



Chapter 12

Three-Phase Circuits

By: Dr. Abdulrahman Almazroui



Learning Objectives

CHAPTER 12 OVERVIEW

01

Understand balanced three-phase voltages

Phase sequences, phasor relationships, why 3-phase dominates power systems

02

Analyze balanced Wye-Wye (Y-Y) circuits

Single-phase equivalent method, line vs. phase quantities

03

Analyze balanced Wye-Delta (Y- Δ) circuits

Line-to-line voltages, $I_L = \sqrt{3} I_p$ relationship

04

Analyze balanced Delta-Delta (Δ - Δ) circuits

Phase and line currents in delta systems

05

Analyze balanced Delta-Wye (Δ -Y) circuits

$\Delta \rightarrow Y$ source transformation, single-phase equivalent

06

Calculate power in balanced 3-phase systems

P, Q, S formulas using line quantities

07

Solve unbalanced three-phase systems

Mesh/nodal analysis; neutral current $\neq 0$

12.1 Introduction

Why Three-Phase? What makes it essential?

Why Three-Phase? Key Advantages

12.1 — INTRODUCTION

A three-phase system = three sinusoidal sources of equal amplitude and frequency, separated by 120° each, combined into one generator.

1

Ubiquitous Power Generation

Nearly all electric power worldwide is generated and distributed as 3-phase AC. Single-phase is derived from it.

2

Constant Instantaneous Power

Unlike single-phase (power pulses twice per cycle), 3-phase delivers constant instantaneous power → less vibration in motors, smoother torque.

3

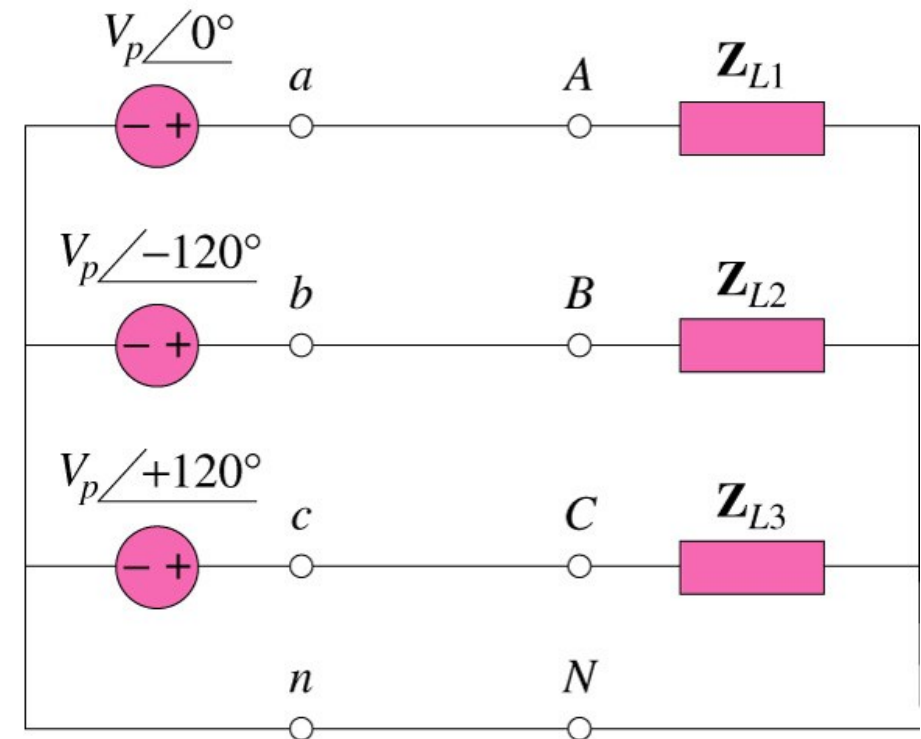
Material Efficiency

3-phase uses only 75% of the conductor material of an equivalent single-phase system for the same power delivered — significant savings at utility scale.

4

Economical for Large Loads

For the same power output, 3-phase generators and motors are smaller, cheaper, and more efficient than equivalent single-phase equipment.



✚ The three-phase system is the international standard. Even when single-phase is needed (homes, small appliances), it is obtained by tapping one phase from a three-phase distribution system.

12.2 Balanced Three-Phase Voltages

Phase Sequences · abc · acb · Phasor Relationships

Balanced Three-Phase Voltages

12.2 — BALANCED VOLTAGES

Balanced phase voltages: equal in magnitude, same frequency, 120° apart from each other $\rightarrow V_{an} + V_{bn} + V_{cn} = 0$

abc SEQUENCE (Positive) — most common

$$V_{an} = V_p \angle 0^\circ \text{ V}$$

Reference phasor — phase a leads all others

$$V_{bn} = V_p \angle -120^\circ \text{ V}$$

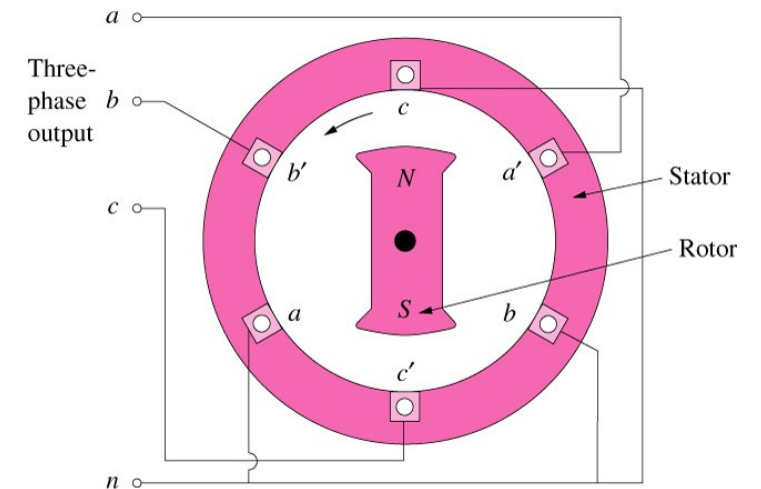
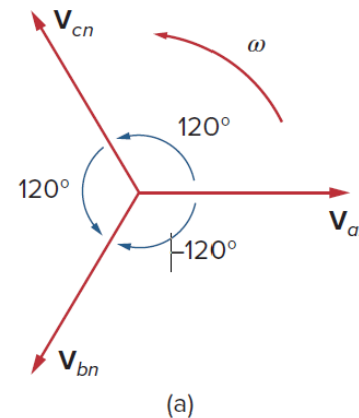
Phase b lags phase a by 120°

$$V_{cn} = V_p \angle 120^\circ \text{ V}$$

Phase c lags phase b by 120°
(leads a by 120°)

Sequence order:

a $\rightarrow$ b $\rightarrow$ c $\rightarrow$ a (counterclockwise rotor)



📌 Phase sequence matters because it determines the direction of rotation of three-phase motors. Swapping any two wires reverses the motor.

