times 1.5 = 4 \times 1 \times 10^{-4} \times 1$$

$$A = \frac{4 \times 10^{-4}}{1.5} = 2.7 \times 10^{-4} \text{ m}^2$$

### Exercise 4

A blood vessel splits into three vessels. If the volume flow rate in each small vessel is  $2 \times 10^{-4} \text{ m}^3/\text{s}$ , what is the volume flow rate in the large vessel?

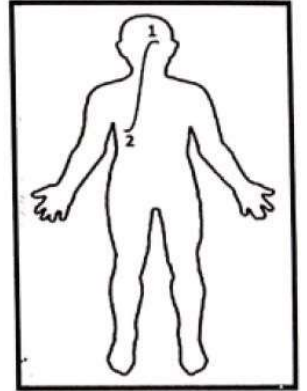
**Solution:**

$$Q = N \times Q_1$$

$$Q = 3 \times 2 \times 10^{-4} = 6 \times 10^{-4} \text{ m}^3/\text{s}$$

### Exercise 5

Blood flows through an S-shaped artery. At one end (position 1 in the figure) the blood in the artery has a pressure of  $4 \times 10^5$  Pa a speed of 10 m/s and a height of 1.8 m above the earth's surface. At the other end (position 2 in the figure) if the speed of the blood is 10.2 m/s and the pressure is  $4.1 \times 10^5$  Pa. What is the height of the second point? Consider  $g$  is  $10 \text{ m/s}^2$  and the density of the blood is  $1025 \text{ kg/m}^3$ . (Assume that the blood acts as an ideal fluid)



### Solution:

$$p + \rho gh + \frac{1}{2} \rho v^2 = \text{Constant}$$

So,

$$p_1 + \rho gh_1 + \frac{1}{2} \rho v_1^2 = p_2 + \rho gh_2 + \frac{1}{2} \rho v_2^2$$

$$\begin{aligned} (4 \times 10^5) + (1025 \times 10 \times 1.8) + \left(\frac{1}{2} \times 1025 \times (10)^2\right) \\ = (4.1 \times 10^5) + (1025 \times 10 \times h_2) + \left(\frac{1}{2} \times 1025 \times (10.2)^2\right) \end{aligned}$$

$$469700 = 10250 h_2 + 463320.5$$

$$6379.5 = 10250 h_2$$

$$h_2 = \frac{6379.5}{10250} = 0.62 \text{ m}$$

**Exercise 6**

What must be the pressure difference (  $\Delta P = ?$  ) between the two ends of a 200 m section of pipe, 0.1 m in radius, if it is to transport oil at a rate of 0.05 m<sup>3</sup>/s? Consider the oil viscosity ( $\eta$ ) is 0.2 Pa.s.

**Solution:**

$$Q = \frac{\pi R^4 \Delta P}{8 \eta L}$$

$$0.05 = \frac{\pi (0.1)^4 \Delta P}{8 \cdot (0.2) \cdot (200)}$$

$$\frac{0.05}{1} = \frac{\pi (0.1)^4 \Delta P}{8 \cdot (0.2) \cdot (200)}$$

$$\pi (0.1)^4 \Delta P = (0.05) \cdot (8) \cdot (0.2) \cdot (200)$$

$$\Delta P = \frac{(0.05) \cdot (8) \cdot (0.2) \cdot (200)}{\pi (0.1)^4} = 5.1 \times 10^4 \text{ Pa}$$

## **Chapter Seven:**

### **Light and optics**

### Exercise 1\*\*

Two plane mirrors meet at a  $135^\circ$  angle. If light rays strike one mirror at  $34^\circ$  as shown below, what is the angle of reflection from the second mirror  $M_2$ ?

**Solution:**



To determine the angle of reflection on the second mirror  $\beta$ , recall that the total of the interior angles of a triangle is  $180^\circ$ .

$$\phi + 135^\circ + \alpha = 180^\circ$$

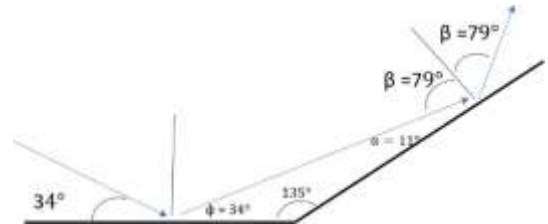
$$\text{But } \phi = 34^\circ$$

$$34^\circ + 135^\circ + \alpha = 180^\circ$$

$$\alpha = 180^\circ - 169^\circ = 11^\circ$$

So,

$$\beta = 90^\circ - 11^\circ = 79^\circ$$



### Exercise 2

The speed of light in ice is  $2.29 \times 10^8$  m/s. What is the index of refraction of ice? Consider the velocity of light in air or vacuum is  $3 \times 10^8$  m/s.

**Solution:**

$$n_{ice} = \frac{c}{v_{in\ ice}} = \frac{3 \times 10^8}{2.9 \times 10^8} = 1.31$$

### Exercise 3

A diver shines a flashlight upward from beneath the water (medium 1) at a  $35.2^\circ$  angle to the vertical. At what angle does the light leave the water into the air (medium 2)?

