

Sheet No-5

Special Curves

Najran University, KSA

Special Curves

- ***Cycloidal Curves***

- Cycloidal curves are generated by a fixed point in the circumference of a circle when it rolls without slipping along a fixed straight line or circular path. The rolling circle is called the generating circle, the fixed straight line, the directing line and the fixed circle, the directing circle.
- Based on the classifications, we have these many types of special curves in engineering drawing:

- **Cycloid**

- Epicycloid
- Hypocycloid

- **Trochoid (Inferior Trochoid and Superior Trochoid)**

- Epitrochoid
- Hypotrochoid

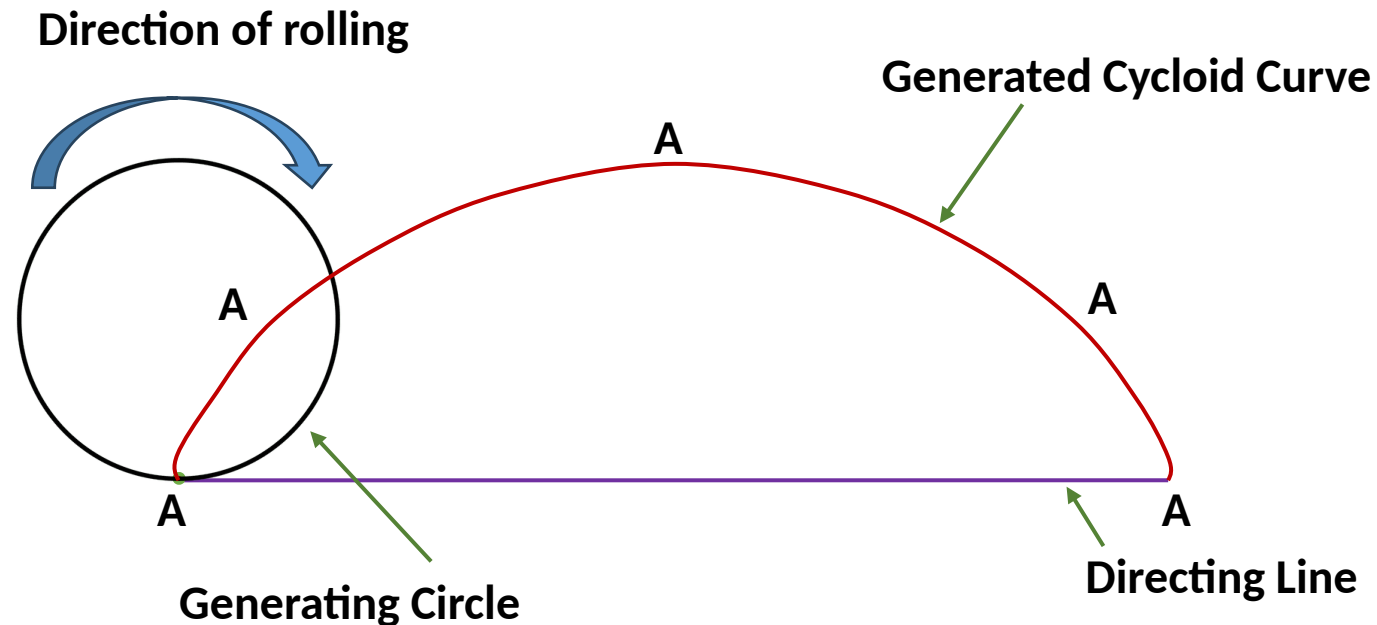
- **Involutes**

- Evolutes
- Spirals (Archimedean Spiral, Logarithmic or Equiangular Spiral)
- Helix
- CAM Profile

Cycloidal Curves: Cycloidal curves are generated by a fixed point on the circumference of a circle, which rolls without slipping along a fixed straight line or circle.

The rolling circle is called ***generating circle*** and the fixed straight line or circle is termed ***directing line*** or ***directing circle***.

Cycloidal curves are used in tooth profile of gears of a dial gauge.

