

Introduction to Chemical Engineering

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Chapter II

Engineering Calculations

❖ Why do we need units ?

➤ We need units because we want to measure the amount or quantity of some things. To make this measurement globally acceptable we need to put some unique measurement value. This value is called a UNIT.

Example: What is best

2 liters of Milk at SR 12, or
1.5 liters of Milk at SR 10 ?

❖ What is the difference between Units & Dimensions ?!

- Dimensions are the basic concepts of measurement such as time, mass, length, temperature,...etc.
- Units are the means of expressing the dimension such as feet or centimeters for length and seconds or hours for time.
- It is useful in most engineering calculations to write both the value and the unit of each quantity appearing in an equation:

2 meters, 13second, 4.29 kilograms

- Units can be treated like algebraic variables when quantities are added, subtracted, multiplied, or divided. The numerical values of two quantities may be added or subtracted only if the units are the same.

$$3\text{ cm} - 1\text{ cm} = 2\text{ cm}$$

$$(3x - x = 2x)$$

5 kilograms + 3 joules → cannot be carried out

$$(5x - 2y = ?)$$

10 pounds + 5 grams → can be performed only after the units are transformed to be the same.

➤ On the other hand, numerical values and their corresponding units may always be combined by multiplication or division.

$$\frac{5.0 \text{ km}}{2.0 \text{ h}} = 2.5 \text{ km/h}$$

$$7.0 \frac{\text{km}}{\text{h}} \times 4 \text{ h} = 28 \text{ km}$$

$$3 \text{ m} \times 4 \text{ m} = 12 \text{ m}^2$$

$$6 \text{ cm} \times 5 \frac{\text{cm}}{\text{s}} = 30 \text{ cm}^2/\text{s}$$

$$\frac{6 \text{ g}}{2 \text{ g}} = 3 \quad (3 \text{ is a dimensionless quantity})$$

$$\left(5.0 \frac{\text{kg}}{\text{s}}\right) \div \left(0.20 \frac{\text{kg}}{\text{m}^3}\right) = 25 \text{ m}^3/\text{s} \quad (\text{Convince yourself})$$

Examples

A student travels with his car from Hail to Braidah with an average speed of 120 km/h, how long it will take him to reach Braidah given that the distance is 265 km?

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}, \quad \text{Time} = \frac{\text{Distance}}{\text{Speed}}, \quad \text{Distance} = \text{Speed} \times \text{Time} \quad (2.5)$$

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}} = \frac{265\text{km}}{120\text{km/h}} = 2.2\text{h} = 2\text{h and } 12\text{ min.}$$

- ✓ Calculate the speed of a swimmer who swims 120 m in 60 seconds.
- ✓ A runner runs with speed 8 m/s. Find the distance covered by the runner in 10 seconds.
- ✓ A train covers 50 meters in two seconds, find its speed in km/h

A man walks a distance of 10 m for 5 s. What is his average speed?

$$\text{Average speed} = \frac{\text{Distance travelled}}{\text{Elapsing time}} = \frac{10\text{m}}{5\text{s}} = 2 \frac{\text{m}}{\text{s}}$$

So, velocity has dimensions of length over time.

✓ Could you calculate the time that light takes from the Sun to reach the Earth, knowing that the distance between the Earth and the Sun equals approximately 149.6 million km, and the light moves in the space at constant speed equals 300 000 km /sec.

✓ In that case, if the sunrise is five o'clock. This means that the light travelled from the Sun at ??? o'clock and ??? minutes and ??? sec.

✓ What does it means one light year?!

A force with magnitude of 10 N is applied on an object. If the object moves 3 m, what will be the work done on the object by this external force?

$$Work = \overrightarrow{force} \cdot \overrightarrow{distance} = 10N \times 3m = 30N.m = 30J \quad (2.4)$$

So, work has dimensions of Newton times meter.

A man has a mass of 70 kg_m, what is his weight in Newton? What is his weight in kg_f?

$$Weight = Mass \times \left(\frac{g}{g_c}\right) = 70kg_m \times \left(\frac{9.8m/s^2}{1kg_m.m/s^2/1N}\right) = 686N$$

$$Weight = Mass \times \left(\frac{g}{g_c}\right) = 70kg_m \times \left(\frac{9.8m/s^2}{9.8kg_m.m/s^2/kg_f}\right) = 70kg_f$$

Notice that: 1kg_f = 9.8N and both represent units of force or weight.