🔑 Key identity: For abc sequence, $V_L(\text{line-to-line}) = \sqrt{3} \cdot V_p$ and V_L LEADS V_p by 30°
Example: $V_{ab} = V_{an} - V_{bn} = \sqrt{3} \cdot V_p \angle 30^\circ$ | $V_{bc} = \sqrt{3} \cdot V_p \angle -90^\circ$ | $V_{ca} = \sqrt{3} \cdot V_p \angle 150^\circ$

Balanced Three-Phase Voltages

12.2 — BALANCED VOLTAGES

Balanced phase voltages: equal in magnitude, same frequency, 120° apart from each other $\rightarrow V_{an} + V_{bn} + V_{cn} = 0$

acb SEQUENCE (Negative) — less common

$$V_{an} = V_p \angle 0^\circ \text{ V}$$

Same reference

$$V_{cn} = V_p \angle -120^\circ \text{ V}$$

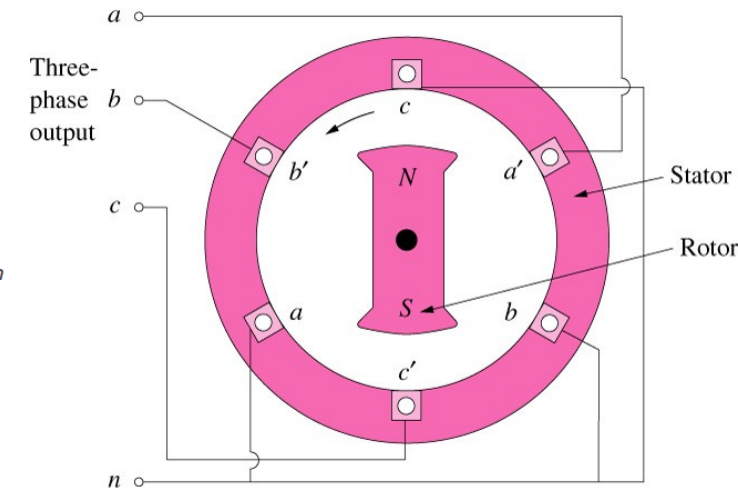
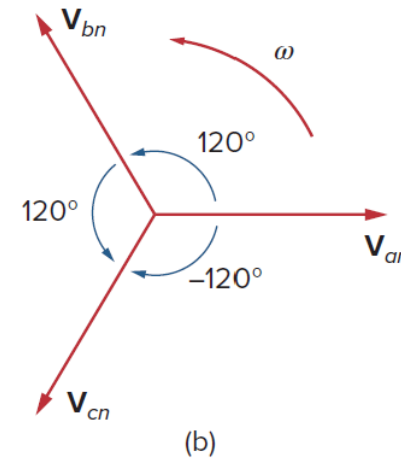
Now c lags a by 120°

$$\begin{aligned} V_{bn} &= V_p \angle 120^\circ \text{ V} \\ &= V_p \angle -240^\circ \text{ V} \end{aligned}$$

b is last in sequence

Sequence order:

$a \rightarrow c \rightarrow b \rightarrow a$ (clockwise rotor)

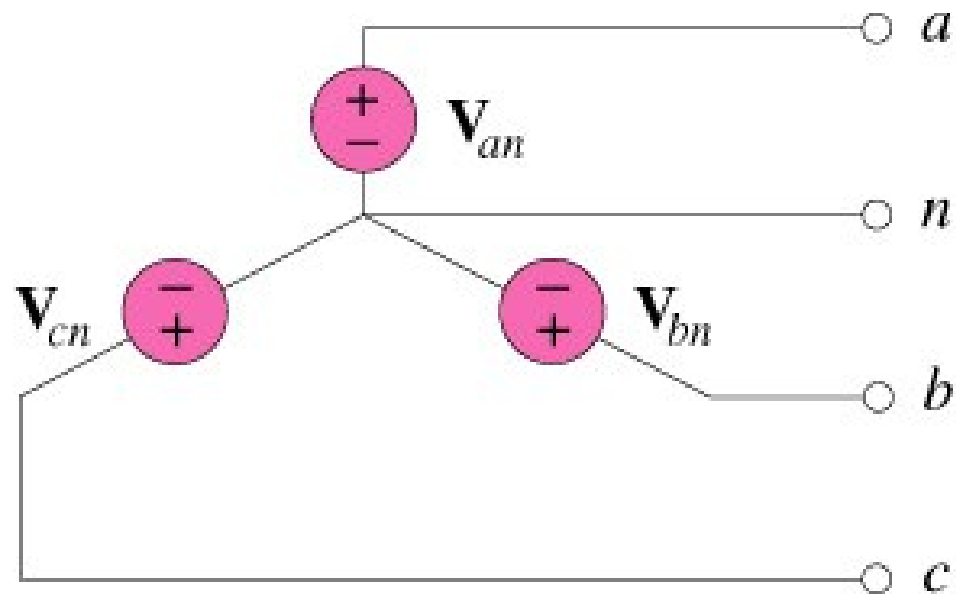


The **phase sequence** is the time order in which the voltages pass through their respective maximum values.

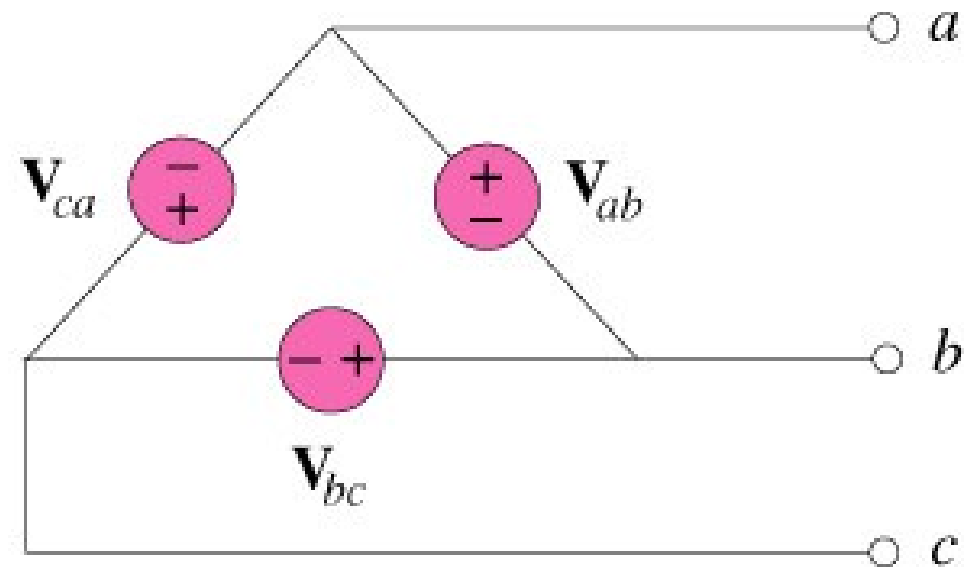
Balanced Three-Phase Voltages

12.2 — BALANCED VOLTAGES

Two possible configurations:



(a)



(b)

Three-phase voltage sources: (a) Y-connected ; (b) Δ -connected

Example 12.1 — Determining Phase Sequence

WORKED EXAMPLE

Problem: Determine the phase sequence of the following set of voltages:

$$V_{an} = 200 \cos(\omega t + 10^\circ) \quad V_{bn} = 200 \cos(\omega t - 230^\circ) \quad V_{cn} = 200 \cos(\omega t - 110^\circ)$$