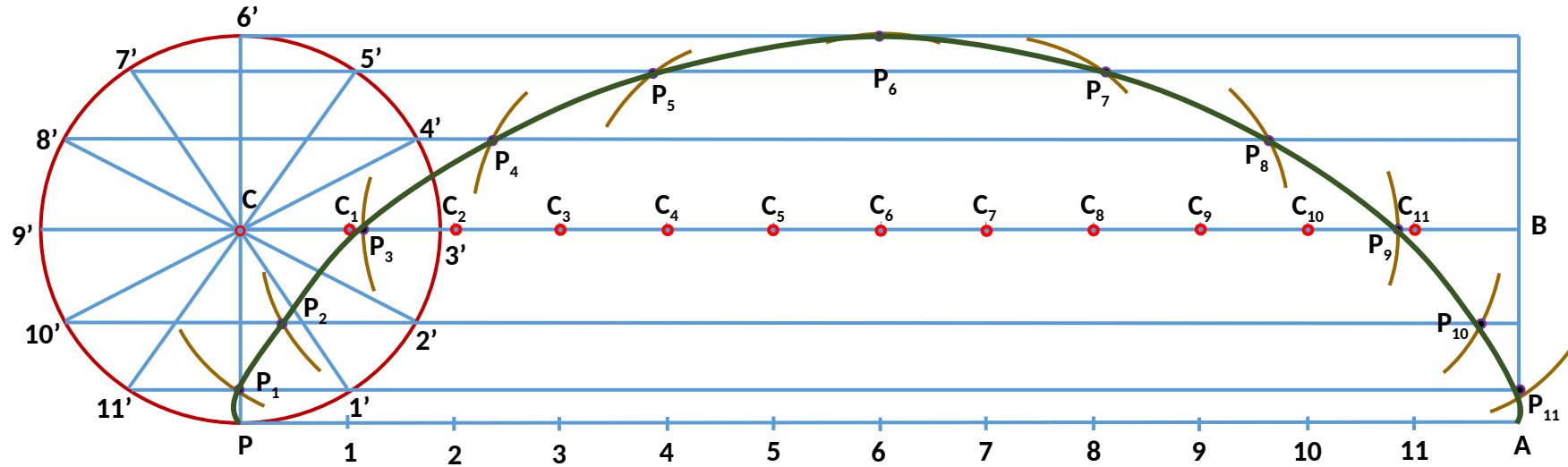


Cycloid Curve: A cycloid is a curve generated by a fixed point on the circumference of a circle, when it rolls without slipping along a straight line.

Sol.: Steps of Construction

- a) With centre C and given radius R , draw a circle. Let P be the generating point.
- b) Draw a line PA tangential to and equal to the circumference of the circle.
- c) Divide the circle and the line PA into the same number of equal parts, say 12, and mark the division-points.
- d) Through C , draw a line CB parallel and equal to PA .
- e) Draw perpendiculars at points 1, 2 etc. cutting CB at points C_1, C_2 etc.
- f) Assume that the circle starts rolling to the right. When point 1' coincides with 1, centre C will move to C_1 . In this position of the circle, the generating point P will have moved to position P_1 on the circle, at a distance equal to $P'1$ from point 1. It is evident that P_1 lies on the horizontal line through 1' and at a distance R from C_1 .
- g) Through the points 1', 2' etc. draw lines parallel to PA .
- h) With centers C_1, C_2 etc. and radius equal to R , radius of generating circle, draw arcs cutting the lines through 1', 2' etc. at points P_1, P_2 etc. respectively. Draw a smooth curve through points $P, P_1, P_2 \dots A$. This curve is the required cycloid.

Sample Problem: To construct a cycloid, given the diameter of the generating circle.



