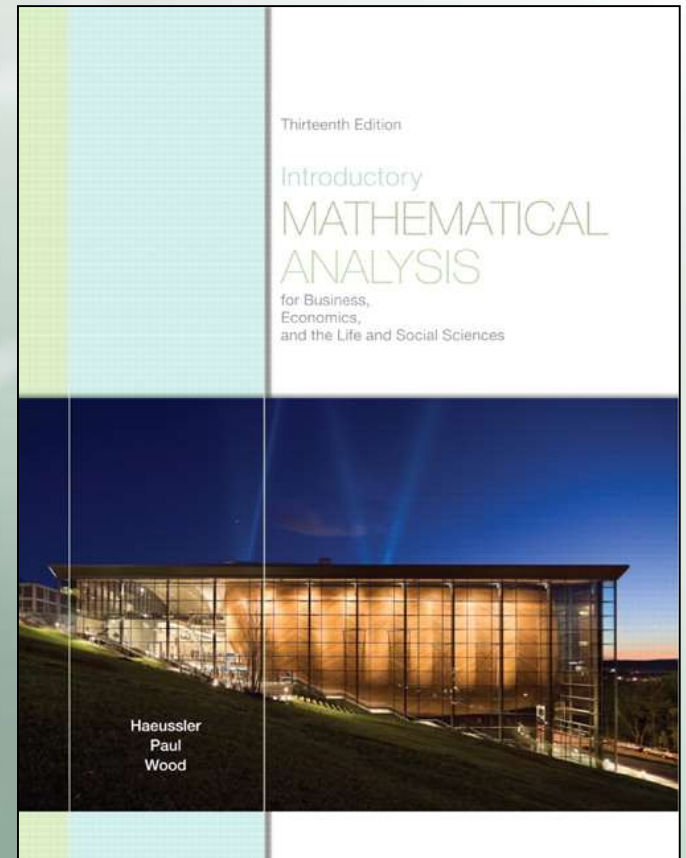


INTRODUCTORY MATHEMATICAL ANALYSIS

For Business, Economics, and the Life and Social Sciences

Chapter 1

Applications and More Algebra



Chapter Objectives

- To model situations described by linear or quadratic equations.
- To solve linear inequalities in one variable and to introduce interval notation.
- To model real-life situations in terms of inequalities.
- To solve equations and inequalities involving absolute values.
- To write sums in summation notation and evaluate such sums.

Chapter Outline

- 1.1) Applications of Equations
- 1.2) Linear Inequalities
- 1.3) Applications of Inequalities
- 1.4) Absolute Value
- 1.5) Summation Notation

1.1 Applications of Equations

- **Modeling:** Translating relationships in the problems to mathematical symbols.

Example 1 - Mixture

A chemist must prepare 350 ml of a chemical solution made up of two parts alcohol and three parts acid. How much of each should be used?

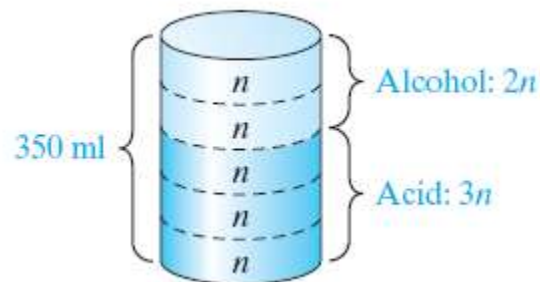
Solution:

Let n = number of milliliters in each part.

$$2n + 3n = 350$$

$$5n = 350$$

$$n = \frac{350}{5} = 70$$



Each part has 70 ml.

Amount of alcohol = $2n = 2(70) = 140$ ml

Amount of acid = $3n = 3(70) = 210$ ml

- **Fixed cost** is the sum of all costs that are independent of the level of production.
- **Variable cost** is the sum of all costs that are dependent on the level of output.
- **Total cost** = variable cost + fixed cost
- **Total revenue** = (price per unit) x
(number of units sold)
- **Profit** = total revenue – total cost

Example 3 – Profit

The Anderson Company produces a product for which the variable cost per unit is \$6 and the fixed cost is \$80,000. Each unit has a selling price of \$10. Determine the number of units that must be sold for the company to earn a profit of \$60,000.

Solution:

Let q = number of sold units.

variable cost = $6q$

total cost = $6q + 80,000$

total revenue = $10q$

Since profit = total revenue – total cost

$$60,000 = 10q - (6q + 80,000)$$

$$140,000 = 4q$$

$$35,000 = q$$

35,000 units must be sold to earn a profit of \$60,000.

Example 5 – Investment

A total of \$10,000 was invested in two business ventures, A and B. At the end of the first year, A and B yielded returns of 6% and 5.75 %, respectively, on the original investments. How was the original amount allocated if the total amount earned was \$588.75?

Solution:

Let x = amount (\$) invested at 6%.

$$(0.06)x + (0.0575)(10,000 - x) = 588.75$$

$$0.06x + 575 - 0.0575x = 588.75$$

$$0.0025x = 13.75$$

$$x = 5500$$

\$5500 was invested at 6%

$\$10,000 - \$5500 = \$4500$ was invested at 5.75%.

Example 7 – Apartment Rent

A real-estate firm owns the Parklane Garden Apartments, which consist of 96 apartments. At \$550 per month, every apartment can be rented. However, for each \$25 per month increase, there will be three vacancies with no possibility of filling them. The firm wants to receive \$54,600 per month from rent. What rent should be charged for each apartment?

Solution 1:

Let r = rent (\$) to be charged per apartment.