Step 1

Convert to Phasor Form

$$V_{an} = 200 \angle 10^\circ \text{ V}$$

$$V_{bn} = 200 \angle -230^\circ \text{ V}$$

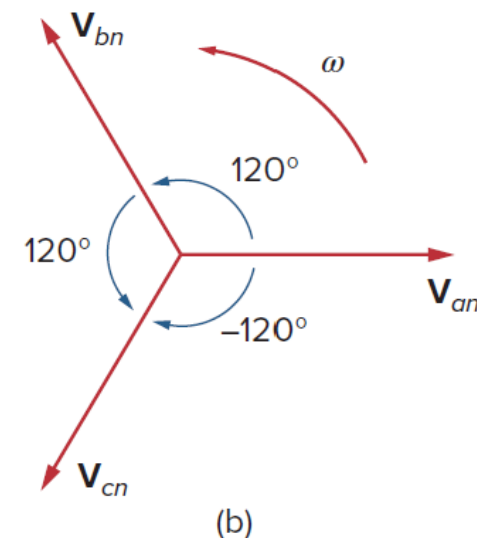
$$V_{cn} = 200 \angle -110^\circ \text{ V}$$

Step 2

Find Angular Differences

$$V_{an} \text{ leads } V_{cn} \text{ by: } 10^\circ - (-110^\circ) = 120^\circ \quad \checkmark$$

$$V_{cn} \text{ leads } V_{bn} \text{ by: } -110^\circ - (-230^\circ) = 120^\circ \quad \checkmark$$



✓ Answer: acb (negative) sequence

Four Possible Three-Phase Configurations

Source and load can each be Y-connected or Δ -connected $\rightarrow$ 4 combinations. All are equivalent via Y- Δ transformation.

Y-Y Wye source, Wye load

Most straightforward. Direct neutral connection possible (4-wire system). Key formulas apply directly.

Y- Δ Wye source, Delta load

Most practical system — utility uses Y source; industrial loads are usually Δ connected. No neutral line at load.

Δ - Δ Delta source, Delta load

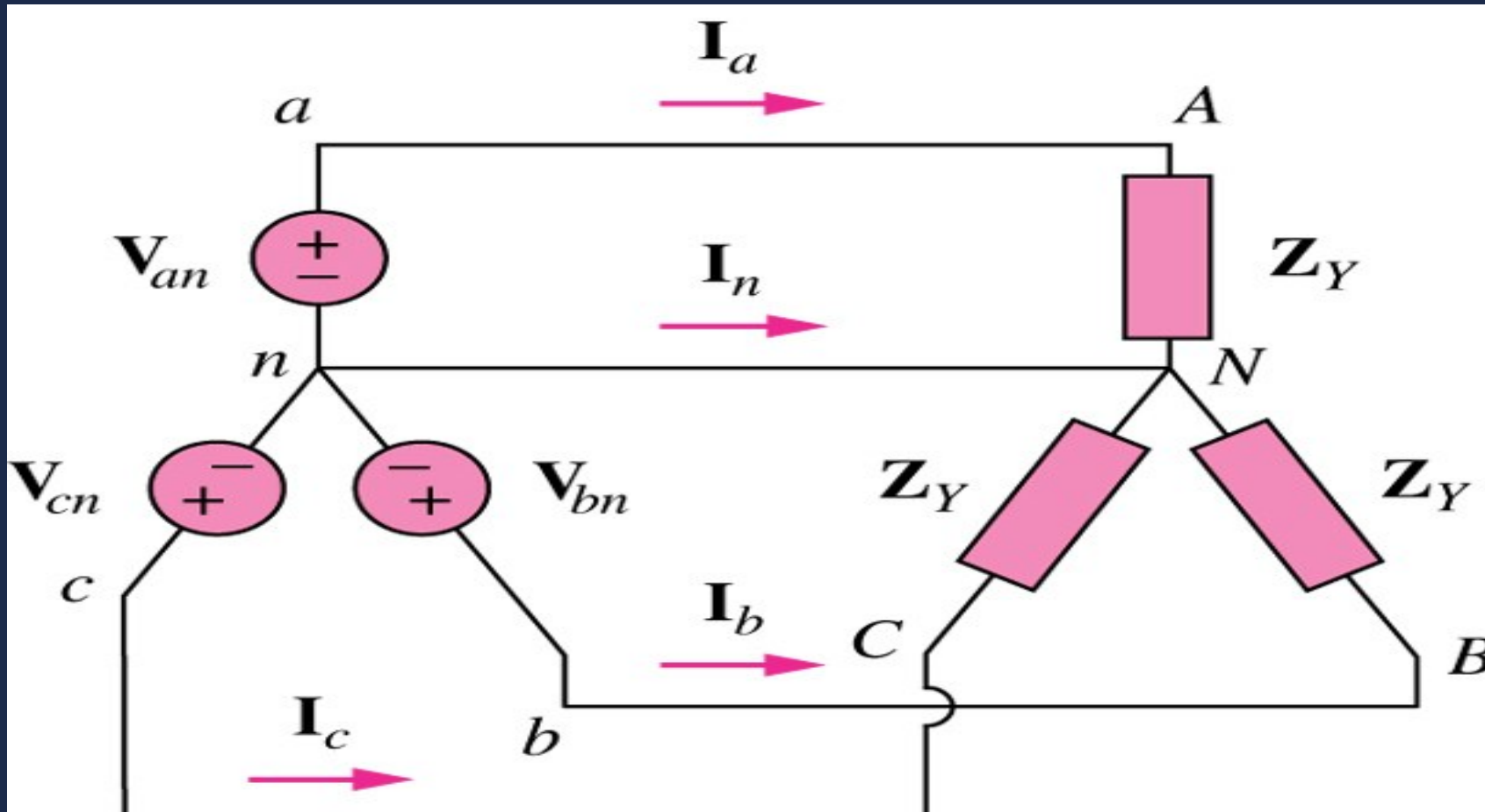
Both source and load delta. Phase voltages = line voltages. Rarely used for sources (circulating currents risk).

Δ -Y Delta source, Wye load

Convert Δ source to Y equivalent, then solve as Y-Y. $Z_Y = Z_{\Delta}/3$ after transformation.

12.3a Balanced Y-Y System

The Foundation — Solve any 3-phase using single-phase equivalent



Balanced Y-Y System — Analysis Method

12.3a — Y-Y CONNECTION

In a balanced Y-Y system, the neutral current $I_n = 0$. The 3-phase circuit reduces to one single-phase circuit for phase a.

Single-Phase Equivalent (Phase a)

$$I_a = V_{an} / Z_Y \text{ where } Z_Y = Z_s + Z_{line} + Z_{load}$$

Then use phase sequence for b and c

$$I_b = I_a \angle -120^\circ \quad I_c = I_a \angle +120^\circ$$

Key Voltage Relationships in Y-Y:

Phase voltages	$V_{an} = V_p \angle 0^\circ, V_{bn} = V_p \angle -120^\circ, V_{cn} = V_p \angle +120^\circ$
Line voltages	$V_{ab} = \sqrt{3} V_p \angle 30^\circ, V_{bc} = \sqrt{3} \cdot V_p \angle -90^\circ, V_{ca} = \sqrt{3} \cdot V_p \angle 150^\circ$
V_line vs V_phase	$V_{L(ab)} = \sqrt{3} \times V_p$ (the $\sqrt{3}$ factor comes from phasor subtraction)
Line currents	$I_L = I_p$ (same current flows through line and load phase in Y)

⚠ Practical note: Z_Y typically lumps source impedance Z_s + line impedance Z_{line} + load impedance Z_{load} . If problem states only load, assume $Z_s = Z_{line} = 0$ and $Z_Y = Z_{load}$ only.