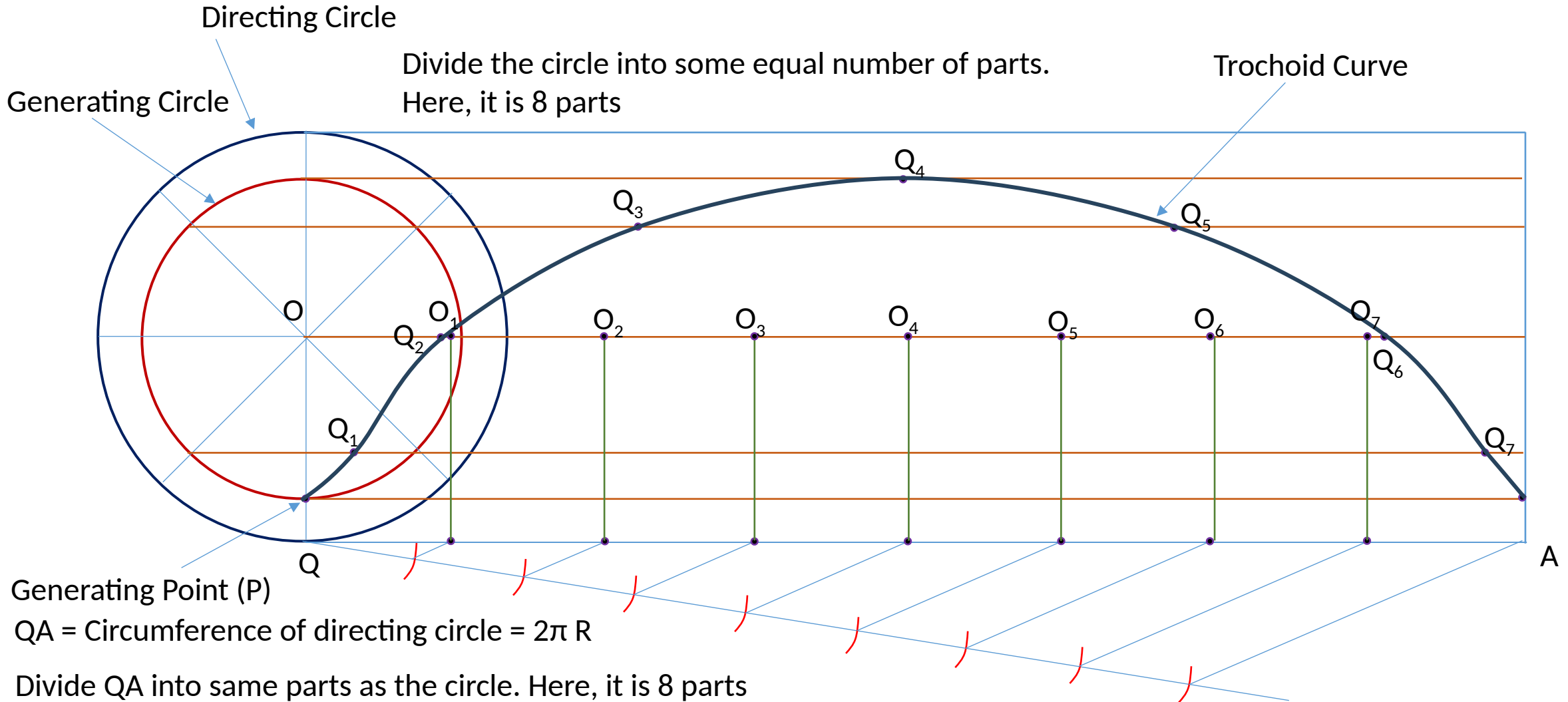
PA is equal to the circumference of the circle.
Divide the Circle and PA into equal number of
parts, say 12 parts.

Trochoid

- Trochoid is a curve generated by a point fixed to a circle, within or outside its circumference, as long as the circle rolls along a straight line without slipping.
- When the point is within the circle, the curve is called an “INFERIOR” trochoid.
- When the point is outside the circle, the curve is called a “SUPERIOR” trochoid.