Consider  $n_{\text{air}} = 1$  and  $n_{\text{water}} = 1.33$ .

#### Solution

$$n_1 = 1.33, \quad n_2 = 1, \quad \theta_1 = 35.2^\circ, \quad \theta_2 = ??$$

$$\frac{\sin \theta_1}{\sin \theta_2} = \frac{n_2}{n_1}$$

$$\frac{\sin 35.2}{\sin \theta_2} = \frac{1}{1.33} \Rightarrow \sin \theta_2 = 1.33 \times \sin 35.2$$

$$\sin \theta_2 = 0.767 \Rightarrow \theta_2 = \sin^{-1} 0.767 = 50.1^\circ$$

### Exercise 4

What is the critical angle between water and glass? Consider  $n_{\text{water}} = 1.33$ , and  $n_{\text{glass}} = 1.5$ .

#### Solution

$$\theta_c = \sin^{-1} \left( \frac{n_2}{n_1} \right) \Rightarrow \theta_c = \sin^{-1} \left( \frac{1.33}{1.5} \right) \Rightarrow \theta_c = 62.5^\circ$$

### Exercise 5

The critical angle for a certain liquid–air surface is  $47.2^\circ$ . What is the index of refraction of the liquid? Consider  $n_{air} = 1$ .

#### Solution

$$\sin \theta_c = \frac{n_2}{n_1}$$

$$\sin 47.2 = \frac{1}{n_1}$$

$$n_1 \cdot \sin 47.2 = 1$$

$$n_1 = \frac{1}{\sin 47.2} = \frac{1}{0.734} = 1.36$$

### Exercise 6

Calculate the refractive power (P) in a diopter for a convex lens that has a focal length of 30 cm.

#### Solution

$$f = 30 \text{ cm} = \frac{30}{100} \text{ m} = 0.3 \text{ m} \quad \text{So,} \quad P = \frac{1}{f} = \frac{1}{0.3} = 3.33 \text{ D}$$

### Exercise 7

Calculate the refractive power (P) in a diopter for a concave lens that has a focal length of 4 m.

#### Solution

$$f = -4 \text{ m} \quad \text{So,} \quad P = \frac{1}{-4} = -0.25 \text{ D}$$

### Exercise 8

A diverging lens has a focal length of 30 cm. If a 20 cm object height is located 90 cm from the lens, find:

- 1) The image distance ( $d_i$  = ??).
- 2) The image is real or virtual?
- 3) The image is upright or inverted?
- 4) The image magnification?
- 5) The image height ( $y'$  = ??).

### Solution

$$2) \quad f = -30 \text{ cm}, \quad d_o = 90 \text{ cm}, \quad d_i = ??$$

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \rightarrow \frac{1}{-30} = \frac{1}{90} + \frac{1}{d_i} \rightarrow \frac{1}{d_i} = \frac{1}{-30} - \frac{1}{90} \rightarrow \frac{1}{d_i} = \frac{-4}{90} \rightarrow$$

$$d_i = -22.5 \text{ cm}$$

3) The image is virtual because  $d_i$  negative.

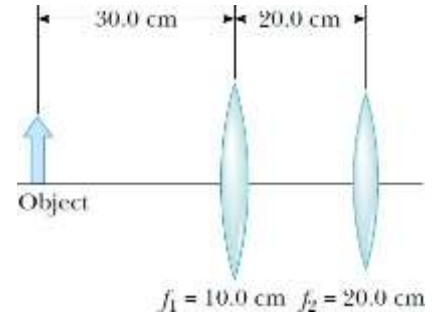
$$3) \quad M = \frac{-d_i}{d_o} = \frac{-(-22.5)}{90} = +0.25 \quad \text{so, the image is upright because } M \text{ is positive.}$$

4) The image is minimized because  $|M| < 1$ . ( $0.25 < 1$ )

$$5) \text{ The image height } y' = ?? \quad M = \frac{y'}{y} \rightarrow y' = M \times y = 0.25 \times 20 = 5 \text{ cm.}$$

### Exercise 9

Two thin converging lenses of focal lengths  $f_1 = 10 \text{ cm}$  and  $f_2 = 20 \text{ cm}$  are separated by  $20 \text{ cm}$ , as illustrated in the Figure below. An object is placed  $30 \text{ cm}$  to the left of lens 1. Find the position and the magnification of the final image.