Example 12.2 — Balanced Y-Y Line Currents

WORKED EXAMPLE — Y-Y

Problem: Three-wire Y-Y system. $V_{an} = 110\angle 0^\circ$ V (abc sequence). Line impedance: $Z_{line} = 5-j2\ \Omega$. Load: $Z_{load} = 10+j8\ \Omega$. Find all line currents.

Step 1

Find total Z_Y per phase

$$\begin{aligned} Z_Y &= Z_{line} + Z_{load} = (5-j2) + (10+j8) = 15+j6\ \Omega \\ |Z_Y| &= 16.155\ \Omega, \quad \theta = \arctan(6/15) = 21.8^\circ \\ \therefore Z_Y &= 16.155\angle 21.8^\circ\ \Omega \end{aligned}$$

Step 2

Apply single-phase equivalent for phase a

$$\begin{aligned} I_a &= V_{an} / Z_Y = 110\angle 0^\circ / 16.155\angle 21.8^\circ \\ I_a &= (110/16.155) \angle (0^\circ - 21.8^\circ) = 6.81\angle -21.8^\circ\ \text{A} \end{aligned}$$

Step 3

Use abc sequence to find I_b and I_c

$$\begin{aligned} I_b &= I_a \angle -120^\circ = 6.81\angle (-21.8^\circ - 120^\circ) = 6.81\angle -141.8^\circ\ \text{A} \\ I_c &= I_a \angle +120^\circ = 6.81\angle (-21.8^\circ + 120^\circ) = 6.81\angle 98.2^\circ\ \text{A} \end{aligned}$$

Check

Verify balanced — sum should = 0

In a balanced system: $I_a + I_b + I_c = 0$ ✓ (all same magnitude, 120° apart)
Neutral current: $I_n = -(I_a + I_b + I_c) = 0$ ✓

✓ **Answers:** $I_a = 6.81\angle -21.8^\circ\ \text{A}$ | $I_b = 6.81\angle -141.8^\circ\ \text{A}$ | $I_c = 6.81\angle 98.2^\circ\ \text{A}$

Example 12.2 — Balanced Y-Y Line Currents

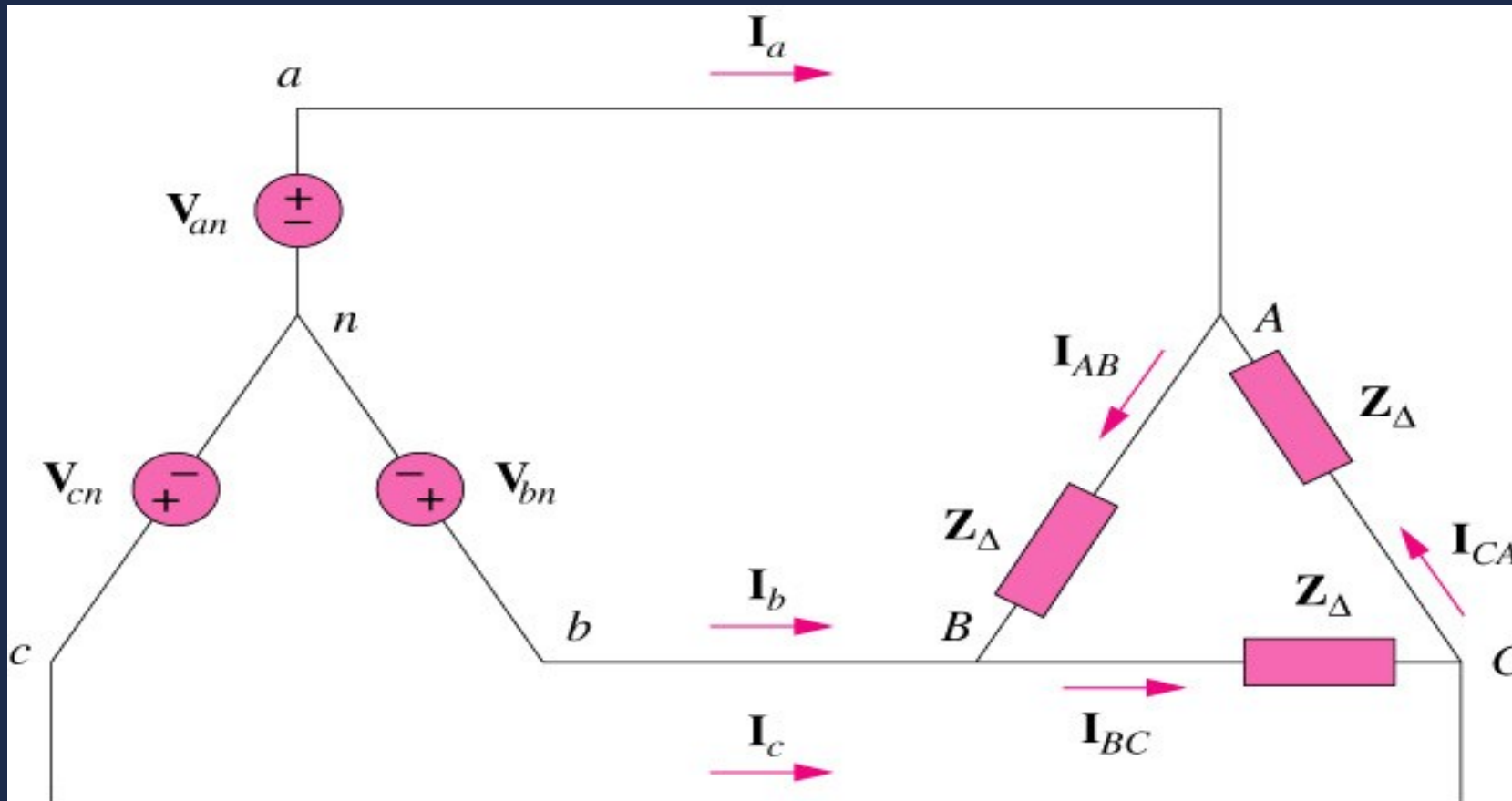
WORKED EXAMPLE — Y-Y

A Y-connected balanced three-phase generator with an impedance of $0.4 + j0.3 \, \Omega$ per phase is connected to a Y-connected balanced load with an impedance of $24 + j19 \, \Omega$ per phase. The line joining the generator and the load has an impedance of $0.6 + j0.7 \, \Omega$ per phase. Assuming a positive sequence for the source voltages and that $V_{an} = 120 \angle 30^\circ \text{ V}$, find:
(a) the line voltages, (b) the line currents.

✓ Answers: a) $207.8 \angle 60^\circ \text{ V}$, $207.8 \angle -60^\circ \text{ V}$, $207.8 \angle -180^\circ \text{ V}$, (b) $3.75 \angle -8.66^\circ \text{ A}$, $3.75 \angle -128.66^\circ \text{ A}$, $3.75 \angle 111.34^\circ \text{ A}$.

12.3b Balanced Y-Δ System

Most practical 3-phase configuration



Balanced Y-Δ System — Analysis Method

12.3b — Y-Δ CONNECTION

Source is Y-connected, load is Δ-connected. No neutral wire. Phase voltages of source → line voltages → applied directly across each Δ load.