❖ Systems of Units

A system of units has the following components:

1. **Base units** for mass, length, time, temperature, electrical current, and light intensity.
2. **Multiple units**, which are defined as multiples or fractions of base units such as minutes, hours, and milliseconds, all of which are defined in terms of the base unit of a second. Multiple units are defined for convenience rather than necessity: it is simply more convenient to refer to **3 yr than to 94,608,000 s**.
3. **Derived units**, obtained in one of two ways:
 - a) By multiplying and dividing base or multiple units (cm^2 , ft/min , $\text{kg}\cdot\text{m}/\text{s}^2$, etc.).
Derived units of this type are referred to as **compound units**.
 - b) As defined equivalents of compound units (e.g., $1 \text{ erg} \equiv (1\text{g}\cdot\text{cm}/\text{s}^2)$, $1\text{lb}_f \equiv 32.174 \text{ lb}_m \cdot \text{ft}/\text{s}^2$).

➤ The two most commonly used systems of units:

❑ SI system of units

❑ AE, or American Engineering system of units

The "Système Internationale d'Unités," or SI for short, has gained widespread acceptance in the scientific and engineering community.

Prefixes are used in SI to indicate powers of ten.

The CGS system (cm-g-second) is almost identical to SI, the principal difference being that grams (g) and centimeters (cm) are used instead of kilograms and meters as the base units of mass and length.

Table 2.3-1: SI and CGS Units

Length	meter (SI)	m	Multiple Unit Preferences	
	centimeter (CGS)	cm	tera (T) = 10^{12}	centi (c) = 10^{-2}
Mass	kilogram (SI)	kg	giga (G) = 10^9	milli (m) = 10^{-3}
	gram (CGS)	g	mega (M) = 10^6	micro (μ) = 10^{-6}
Moles	gram-mole	mol or g-mole	kilo (k) = 10^3	nano (n) = 10^{-9}
Time	second	s		
Temperature	kelvin	K		
Electric current	ampere	A		
Light intensity	candela	cd		

Derived Quantities			
Quantity	Unit	Symbol	Equivalent in Terms of Base Units
Volume	liter	L	0.001 m^3
			1000 cm^3
Force	newton (SI)	N	$1 \text{ kg}\cdot\text{m}/\text{s}^2$
	dyne (CGS)		$1 \text{ g}\cdot\text{cm}/\text{s}^2$
Pressure	pascal (SI)	Pa	$1 \text{ N}/\text{m}^2$
Energy, work	joule (SI)	J	$1 \text{ N}\cdot\text{m} = 1 \text{ kg}\cdot\text{m}^2/\text{s}^2$
	erg (CGS)		$1 \text{ dyne}\cdot\text{cm} = 1 \text{ g}\cdot\text{cm}^2/\text{s}^2$
	gram-calorie	cal	$4.184 \text{ J} = 4.184 \text{ kg}\cdot\text{m}^2/\text{s}^2$
Power	watt	W	$1 \text{ J}/\text{s} = 1 \text{ kg}\cdot\text{m}^2/\text{s}^3$

SI Prefixes

<u>Factor</u>	<u>Prefix</u>	<u>Symbol</u>	<u>Factor</u>	<u>Prefix</u>	<u>Symbol</u>
10^9	giga	G	10^{-1}	deci	d
10^6	mega	M	10^{-2}	centi	c
10^3	kilo	k	10^{-3}	milli	m
10^2	hekta	h	10^{-6}	micro	μ
10^1	deka	da	10^{-9}	nano	n

The base units of the American engineering system are the foot (ft) for length, the pound-mass (lb_m) for mass, and the second (s) for time. This system has two principal difficulties. The first is the occurrence of conversion factors (such as 1 ft/12 in), which, unlike those in the metric systems, are not multiples of 10; the second, which has to do with the unit of force, is discussed in the next section.

Physical Quantity	Name of Unit	Symbol
<i>Some Basic Units</i>		
Length	foot	ft
Mass	pound (mass)	lb_m
Time	second, minute, hour, day	s, min, h (hr), day
Temperature	degree Rankine or degree Fahrenheit	$^{\circ}R$ or $^{\circ}F$
Molar amount	pound mole	$lb\ mol$
<i>Derived Units</i>		
Force	pound (force)	lb_f
Energy	British thermal unit, foot pound (force)	Btu, (ft)(lb_f)
Power	horsepower	hp
Density	pound (mass) per cubic foot	lb_m/ft^3
Velocity	feet per second	ft/s
Acceleration	feet per second squared	ft/s^2
Pressure	pound (force) per square inch	$lb_f/in.^2$, psi
Heat capacity	Btu per pound (mass) per degree F	$Btu/(lb_m)(^{\circ}F)$