### Solution

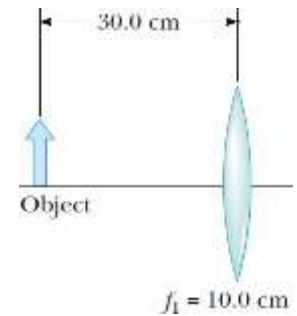
First, we find the image formed by the first lens (ignore the second lens)

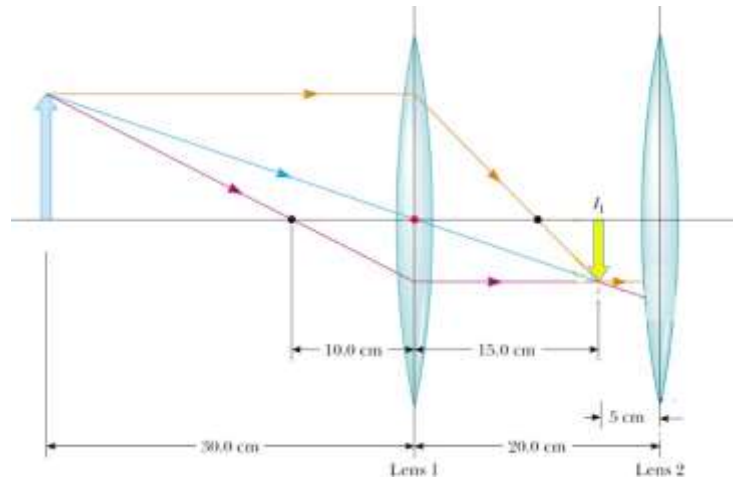
$$f_1 = +10 \text{ cm}, \quad d_{o1} = 30 \text{ cm}, \quad d_{i1} = ??$$

$$\frac{1}{f_1} = \frac{1}{d_{o1}} + \frac{1}{d_{i1}} \rightarrow \frac{1}{10} = \frac{1}{30} + \frac{1}{d_{i1}} \rightarrow \frac{1}{d_{i1}} = \frac{1}{10} - \frac{1}{30} \rightarrow \frac{1}{d_{i1}} = \frac{2}{30} \rightarrow$$

$$d_{i1} = +15 \text{ cm}$$

$$\text{The magnification through the first lens } M_1 = \frac{-d_{i1}}{d_{o1}} = \frac{-(15)}{30} = -0.5$$





Now, the image formed by this lens (the first lens) acts as the object for the second lens. Thus, the object distance for the second lens is  $20.0 \text{ cm} - 15.0 \text{ cm} = 5.00 \text{ cm}$ . We again apply the thin lens equation to find the location of the final image:

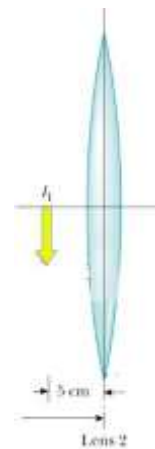
So,

$$f_2 = +20 \text{ cm}, \quad d_{o2} = 5 \text{ cm}, \quad d_{i2} = ??$$

$$\frac{1}{f_2} = \frac{1}{d_{o2}} + \frac{1}{d_{i2}} \rightarrow \frac{1}{20} = \frac{1}{5} + \frac{1}{d_{i2}} \rightarrow \frac{1}{d_{i2}} = \frac{1}{20} - \frac{1}{5} \rightarrow$$

$$\frac{1}{d_{i2}} = \frac{-3}{20} \rightarrow d_{i2} = -6.67 \text{ cm}$$

The magnification through the first lens  $M_2 = \frac{-d_{i2}}{d_{o2}} = \frac{-(-6.67)}{5} = +1.33$



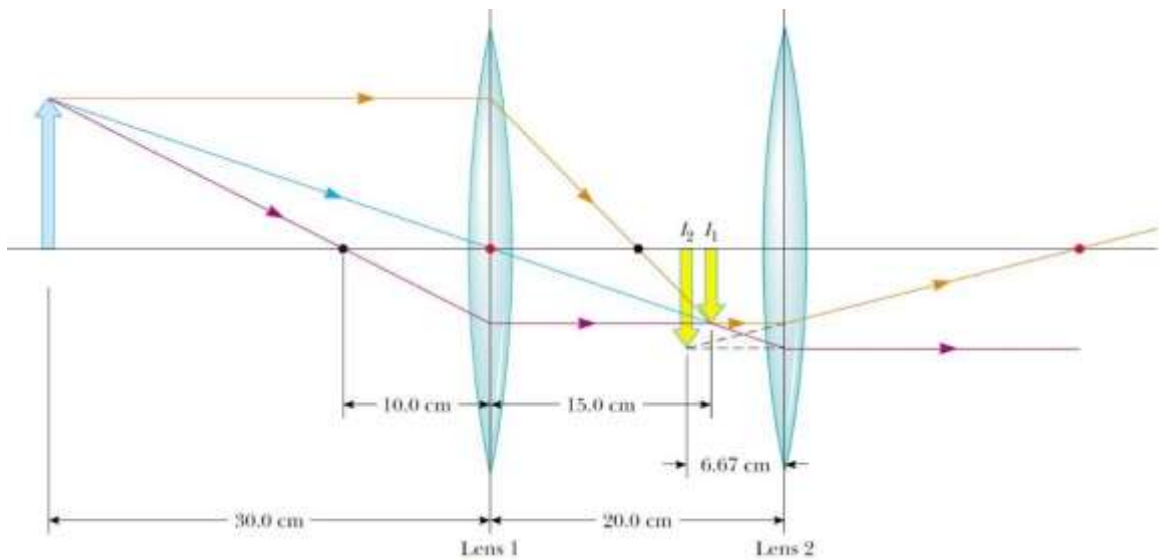
Thus, the overall magnification of the system is

$$M_{final} = M_1 \cdot M_2 = (-0.5) \cdot (1.33) = -0.67$$

The final image is minimized (with respect to the original object) because

$$|M| < 1. \quad (0.67 < 1)$$

To finalize the problem, note that the negative sign on the overall magnification indicates that the final image is inverted with respect to the initial object. The fact that the absolute value of the magnification is less than one tells us that the final image is smaller than the object. The fact that  $d_{i2}$  is negative tells us that the final image is on the front, or left, side of lens 2. All of these conclusions are shown in the Figure below.