METHOD 1 — Direct Phase Current Approach

Line voltages from source

$$V_{AB} = \sqrt{3} \cdot V_p \angle 30^\circ, V_{BC} = \sqrt{3} \cdot V_p \angle -90^\circ, V_{CA} = \sqrt{3} \cdot V_p \angle 150^\circ$$

Phase currents in Δ load

$$I_{AB} = V_{AB} / Z_{\Delta}, I_{BC} = V_{BC} / Z_{\Delta}, I_{CA} = V_{CA} / Z_{\Delta}$$

Line currents from KCL

$$I_a = I_{AB} - I_{CA} \text{ (apply KCL at each } \Delta \text{ node)}$$

Result (abc sequence)

$$I_L = \sqrt{3} \cdot I_p \angle -30^\circ, \quad \text{and} \quad I_L \text{ lags } I_p \text{ by } 30^\circ$$

Critical Formula — Y-Δ

$$I_L = \sqrt{3} \cdot I_{p\Delta} \angle -30^\circ, \\ I_L \text{ lags } I_p \text{ by } 30^\circ$$

METHOD 2 — Convert Δ→Y, Solve as Y-Y

Convert load

$$Z_{Yeq} = Z_{\Delta} / 3$$

Now have Y-Y system

Solve using single-phase equivalent

Line current directly

$$I_a = V_{an} / Z_{Yeq}$$

Phase currents in Δ

$$I_p = \frac{I_L}{\sqrt{3}} \angle 30^\circ, \quad I_p \text{ leads } I_L \text{ by } 30^\circ$$

Preferred Method

Method 2 is faster: $Z_Y = Z_{\Delta} / 3$
then single-phase: $I_a = V_{an} / Z_{Yeq}$

⚠ In Y-Δ, there is NO neutral wire — and you cannot put one in (delta load has no neutral point). Three-wire system only.

Example 12.3 — Balanced Y-Δ Phase and Line Currents

WORKED EXAMPLE — Y-Δ

Problem: Balanced abc-sequence Y-connected source, $V_{an} = 100\angle 10^\circ$ V. Δ -connected load: $Z_\Delta = 8+j4\ \Omega$ per phase. Find phase and line currents.

Step 1

Find line voltage V_{AB}

$$V_{AB} = \sqrt{3} \cdot V_p \angle 30^\circ = \sqrt{3} \times 100 \angle 40^\circ = 173.2 \angle 40^\circ \text{ V}$$

(Line voltage = $\sqrt{3}$ × phase voltage, and leads it by 30°)

Step 2

Find Δ impedance in polar form

$$Z_\Delta = 8+j4\ \Omega \rightarrow |Z_\Delta| = \sqrt{64+16} = \sqrt{80} = 8.944\ \Omega, \theta = \arctan(4/8) = 26.57^\circ$$
$$Z_\Delta = 8.944 \angle 26.57^\circ\ \Omega$$

Step 3

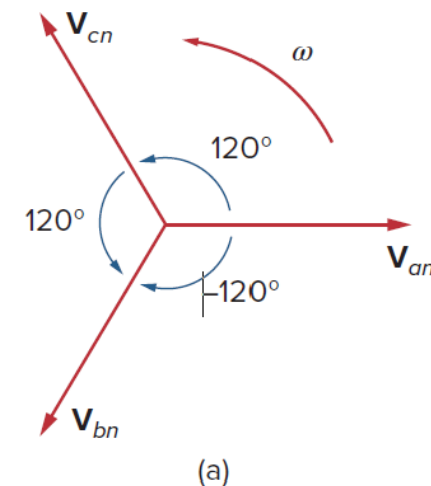
Phase currents in Δ load

$$I_{AB} = V_{AB} / Z_\Delta = 173.2 \angle 40^\circ / 8.944 \angle 26.57^\circ = 19.36 \angle 13.43^\circ \text{ A}$$
$$I_{BC} = I_{AB} \angle -120^\circ = 19.36 \angle -106.57^\circ \text{ A} \quad I_{CA} = I_{AB} \angle +120^\circ = 19.36 \angle 133.43^\circ \text{ A}$$

Step 4

Line currents ($I_L = \sqrt{3} \cdot I_p \angle 30^\circ$)

$$I_a = \sqrt{3} \times I_{AB} \angle -30^\circ = 1.732 \times 19.36 \angle (13.43^\circ - 30^\circ) = 33.53 \angle -16.57^\circ \text{ A}$$
$$I_b = 33.53 \angle -136.57^\circ \text{ A} \quad I_c = 33.53 \angle 103.43^\circ \text{ A}$$



✓ Phase currents: $I_{AB} = 19.36 \angle 13.43^\circ$ A, $I_{BC} = 19.36 \angle -106.57^\circ$ A, $I_{CA} = 19.36 \angle 133.43^\circ$ A
Line currents: $I_a = 33.53 \angle -16.57^\circ$ A, $I_b = 33.53 \angle -136.57^\circ$ A, $I_c = 33.53 \angle 103.43^\circ$ A

Example 12.3 — Balanced Y- Δ Phase and Line Currents

WORKED EXAMPLE — Y- Δ

One line voltage of a balanced Y-connected source is $V_{AB} = 120 \angle -20^\circ$ V. If the source is connected to a Δ -connected load of $20 \angle 40^\circ \Omega$, find the phase and line currents. Assume the abc sequence.

Answer: $6 \angle -60^\circ$ A, $6 \angle -180^\circ$ A, $6 \angle 60^\circ$ A, $10.392 \angle -90^\circ$ A, $10.392 \angle 150^\circ$ A, $10.392 \angle 30^\circ$ A.

Δ - Δ and Δ -Y Balanced Systems

12.3c & 12.3d — Δ - Δ

DELTA-DELTA (Δ - Δ)

Source

$$V_{ab} = V_p, V_{bc} = V_p \angle -120^\circ, V_{ca} = V_p \angle +120^\circ$$

Load voltages

$$V_{AB} = V_{ab}, V_{BC} = V_{bc}, V_{CA} = V_{ca} \text{ (line = phase in } \Delta \text{)}$$

Phase currents

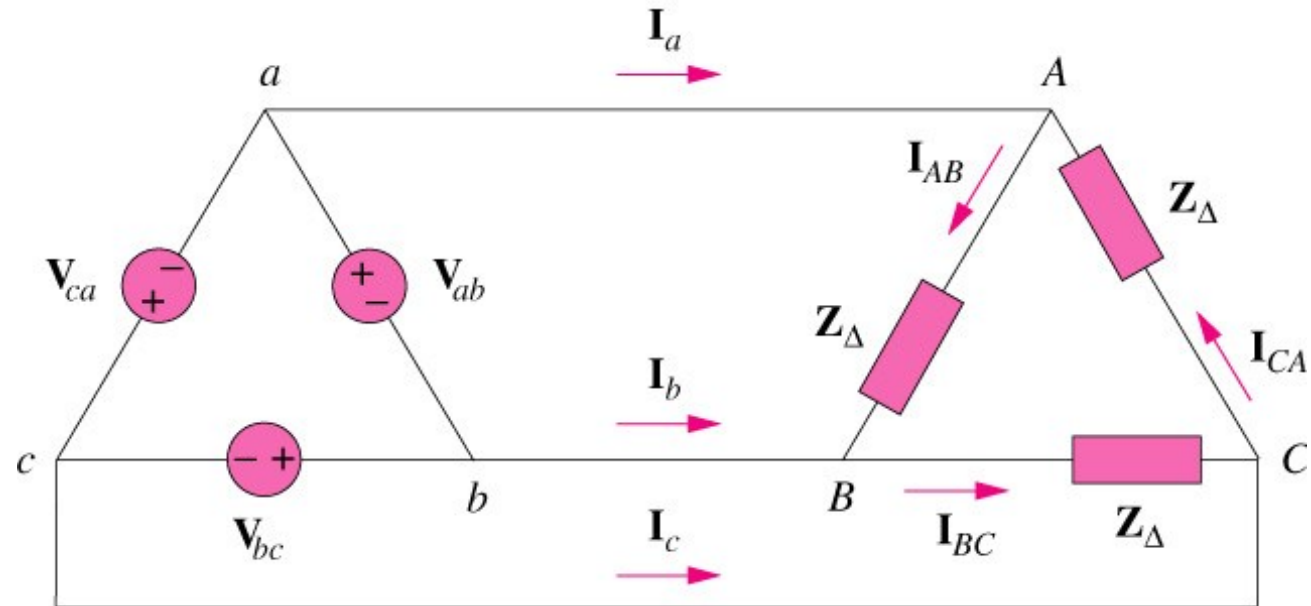
$$I_{AB} = V_{AB} / Z_{\Delta}, \text{ (sequence gives } I_{BC}, I_{CA} \text{)}$$