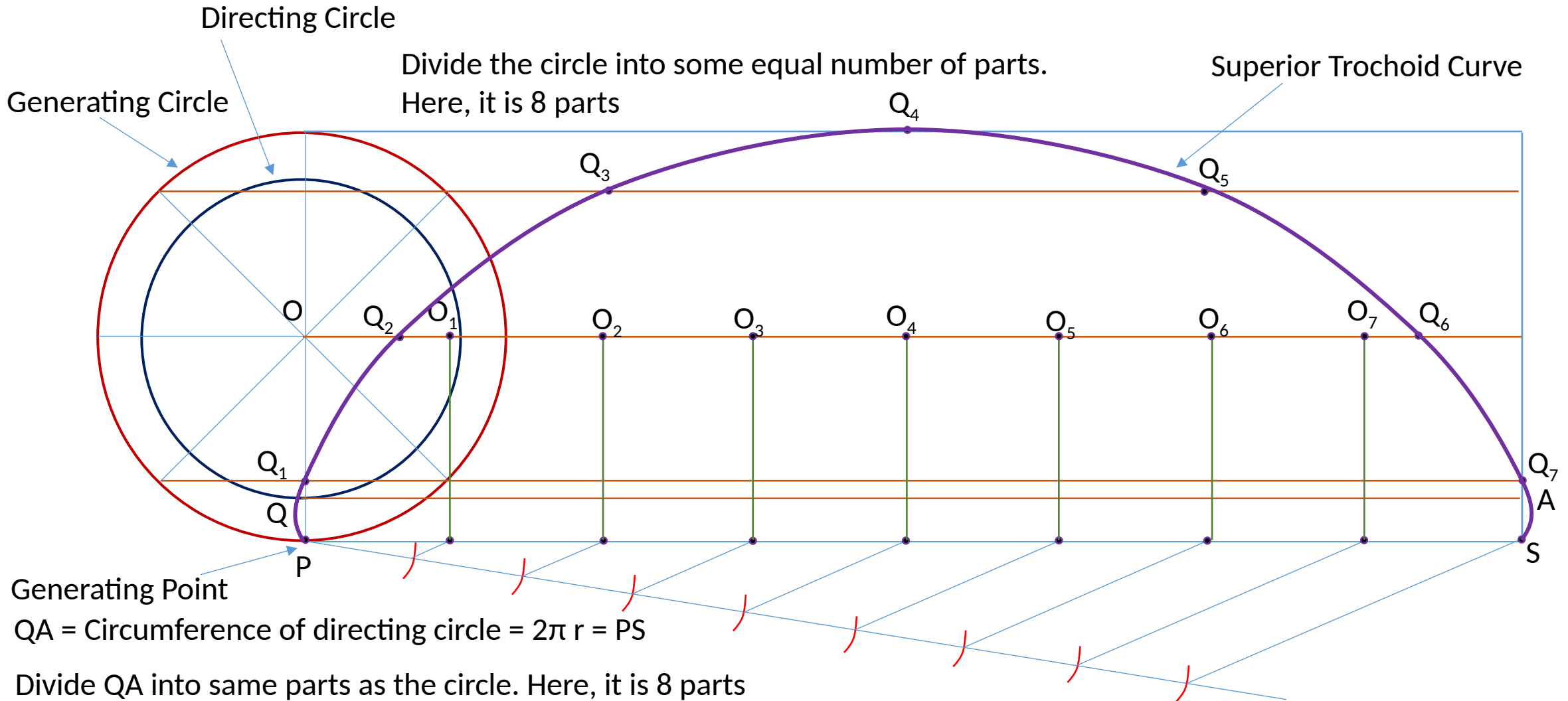
Inferior Trochoid: When the generating point is inside the circumference of the directing circle (Radius R)

Take OP as the radius and $O_1, O_2, O_3, \dots$ as the centre to get the points Q_1, Q_2, Q_3, Q_4 , etc.



Superior Trochoid: When the generating point is outside the circumference of the directing circle (**Radius r**)

Take OP as the radius and $O_1, O_2, O_3, \dots$ as the centre to get the points Q_1, Q_2, Q_3, Q_4 , etc.



Involutes

- The involute is a curve traced out by an end of a piece of thread unwound from a circle or a polygon, the thread being kept tight. It may also be defined as a curve traced out by a point in a straight line which rolls without slipping along a circle or a polygon.
- Involute of a circle is used in teeth profile of gear wheel.
- Mathematically, it can be represented as

$$x = r \cos \theta + r \theta \sin \theta; y = r \sin \theta - r \theta \cos \theta$$