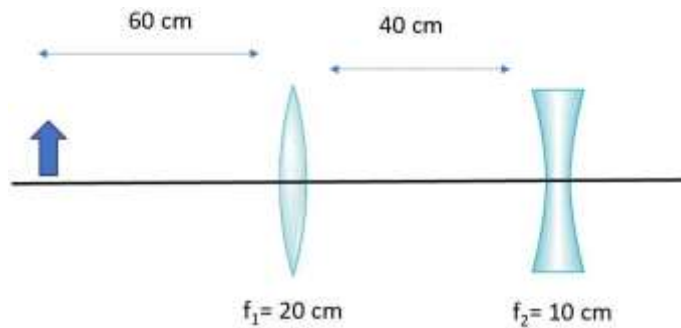
### Exercise 10

Two lenses, one converging with focal length 20.0 cm and one diverging with focal length 10.0 cm, are placed 40.0 cm apart. An object is placed 60.0 cm in front of the converging lens. Determine (a) the position and (b) the magnification of the final image formed.

**The answer:**

The final position of the image is  $d_{i2} = +10 \text{ cm}$

The final magnification of the system is  $M_{final} = -1$

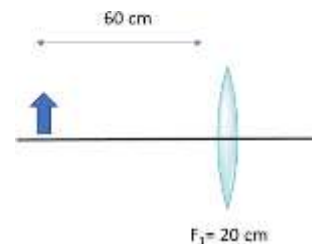


First, we find the image formed by the first lens (ignore the second lens)

$$f_1 = +20 \text{ cm}, \quad d_{o1} = 60 \text{ cm}, \quad d_{i1} = ??$$

$$\frac{1}{f_1} = \frac{1}{d_{o1}} + \frac{1}{d_{i1}} \rightarrow \frac{1}{20} = \frac{1}{60} + \frac{1}{d_{i1}} \rightarrow \frac{1}{d_{i1}} = \frac{1}{20} - \frac{1}{60} \rightarrow \frac{1}{d_{i1}} = \frac{2}{60} \rightarrow$$

$$d_{i1} = +30 \text{ cm}$$



$$\text{The magnification through the first lens } M_1 = \frac{-d_{i1}}{d_{o1}} = \frac{-(30)}{60} = -0.5$$

Now, the image formed by this lens (the first lens) acts as the object for the second lens. Thus, the object distance for the second lens is  $40 \text{ cm} - 30 \text{ cm} = 10 \text{ cm}$ . We again apply the thin lens equation to find the location of the final image:

So,

$$f_2 = -10 \text{ cm}, \quad d_{o2} = 10 \text{ cm}, \quad d_{i2} = ??$$

$$\frac{1}{f_2} = \frac{1}{d_{o2}} + \frac{1}{d_{i2}} \rightarrow \frac{1}{-10} = \frac{1}{10} + \frac{1}{d_{i2}} \rightarrow \frac{1}{d_{i2}} = \frac{-1}{10} - \frac{1}{10} \rightarrow$$

$$\frac{1}{d_{i2}} = \frac{-2}{10} \rightarrow d_{i2} = -5 \text{ cm}$$

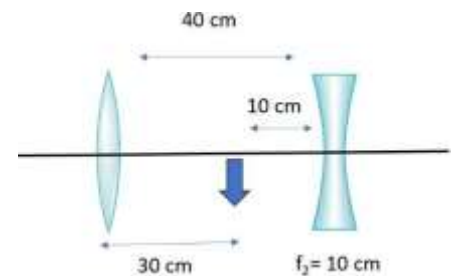
$$\text{The magnification through the second lens } M_2 = \frac{-d_{i2}}{d_{o2}} = \frac{-(-5)}{10} = 0.5$$

Thus, the overall magnification of the system is

$$M_{final} = M_1 \cdot M_2 = (-0.5) \cdot (0.5) = -0.25$$

The final image is minimized (with respect to the original object) because

$$|M| < 1. \quad (0.25 < 1)$$



**Chapter Eight:**  
**Nuclear physics**

### Exercise 1

What is the decay constant of  ${}^{238}_{92}\text{U}$  whose half-life is  $4.5 \times 10^9 \text{ year}$ ?