Total rent = (rent per apartment) x
(number of apartments rented)

Solution 1 (Con't):

$$54,600 = r \left[96 - \frac{3(r - 550)}{25} \right]$$

$$54,600 = r \left[\frac{2400 - 3r + 1650}{25} \right]$$

$$54,600 = r \left[\frac{4050 - 3r}{25} \right]$$

$$1,365,000 = r(4050 - 3r)$$

$$3r^2 - 4050r + 1,365,000 = 0$$

$$r = \frac{4050 \pm \sqrt{(-4050)^2 - 4(3)(1,365,000)}}{2(3)}$$

$$= \frac{4050 \pm \sqrt{22,500}}{6} = 675 \pm 25$$

Rent should be \$650 or \$700.

Solution 2:

Let n = number of \$25 increases.

Total rent = (rent per apartment) x
(number of apartments rented)

Solution 2 (Con't):

$$54,600 = (550 + 25n)(96 - 3n)$$

$$54,600 = 52,800 + 750n - 75n^2$$

$$75n^2 - 750n + 1800 = 0$$

$$n^2 - 10n + 24 = 0$$

$$(n - 6)(n - 4) = 0$$

$$n = 6 \text{ or } 4$$

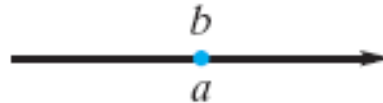
The rent charged should be either

$$550 + 25(6) = \$700 \text{ or}$$

$$550 + 25(4) = \$650.$$

1.2 Linear Inequalities

- Supposing a and b are two points on the real-number line, the relative positions of two points are as follows:



$$a = b$$



$$a < b, a \text{ is less than } b$$
$$b > a, b \text{ is greater than } a$$

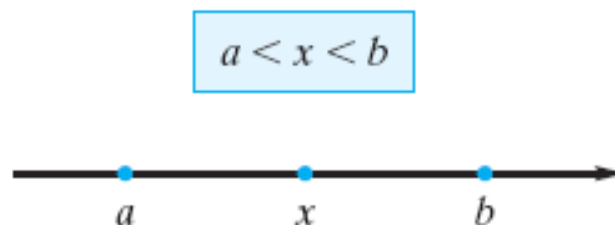


$$a > b, a \text{ is greater than } b$$
$$b < a, b \text{ is less than } a$$

- We use dots to indicate points on a number line.



- Suppose that $a < b$ and x is between a and b .



- **Inequality** is a statement that one number is less than another number.

- Rules for Inequalities:
 1. If $a < b$, then $a + c < b + c$ and $a - c < b - c$.
 2. If $a < b$ and $c > 0$, then $ac < bc$
and $a/c < b/c$.
 3. If $a < b$ and $c < 0$, then $a(c) > b(c)$ and $a/c > b/c$.
 4. If $a < b$ and $a = c$, then $c < b$.
 5. If $0 < a < b$ or $a < b < 0$, then $1/a > 1/b$.
 6. If $0 < a < b$ and $n > 0$, then $a^n < b^n$.
If $0 < a < b$, then $\sqrt[n]{a} < \sqrt[n]{b}$.

- **Linear inequality** can be written in the form
$$ax + b < 0$$
where a and b are constants and $a \neq 0$
- To **solve** an inequality involving a variable is to find all values of the variable for which the inequality is true.

Example 1 – Solving a Linear Inequality

Solve $2(x - 3) < 4$.

Solution:

Replace inequality by equivalent inequalities.

$$2(x - 3) < 4$$

$$2x - 6 < 4$$

$$2x - 6 + 6 < 4 + 6$$

$$2x < 10$$

$$\frac{2x}{2} < \frac{10}{2}$$

$$x < 5$$

Example 3 – Solving a Linear Inequality

Solve $(3/2)(s - 2) + 1 > -2(s - 4)$.

Solution:

$$\frac{3}{2}(s - 2) + 1 > -2(s - 4)$$

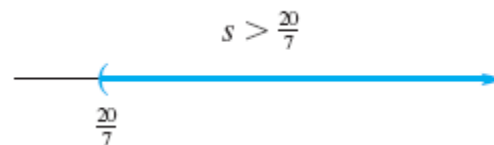
$$2\left[\frac{3}{2}(s - 2) + 1\right] > 2[-2(s - 4)]$$

$$2[3(s - 2) + 2] > -4(s - 4)$$

$$3s - 4 > -4s + 16$$

$$7s > 20$$

$$s > \frac{20}{7}$$



The solution is $(\frac{20}{7}, \infty)$.

1.3 Applications of Inequalities

- Solving word problems may involve inequalities.

Example 1 - Profit

For a company that manufactures aquarium heaters, the combined cost for labor and material is \$21 per heater. Fixed costs (costs incurred in a given period, regardless of output) are \$70,000. If the selling price of a heater is \$35, how many must be sold for the company to earn a profit?

Solution:

$$\text{profit} = \text{total revenue} - \text{total cost}$$

Let q = number of heaters sold.

$$\text{total revenue} - \text{total cost} > 0$$

$$35q - (21q + 70,000) > 0$$

$$14q > 70,000$$

$$q > 5000$$

Example 3 – Current Ratio

After consulting with the comptroller, the president of the Ace Sports Equipment Company decides to take out a short-term loan to build up inventory. The company has current assets of \$350,000 and current liabilities of \$80,000. How much can the company borrow if the current ratio is to be no less than 2.5? (Note: The funds received are considered as current assets and the loan as a current liability.)

Example 3 – Current Ratio

Solution:

Let x = amount the company can borrow.

Current ratio = Current assets / Current liabilities

We want,

$$\frac{350,000 + x}{80,000 + x} \geq 2.5$$

$$350,000 + x \geq 2.5(80,000 + x)$$

$$150,000 \geq 1.5x$$

$$100,000 \geq x$$

The company may borrow up to \$100,000.