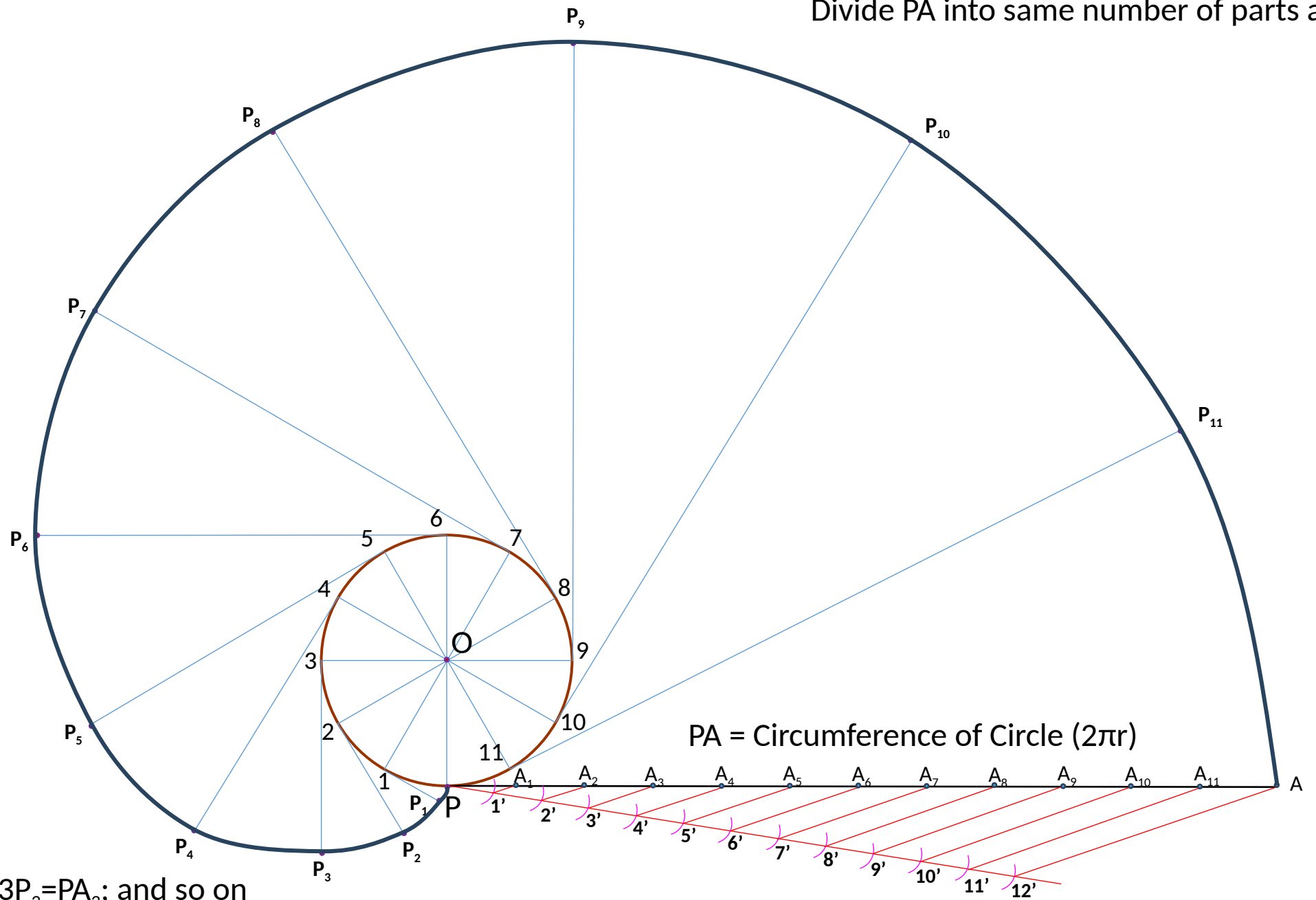
Problem: Draw the involute curve of circle of radius 25 mm.

- Steps of Construction:

- i) Draw a circle with center O and radius 25 mm.
- ii) Divide the circle into 12 equal parts. Draw a line PQ tangent to the circle and equal to the length of the circumference of the circle ($2\pi r$).
- iii) Divide the line PQ into 12 equal parts. Mark the points on PQ as 1', 2', 3', 4', 5'.....11'.
- iv) Draw tangents at points 1, 2, 3, 4, ... etc. and mark on them points P_1 , P_2 , P_3 , ...etc. such that $1P_1 = P1'$, $2P_2 = P2'$, and so on.
- v) From the point P_1 , P_2 , P_3 , ..etc. trace the required involute.