### Solution

$$\tau_{\frac{1}{2}} = \frac{0.693}{\lambda} \quad \Rightarrow \quad 4.5 \times 10^9 = \frac{0.693}{\lambda}$$

$$4.5 \times 10^9 \times \lambda = 0.693 \quad \Rightarrow \quad \lambda = \frac{0.693}{4.5 \times 10^9} \quad \Rightarrow \quad \lambda = 0.154 \times 10^{-9} \text{ year}^{-1}$$

### Exercise 2

The decay constant of  ${}^{238}_{92}\text{U}$  is  $3.2 \times 10^{-5} \text{ s}^{-1}$ . What is its half-life?

### Solution

$$\tau_{\frac{1}{2}} = \frac{0.693}{\lambda} \quad \Rightarrow \quad \tau_{\frac{1}{2}} = \frac{0.693}{3.2 \times 10^{-5}} = 0.22 \times 10^5 \text{ s}$$

### Exercise 3

The radioactive isotope  $^{198}_{79}\text{Au}$  has a decay constant of  $0.01 \text{ h}^{-1}$ . A sample containing this isotope has  $9 \times 10^{12}$  nuclei. Calculate the number of nuclei after 120 h.

### Solution

$$N = N_0 \times e^{-\lambda \times t}$$

$$N = 9 \times 10^{12} \times e^{-0.01 \times 120}$$

$$N = 9 \times 10^{12} \times e^{-1.2}$$

$$N = 9 \times 10^{12} \times 0.3$$

$$N = 2.7 \times 10^{12} \text{ nuclei}$$

### Exercise 4

Find the value of x in each of the following equations.

$$1) \quad ^{226}_{88}\text{Ra} \rightarrow ^{222}_x\text{Rn} + ^4_2\text{He}$$

$$2) \quad ^{12}_x\text{C} \rightarrow ^{12}_7\text{N} + e^{-1} + \bar{\nu} \text{ (antineutrino)}$$

$$3) \quad ^{12}_6\text{C}^* \rightarrow ^x_6\text{C} + \gamma$$

**The answer:**    1)  $x = 86$                       2)  $x = 6$                       3)  $x = 12$

### Exercise 5

The radioactive isotope  $^{198}_{79}\text{Au}$  has a decay constant of  $0.01 \text{ h}^{-1}$ . A sample containing this isotope has  $8 \times 10^{15} \text{ nuclei}$ . Calculate the number of nuclei after 3600 s.

### Solution

$$\lambda = 0.01 \text{ h}^{-1}$$

$$N_0 = 8 \times 10^{15} \text{ nuclei}$$

$$N = ??$$

$$t = 3600 \text{ s}$$

$$t = \frac{3600}{3600} = 1 \text{ h}$$

$$N = N_0 \times e^{-\lambda \times t}$$

$$N = 8 \times 10^{15} \times e^{-0.01 \text{ h}^{-1} \times 1 \text{ h}}$$

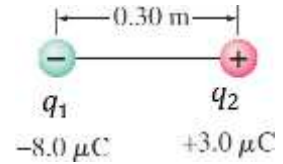
$$N \approx 7.92 \times 10^{15} \text{ nuclei}$$

## **Chapter Nine:**

### **Electricity**

### Exercise 1

Two point charges  $q_1$  and  $q_2$  are 0.3 m apart as shown in the Figure below. Calculate the electrical force on  $q_2$  due to  $q_1$ .



### Solution

The quantities should be in SI units

$$q_1 = -8 \mu\text{C} = -8 \times 10^{-6} \text{C}$$

$$q_2 = 3 \mu\text{C} = 3 \times 10^{-6} \text{C}$$

$$r = 0.3 \text{ m}$$

The electrical force on  $q_2$  due to  $q_1$  ( $F_{12} = ??$ ) is

$$F_{12} = 9 \times 10^9 \times \frac{q_1 \times q_2}{r^2} = 9 \times 10^9 \times \frac{8 \times 10^{-6} \times 3 \times 10^{-6}}{(0.3)^2}$$

$$F_{12} = 2.4 \text{ N to the left}$$

## Exercise 2

Two identical positive charges exert a repulsive force of  $36 \times 10^{19} \text{ N}$  when separated by a distance of 2 m. Calculate the charge of each.

### Solution

$$F = 36 \times 10^{19} \text{ N}$$

$$r = 2 \text{ m}$$

$$q_1 = q_2 = ??$$

$$F = 9 \times 10^9 \times \frac{q_1 \times q_2}{r^2}$$

$$36 \times 10^{19} = 9 \times 10^9 \times \frac{q_1 \times q_1}{(2)^2}$$

$$\frac{36 \times 10^{19}}{1} = \frac{9 \times 10^9 \times q_1^2}{4}$$

$$9 \times 10^9 \times q_1^2 = 4 \times 36 \times 10^{19}$$

$$q_1^2 = \frac{4 \times 36 \times 10^{19}}{9 \times 10^9}$$

$$q_1^2 = 16 \times 10^{10}$$

$$q_1 = \sqrt{16 \times 10^{10}}$$

$$q_1 = 4 \times 10^5 \text{ C} = q_2$$

### Exercise 3

Given two charges  $q_1$  and  $q_2$ , as shown in the Figure below, and a positive charge  $q_3 = 1.8 \times 10^{-18} \text{ C}$  at the origin, what is the net force exerted on the charge  $q_3$  by the two charges ( $q_1$  and  $q_2$ )?