Line currents

$$I_a = I_{AB} - I_{CA}, I_L = \sqrt{3} \cdot I_p \angle -30^\circ,$$

Key rule

$$V_L = V_p \quad \text{and} \quad I_L = \sqrt{3} I_p$$



Example 12.2 — Balanced

WORKED EXAMPLE — Δ - Δ

A balanced Δ -connected load having an impedance $20-j15$ ohm is connected to a Δ -connected positive-sequence generator having ($V_{ab} = 330\angle 0^\circ$ V). Calculate the phase currents of the load and the line currents.

Step 1

The phase currents

$$I_{AB} = V_{AB} / Z_{\Delta}$$

$$I_{AB} = 13.2\angle 36.87^\circ \text{ A}; I_{BC} = 13.2\angle -81.13^\circ \text{ A}; I_{CA} = 13.2\angle 156.87^\circ \text{ A}$$

Step 2

The line currents

$$I_L = \sqrt{3} \cdot I_p \angle -30^\circ,$$

$$I_a = 22.86\angle 6.87^\circ \text{ A}; I_b = 22.86\angle -113.13^\circ \text{ A}; I_c = 22.86\angle 126.87^\circ \text{ A}$$

Δ-Δ and Δ-Y Balanced Systems

DELTA-WYE (Δ-Y)

Δ source voltages

$$V_{ab} = V_p, V_{bc} = V_p \angle -120^\circ, V_{ca} = V_p \angle +120^\circ$$

Convert Δ source to Y

$$V_{p(an)} = V_{L(ab)} / \sqrt{3} \angle -30^\circ$$

Now solve as Y-Y

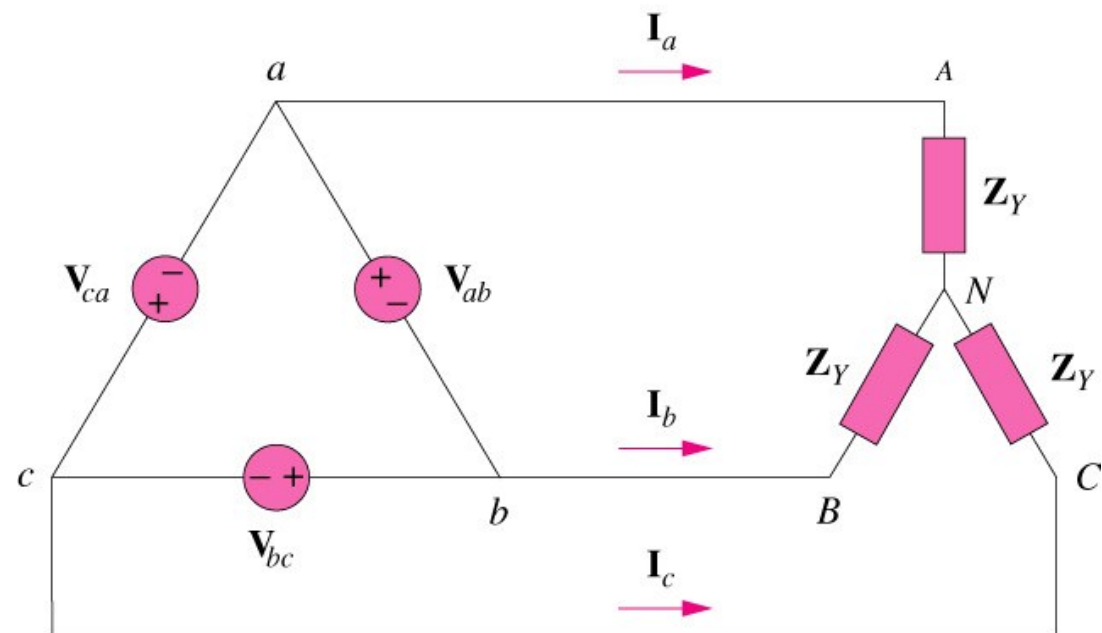
$$I_a = V_{(an)} / Z_Y \text{ (single-phase equivalent)}$$

Line current = phase current

$$I_L = I_p(\text{wye}), I_b = I_a \angle -120^\circ, I_c = I_a \angle +120^\circ$$

Key rule

$$V_L = V_p \quad (\text{source}) \quad \cdot \quad I_L = I_p \quad (\text{load, Y side})$$



Example 12.2 — Balanced Y-Y Line Currents

WORKED EXAMPLE — Δ -Y

Problem: A balanced Y-connected load with a phase impedance $40+j25$ ohm is supplied by a balanced, positive-sequence Δ -connected source with a line voltage of 210V. Calculate the phase currents. Use V_{ab} as reference.

✓ **Answers:** $I_a \rightarrow I_a = 121.2\angle-30^\circ / 47.17\angle32^\circ = 2.57\angle-62^\circ$ A

Line vs. Phase Quantities — The Critical Relationships

12.3 — KEY FORMULAS

Line quantity = measured between two line wires (A-B, B-C, C-A) | Phase quantity = measured across one element

Configuration	V_line vs V_phase	I_line vs I_phase	Phase Shift
Y-Y	$V_L = \sqrt{3} \cdot V_p$	$I_L = I_p(\text{same})$	V_L leads V_p by 30°
Y-Δ	$V_L = \sqrt{3} \cdot V_p$	$I_L = \sqrt{3}I_p \angle -30^\circ$	I_L lags I_p by 30°
Δ-Δ	$V_L = V_p$	$I_L = \sqrt{3}I_p \angle -30^\circ$	I_L lags I_p by 30°
Δ-Y	$V_{p(an)} = V_{L(ab)} / \sqrt{3} \angle -30^\circ$	$I_L = I_p(\text{wye})$	Use Δ→Y transform