Solution

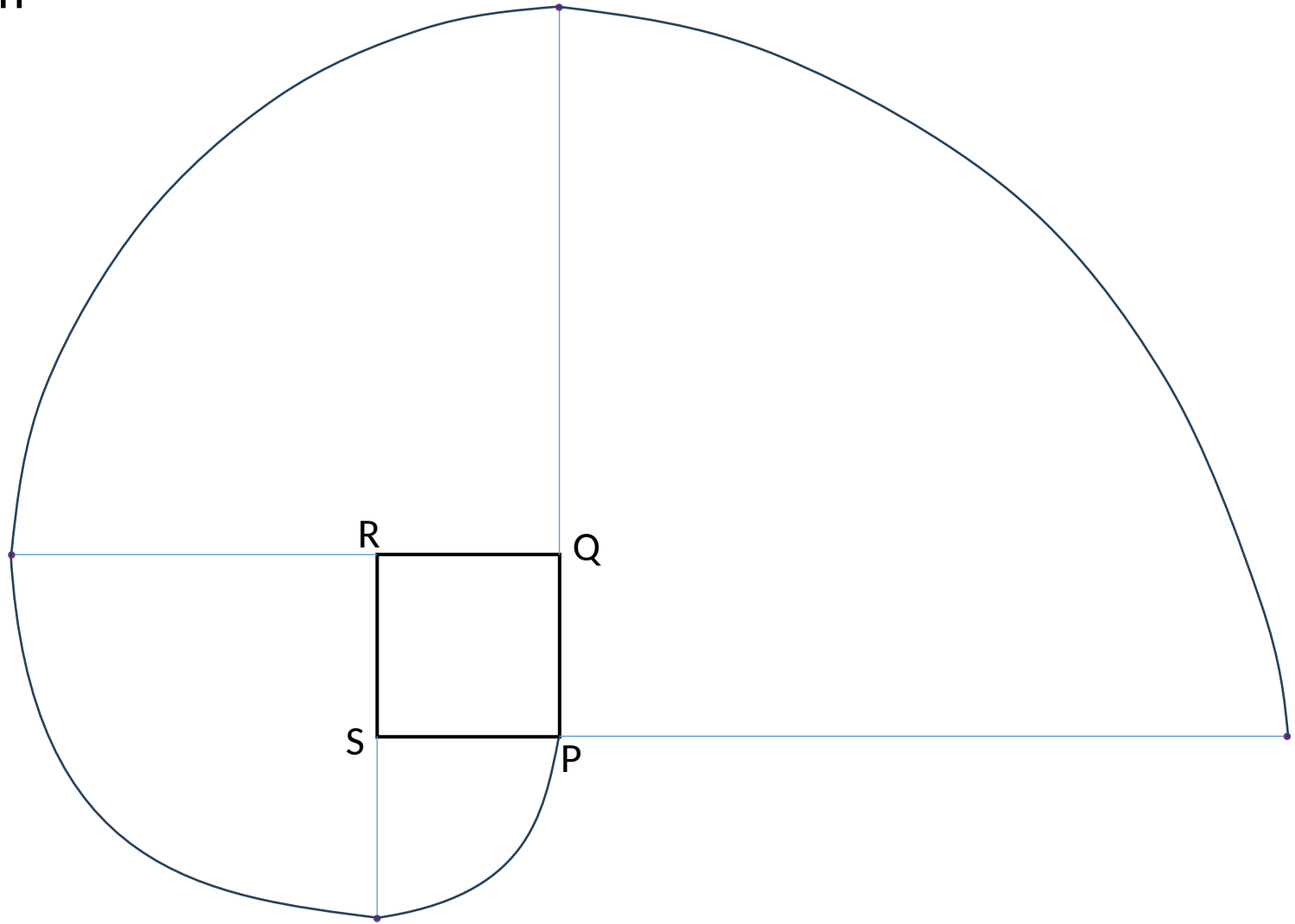
Divide PA into same number of parts as Circle



$$1P_1 = PA_1; 2P_2 = PA_2; 3P_3 = PA_3; \text{ and so on}$$

Involute of a Square

Side of the square: 20 mm



Problems for Sheet

- 1. Draw cycloid curve for a point P which is lying at the top of a circle of diameter 40 mm and it is rolling on a straight line in clockwise direction.
- 2. There is a point B at the top of the circumference of a circle such that it is 6 mm above the circle. If the radius of the circle is 25 mm and it is rolling in the clockwise direction, then trace the trochoid curve for the point.
- 3. The diameter of a rolling circle is 30 mm and a point Q is at the bottom of the circle such that it is at 3 mm distance inside from the circle. Trace the trochoid for the point.
- 4. Trace the involute of a triangle for one revolution if the side of the triangle is 20 mm.