#### Solution

The quantities should be in SI units

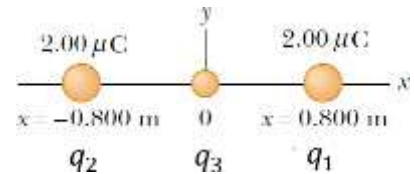
$$q_1 = 2 \mu\text{C} = 2 \times 10^{-6} \text{ C}$$

$$q_2 = 2 \mu\text{C} = 2 \times 10^{-6} \text{ C}$$

$$q_3 = 1.8 \times 10^{-18} \text{ C}$$

$$r_{13} = 0.8 \text{ m}$$

$$r_{23} = -0.8 \text{ m}$$



The electrical forces on  $q_3$  due to  $q_1$  and  $q_2$  ( $F_{13} = ??$  and  $F_{23} = ??$ ) are

$$F_{13} = 9 \times 10^9 \times \frac{q_1 \times q_3}{r_{13}^2} = 9 \times 10^9 \times \frac{2 \times 10^{-6} \times 1.8 \times 10^{-18}}{(0.8)^2}$$

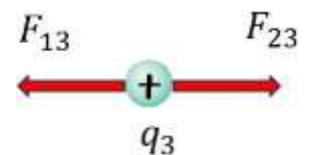
$$F_{13} = 5.1 \times 10^{-18} \text{ N (to the left)}$$

$$F_{23} = 9 \times 10^9 \times \frac{q_2 \times q_3}{r_{23}^2} = 9 \times 10^9 \times \frac{2 \times 10^{-6} \times 1.8 \times 10^{-18}}{(-0.8)^2}$$

$$F_{23} = 5.1 \times 10^{-18} \text{ N (to the right)}$$

The net force on  $q_3$  is

$$\sum F_x = F_{23} - F_{13} = 5.1 \times 10^{-18} - 5.1 \times 10^{-18} = 0$$



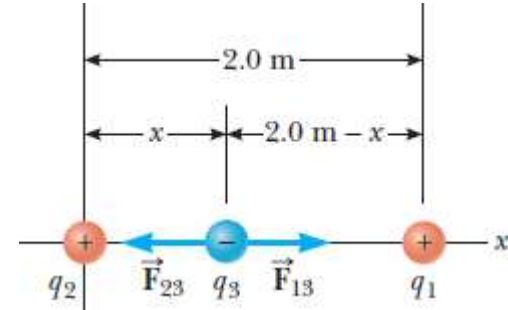
#### Exercise 4

Three charges lie along the x-axis as in the Figure below. The positive charge  $q_1 = 4 \mu\text{C}$  is at 2.0 m to the right of the origin point, and the positive charge  $q_2 = 9 \mu\text{C}$  is at the origin. Where must a negative charge  $q_3$  be placed on the x-axis so that the resultant electric force on it is zero?

#### Solution

If  $q_3$  is to the right or left of the other two charges, the net force on  $q_3$  can't be zero because then  $F_{13}$  and  $F_{23}$  act in the same direction. Consequently,  $q_3$  must lie between the two other charges. Write  $F_{13}$  and  $F_{23}$  in terms of the unknown coordinate position  $x$ , then sum them and set them equal to zero, solving for the unknown. The solution can be obtained with the quadratic formula.

The resultant force on  $q_3$  is zero when



$$F_{23} = F_{13}$$

$$9 \times 10^9 \times \frac{q_2 \times q_3}{r_{23}^2} = 9 \times 10^9 \times \frac{q_1 \times q_3}{r_{13}^2}$$

$$\frac{9 \times 10^9 \times 9 \times 10^{-6} \times q_3}{(x)^2} = \frac{9 \times 10^9 \times 4 \times 10^{-6} \times q_3}{(0.2 - x)^2}$$

$$\frac{9}{(x)^2} = \frac{4}{(0.2 - x)^2}$$

Taking the square root for both sides

$$\frac{3}{x} = \frac{2}{0.2 - x}$$

$$\text{So, } 2x = 0.6 - 3x \Rightarrow x = 0.12 \text{ m}$$

### Exercise 5\*\*

Calculate the net electrical force exerted on the charge  $q_2$  by the two charges ( $q_1$  and  $q_3$ ) in the Figure shown below.