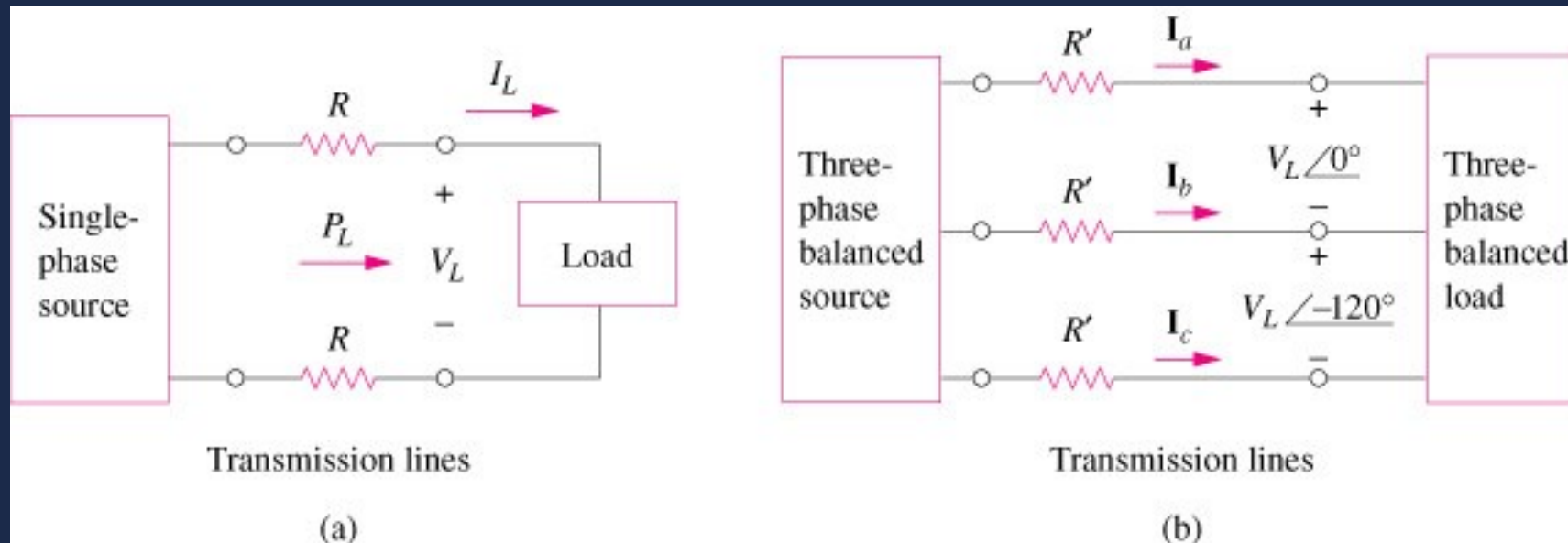
 $\sqrt{3} \approx 1.732$ is the universal conversion factor between line and phase.
For DELTA loads: current divides into 3 branches $\rightarrow I_L = \sqrt{3}I_p \angle -30^\circ$,
For WYE: voltage adds through two phases $\rightarrow V_L = \sqrt{3} \cdot V_p \angle 30^\circ$,

Transform rule:
 $Z_Y = Z_\Delta / 3$
 $Z_\Delta = 3Z_Y$

 Master Strategy: ANY balanced 3-phase circuit can be converted to an equivalent Y-Y and solved using the single-phase equivalent. This is the most efficient approach.

12.7 Power in Balanced 3-Phase Systems

Real · Reactive · Apparent · Complex Power



Power in a Balanced Three-Phase System

For a balanced load, total 3-phase power = 3 × (power per phase). Key insight: instantaneous total power is **CONSTANT** (no pulsation).

Formulas per Phase (Y-connected load, V_p = phase voltage, I_p = phase current):

Real Power (per phase)

$$P_{\phi} = V_{rms} I_{rms} \cos \phi \text{ [W]}$$

Reactive Power (per phase)

$$Q_{\phi} = V_{rms} I_{rms} \sin \theta \text{ [VAR]}$$

Complex Power (per phase)

$$S_{\phi} = V_{rms} I_{rms}^* = P_{\phi} + jQ_{\phi}$$

Total Three-Phase Power (all configurations — use LINE quantities for convenience):

Total Real Power P

$$P = \sqrt{3} V_L I_L \cos \theta \text{ [W]}$$
$$(= 3 V_P I_P \cos \theta)$$

Total Reactive Power Q

$$Q = \sqrt{3} V_L I_L \sin \theta \text{ [VAR]}$$
$$(= 3 V_P I_P \sin \theta)$$

Total Apparent Power |S|

$$|S| = \sqrt{3} V_L I_L \text{ [VA]}$$
$$|S| = \sqrt{P^2 + Q^2}$$

Where θ = angle of load impedance Z = impedance angle = $\theta_v - \theta_i$

Y-load: $V_p = V_L / \sqrt{3}, I_L = I_p$	Substitute to get same formula: $P = \sqrt{3} V_L I_L \cos \theta$
Δ -load: $V_p = V_L, I_p = I_L / \sqrt{3}$	Substitute: $P = 3 V_L (I_L / \sqrt{3}) \cos \theta = \sqrt{3} V_L I_L \cos \theta \checkmark$
Universal formula	$P = \sqrt{3} V_L I_L \cos \theta$ works for BOTH Y and Δ loads
Complex power total	$S_T = 3 S_{\phi} = 3^* V_P I_P^*$

Power factor: $pf = \cos \theta = P / |S|$. Lagging pf → inductive load. Leading pf → capacitive load.

Example 12.6 — Power at Source and Load (Y-Y)

Problem (from Example 12.2): Y-Y system, $V_{an} = 110\angle 0^\circ$ V, $Z_{line} = 5-j2\ \Omega$, $Z_{load} = 10+j8\ \Omega$, $I_a = 6.81\angle -21.8^\circ$ A
Find: total average power, reactive power, and complex power at SOURCE and at LOAD.

At the LOAD

$$S_{load} = 3 \times |I_p|^2 \times Z_{load}$$

$$\begin{aligned} Z_{load} &= 10+j8 = 12.81\angle 38.66^\circ\ \Omega \\ S_L &= 3 \times (6.81)^2 \times 12.81\angle 38.66^\circ = 3 \times 46.38 \times 12.81\angle 38.66^\circ \\ S_L &= 1782\angle 38.66^\circ = (1392 + j1113)\text{ VA} \\ \rightarrow P_{load} &= 1392\text{ W}, Q_{load} = +1113\text{ VAR (lagging, inductive)} \end{aligned}$$

At the SOURCE

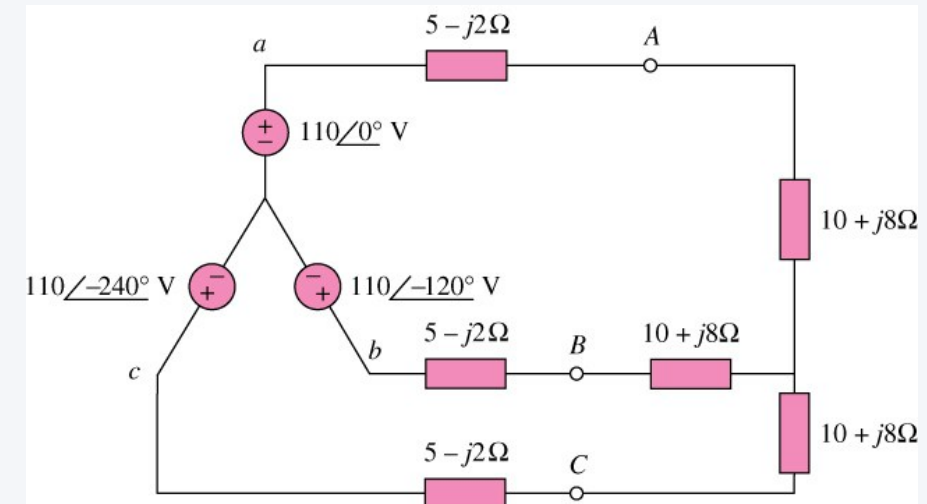
$$S_{source} = -3 V_p I_p^* \text{ (source supplies)}$$

$$\begin{aligned} S_s &= -3 \times V_{an} \times I_a^* = -3 \times (110\angle 0^\circ) \times (6.81\angle +21.8^\circ) \text{ [conjugate flips sign]} \\ S_s &= -3 \times 748.6\angle 21.8^\circ = -2247\angle 21.8^\circ = -(2087 + j834.6)\text{ VA} \\ \rightarrow P_{source} &= -2087\text{ W (supplies 2087 W)}, Q_{source} = -834.6\text{ VAR} \end{aligned}$$