### Solution

The quantities should be in SI units

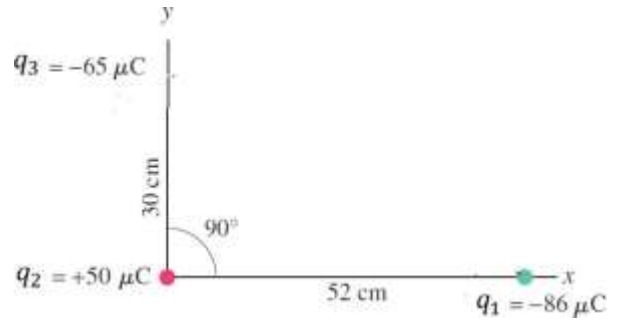
$$q_1 = -86 \mu C = -86 \times 10^{-6} C$$

$$q_2 = 50 \mu C = 50 \times 10^{-6} C$$

$$q_3 = -65 \mu C = -65 \times 10^{-6} C$$

$$r_{12} = 52 cm = 52 \times 10^{-2} m$$

$$r_{23} = 30 cm = 30 \times 10^{-2} m$$



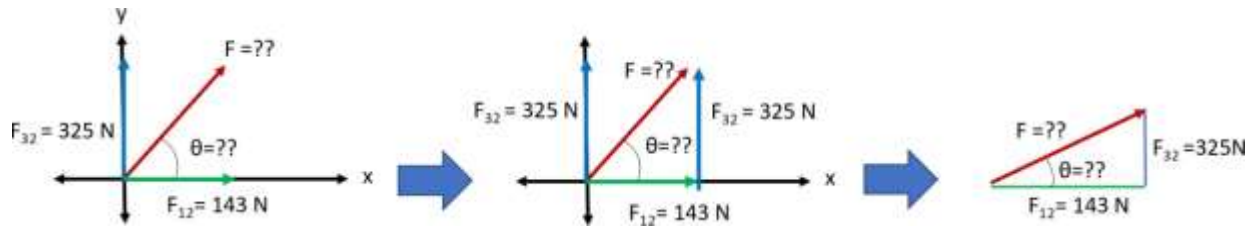
The electrical forces on  $q_2$  due to  $q_1$  and  $q_3$  ( $F_{12} = ??$  and  $F_{32} = ??$ ) are

$$F_{12} = 9 \times 10^9 \times \frac{q_1 \times q_2}{r_{12}^2} = 9 \times 10^9 \times \frac{86 \times 10^{-6} \times 50 \times 10^{-6}}{(52 \times 10^{-2})^2}$$

$$F_{12} = 143 N \text{ ( to the right)}$$

$$F_{32} = 9 \times 10^9 \times \frac{q_2 \times q_3}{r_{23}^2} = 9 \times 10^9 \times \frac{50 \times 10^{-6} \times 65 \times 10^{-6}}{(30 \times 10^{-2})^2}$$

$$F_{32} = 325 N \text{ ( Upward direction)}$$



The net force magnitude ( $F = ??$ )

$$F = \sqrt{F_{12}^2 + F_{32}^2} = \sqrt{(143)^2 + (325)^2} = 355.1 \text{ N}$$

The direction of  $F$  (the angle with respect to  $x$  axis) ( $\theta_x = ??$ )

$$\tan \theta = \frac{F_{32}}{F_{12}}$$

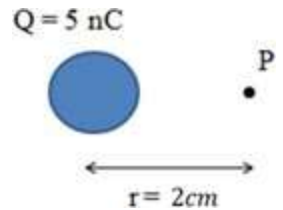
So,

$$\theta = \tan^{-1} \frac{F_{32}}{F_{12}}$$

$$\theta_x = \tan^{-1} \left( \frac{325}{143} \right) = 66.3^\circ$$

### Exercise 6\*\*

In the following figure, calculate the electric field due to the charge Q at point p.



### Solution

$$q = 5 \text{ nC} = 5 \times 10^{-9} \text{ C}$$

$$r = 2 \text{ cm} = 2 \times 10^{-2} \text{ m} = 0.02 \text{ m}$$

$$\vec{E} = 9 \times 10^9 \times \frac{q}{r^2} = 9 \times 10^9 \times \frac{5 \times 10^{-9}}{(0.02)^2}$$

$$\vec{E} = 11.25 \times 10^4 \text{ N/C away from the charge Q.}$$

### Exercise 7

Determine the potential at a point 0.50 m (a) from a  $20 \mu\text{C}$  point charge, (b) from a  $-20 \mu\text{C}$  point charge.

### Solution

$$a) q = 20 \mu\text{C} = 20 \times 10^{-6} \text{ C}$$

$$V = 9 \times 10^9 \times \frac{20 \times 10^{-6}}{0.5} = 3.6 \times 10^5 \text{ V}$$

$$b) q = -20 \mu\text{C} = -20 \times 10^{-6} \text{ C}$$

$$V = 9 \times 10^9 \times \frac{-20 \times 10^{-6}}{0.5} = -3.6 \times 10^5 \text{ V}$$

### Exercise 8

What is the electric potential 15.0 cm from a  $20 \mu C$  point charge?

#### Solution

$$q = 20 \mu C = 20 \times 10^{-6} C$$

$$r = 15 \text{ cm} = 15 \times 10^{-2} m$$

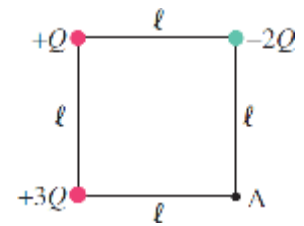
$$V = 9 \times 10^9 \times \frac{20 \times 10^{-6}}{15 \times 10^{-2}} = 12 \times 10^5 V$$

### Exercise 9\*\*

Three-point charges are arranged at the corners of a square of side as shown in Figure below.  
What is the potential at the fourth corner (point A)?

$$\text{Solution: } V = 9 \times 10^9 \frac{Q(2+\sqrt{2})}{2l}$$

The potential at the corner is the sum of the potentials due to each of the charges.



$$V = 9 \times 10^9 \times \frac{3Q}{l} + 9 \times 10^9 \times \frac{Q}{\sqrt{2}l} + 9 \times 10^9 \times \frac{-2Q}{l}$$

$$V = 9 \times 10^9 \times \frac{3Q}{l} \times \frac{2}{2} + 9 \times 10^9 \times \frac{Q}{\sqrt{2}l} \times \frac{\sqrt{2}}{\sqrt{2}} + 9 \times 10^9 \times \frac{-2Q}{l} \times \frac{2}{2}$$

$$V = 9 \times 10^9 \times \frac{6Q}{2l} + 9 \times 10^9 \times \frac{\sqrt{2} Q}{2l} + 9 \times 10^9 \times \frac{-4Q}{2l}$$

$$V = 9 \times 10^9 \times \frac{Q}{2l} (6 + \sqrt{2} - 4) = 9 \times 10^9 \times \frac{Q}{2l} (2 + \sqrt{2})$$

**Exercise 10**

10 coulombs of charge flow through the bulb in 50 seconds. Calculate the current flowing through the bulb.

**Solution**

$$\Delta Q = 10 \text{ C}$$

$$\Delta t = 50 \text{ s}$$

$$I = ??$$

$$I = \frac{\Delta Q}{\Delta t}$$

$$I = \frac{10}{50} = 0.2 \text{ A}$$

**Exercise 11**

A service station charges a battery using a current of 6.7 A for 5.0 h. How much charge passes through the battery?

**Solution**

$$I = 6.7 \text{ A}$$

$$\Delta t = 5 \times 60 \times 60 = 18000 \text{ s}$$

$$\Delta Q = ??$$

$$I = \frac{\Delta Q}{\Delta t}$$

$$6.7 = \frac{\Delta Q}{18000} \Rightarrow \Delta Q = 6.7 \times 18000 = 120600 \text{ C}$$

**Exercise 12**

A hair dryer draws 13.5 A when plugged into a 120 V. What is its resistance?

**Solution**

$$\Delta V = 120 \text{ V}$$

$$I = 13.5 \text{ A}$$

$$R = ??$$

$$R = \frac{\Delta V}{I}$$

$$R = \frac{120}{13.5} \Rightarrow R = 8.9 \text{ } \Omega$$

**Exercise 13**

What is the area cross-section of a 1.00-m length of tungsten wire whose resistance is 0.32  $\Omega$ ? Consider the resistivity of the tungsten is  $5.6 \times 10^{-8} \text{ } \Omega \cdot m$ .

**Solution**

$$R = \frac{\rho \times L}{A}$$

$$0.32 = \frac{5.6 \times 10^{-8} \times 1}{A}$$

$$0.32 A = 5.6 \times 10^{-8}$$

$$A = \frac{5.6 \times 10^{-8}}{0.32} = 17.5 \times 10^{-8} \text{ } m^2$$

**Exercise 14**

An 8500-pF capacitor holds plus and minus charges of  $16.5 \times 10^{-8}$  C. What is the voltage across the capacitor?

**Solution:**

$$Q = 16.5 \times 10^{-8} \text{ C}$$

$$C = 8500 \text{ pF} = 8500 \times 10^{-12} \text{ F}$$

$$\Delta V = ??$$

$$C = \frac{|Q|}{\Delta V} =$$

$$8500 \times 10^{-12} = \frac{16.5 \times 10^{-8}}{\Delta V}$$

$$8500 \times 10^{-12} \times \Delta V = 16.5 \times 10^{-8}$$

$$\Delta V = \frac{16.5 \times 10^{-8}}{8500 \times 10^{-12}} = 19.4 \text{ V}$$

**Exercise 15**

A parallel Plate Capacitor has a plate area of  $0.04 \text{ m}^2$  separated by a  $1.8 \text{ mm}$  thickness of paraffin. Find its capacitance. Consider:  $\kappa = 2.2$  for paraffin, and  $\epsilon_0 = 8.8542 \times 10^{-12} \frac{\text{C}^2}{\text{N.m}^2}$

**Solution**

$$C = k \times \frac{A \times \epsilon_0}{d}$$

$$C = 2.2 \times \frac{0.04 \times 8.8542 \times 10^{-12}}{1.8 \times 10^{-3}}$$

$$C = 2.2 \times \frac{0.04 \times 8.8542 \times 10^{-12}}{1.8 \times 10^{-3}}$$

$$C = 43.3 \times 10^{-11} \text{ F}$$

**The End**

**"Our greatest weakness lies in giving up. The most certain way to succeed is always to try  
just one more time."**