Line losses

$$S_{line} = 3 \times |I_p|^2 \times Z_{line}$$

$$\begin{aligned} Z_{line} &= 5-j2\ \Omega \\ S_{line} &= 3 \times (6.81)^2 \times (5-j2) = 3 \times 46.38 \times (5-j2) = (695.7 - j278.3)\text{ VA} \\ \text{Check: } S_{source} + S_{line} + S_{load} &= 0 \quad \checkmark \quad (-2087 + 695.7 + 1392 \approx 0) \end{aligned}$$



Conservation check: $S_{source} + S_{line} + S_{load} = 0$ always holds. Negative power at source = it is SUPPLYING power.

12.8 Unbalanced Three-Phase Systems

When the math gets harder — mesh & nodal analysis

Unbalanced Three-Phase Systems

- An unbalanced system results from: (1) unequal source voltages, OR (2) unequal load impedances. Cannot use single-phase equivalent — must solve directly.
- To calculate power in an unbalanced three-phase system requires that we find the power in each phase.
- The total power is not simply three times the power in one phase but the sum of the powers in the three phases.

4-WIRE UNBALANCED Y-LOAD (neutral present)

Line currents

Each phase solved independently:

$$I_a = \frac{V_{AN}}{Z_A}$$

Phase a uses its own impedance Z_A

$$I_b = \frac{V_{BN}}{Z_B}$$

Phase b uses its own impedance Z_B

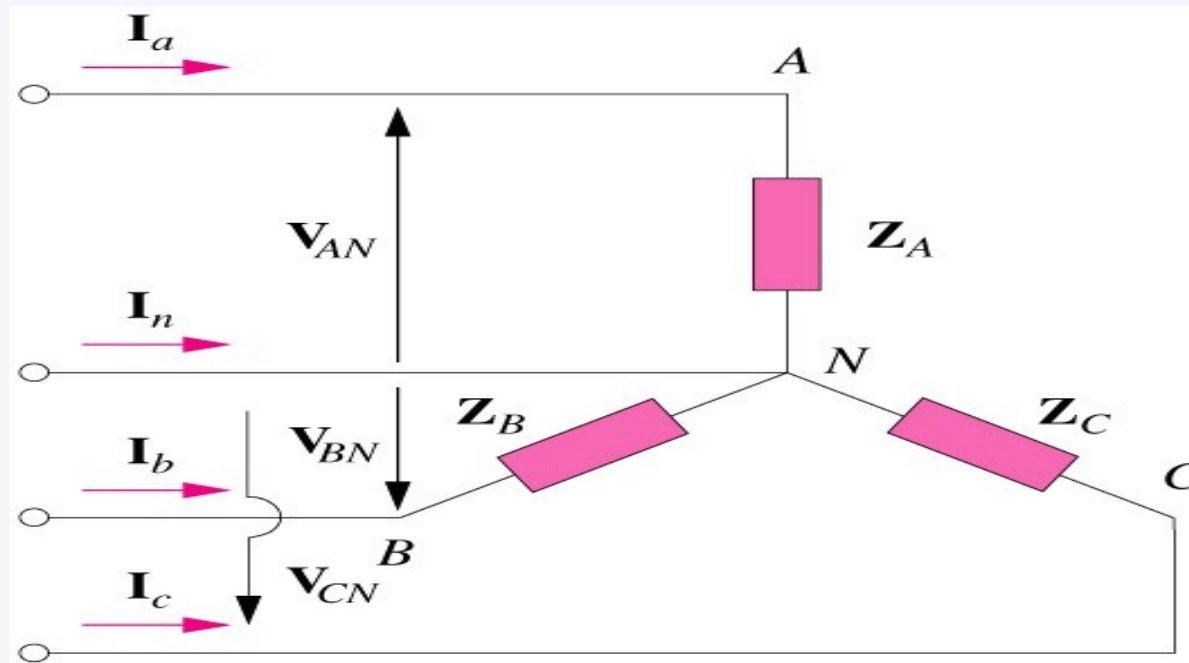
$$I_c = \frac{V_{CN}}{Z_C}$$

Phase c uses its own impedance Z_C

Neutral current

$$I_n = -(I_a + I_b + I_c)$$

✦ Ex 12.9: $V = 100$ V, acb sequence
 $Z_A = 15\Omega$, $Z_B = 10 + j5\Omega$, $Z_C = 6 - j8\Omega$
 $I_n = -(I_a + I_b + I_c) = 10.06 \angle 178.4^\circ$ A



✦ Total power in unbalanced = sum per phase:
 $P = \text{Re}(S_A) + \text{Re}(S_B) + \text{Re}(S_C)$
NOT simply $3 \times P_{\text{one phase}}$

Three-Phase Circuit Problem-Solving Strategy

Step 1

Identify the configuration

Determine: Is source Y or Δ ? Is load Y or Δ ? $\rightarrow$ Gives you Y-Y, Y- Δ , Δ - Δ , or Δ -Y.

Step 2

Is it balanced or unbalanced?

Balanced: All phase impedances equal. Unbalanced: Use mesh or nodal analysis directly.

Step 3

Convert everything to Y (if balanced)

Use $Z_Y = Z_\Delta/3$ for loads. Use $V_{an} = V_{ab}/\sqrt{3} \angle -30^\circ$ for Δ sources.

Step 4

Solve single-phase equivalent (phase a only)

$I_a = V_{an} / Z_{total}$. Find any unknown voltage or current in phase a.

Step 5

Use phase sequence for phases b and c

$I_b = I_a \angle -120^\circ$, $I_c = I_a \angle +120^\circ$ (abc). Done for currents.

Step 6

Calculate power if required

$P = \sqrt{3} V_L I_L \cos \theta$ or $P = 3 |I_p|^2 \operatorname{Re}(Z_{load})$



The Y-Y single-phase equivalent is the master key — every balanced 3-phase problem reduces to it through $Y \leftrightarrow \Delta$ transformations.

Chapter 12 — Key Takeaways

Three-Phase Circuits

- $\sqrt{3}$ is the universal bridge between line and phase quantities
- Single-phase equivalent unlocks any balanced 3-phase problem
- Δ -Y or Y- Δ transformation ($Z_Y = Z_\Delta/3$) normalizes the circuit
- $P = \sqrt{3} V_L I_L \cos \theta$ works for any balanced configuration
- Unbalanced: no shortcuts — mesh/nodal analysis required