❖ CONVERSION OF UNITS

- ✓ A measured quantity can be expressed in terms of any units having the appropriate dimension. A particular velocity, for instance, may be expressed in ft/s, miles/h, km/yr, cm/s, or any other ratio of a length unit to a time unit. The numerical value of the velocity naturally depends on the units chosen.
- ✓ The equivalence between two expressions of the same quantity may be defined in terms of a ratio (conversion factors):

$$\left(\frac{1\text{cm}}{10\text{mm}}\right), \quad \left(\frac{1\text{m}}{100\text{cm}}\right), \quad \left(\frac{100\text{cm}}{1\text{m}}\right), \quad \left(\frac{1\text{km}}{1000\text{m}}\right), \quad \left(\frac{1\text{ft}}{30.48\text{cm}}\right), \quad \left(\frac{1\text{ft}}{0.3048\text{m}}\right)$$

$$\left[\frac{10\text{ mm}}{1\text{ cm}}\right]^2 = \frac{100\text{ mm}^2}{1\text{ cm}^2}$$

FACTORS FOR UNIT CONVERSIONS

Quantity	Equivalent Values
Mass	$1 \text{ kg} = 1000 \text{ g} = 0.001 \text{ metric ton} = 2.20462 \text{ lb}_m = 35.27392 \text{ oz}$ $1 \text{ lb}_m = 16 \text{ oz} = 5 \times 10^{-4} \text{ ton} = 453.593 \text{ g} = 0.453593 \text{ kg}$
Length	$1 \text{ m} = 100 \text{ cm} = 1000 \text{ mm} = 10^6 \text{ microns } (\mu\text{m}) = 10^{10} \text{ angstroms } (\text{\AA})$ $= 39.37 \text{ in.} = 3.2808 \text{ ft} = 1.0936 \text{ yd} = 0.0006214 \text{ mile}$ $1 \text{ ft} = 12 \text{ in.} = 1/3 \text{ yd} = 0.3048 \text{ m} = 30.48 \text{ cm}$
Volume	$1 \text{ m}^3 = 1000 \text{ L} = 10^6 \text{ cm}^3 = 10^6 \text{ mL}$ $= 35.3145 \text{ ft}^3 = 220.83 \text{ imperial gallons} = 264.17 \text{ gal}$ $= 1056.68 \text{ qt}$ $1 \text{ ft}^3 = 1728 \text{ in.}^3 = 7.4805 \text{ gal} = 0.028317 \text{ m}^3 = 28.317 \text{ L}$ $= 28,317 \text{ cm}^3$
Force	$1 \text{ N} = 1 \text{ kg}\cdot\text{m/s}^2 = 10^5 \text{ dynes} = 10^5 \text{ g}\cdot\text{cm/s}^2 = 0.22481 \text{ lb}_f$ $1 \text{ lb}_f = 32.174 \text{ lb}_m\cdot\text{ft/s}^2 = 4.4482 \text{ N} = 4.4482 \times 10^5 \text{ dynes}$
Pressure	$1 \text{ atm} = 1.01325 \times 10^5 \text{ N/m}^2 (\text{Pa}) = 101.325 \text{ kPa} = 1.01325 \text{ bar}$ $= 1.01325 \times 10^6 \text{ dynes/cm}^2$ $= 760 \text{ mm Hg at } 0^\circ\text{C (torr)} = 10.333 \text{ m H}_2\text{O at } 4^\circ\text{C}$ $= 14.696 \text{ lb}_f/\text{in.}^2 (\text{psi}) = 33.9 \text{ ft H}_2\text{O at } 4^\circ\text{C}$ $= 29.921 \text{ in. Hg at } 0^\circ\text{C}$
Energy	$1 \text{ J} = 1 \text{ N}\cdot\text{m} = 10^7 \text{ ergs} = 10^7 \text{ dyne}\cdot\text{cm}$ $= 2.778 \times 10^{-7} \text{ kW}\cdot\text{h} = 0.23901 \text{ cal}$ $= 0.7376 \text{ ft}\cdot\text{lb}_f = 9.486 \times 10^{-4} \text{ Btu}$
Power	$1 \text{ W} = 1 \text{ J/s} = 0.23901 \text{ cal/s} = 0.7376 \text{ ft}\cdot\text{lb}_f/\text{s} = 9.486 \times 10^{-4} \text{ Btu/s}$ $= 1.341 \times 10^{-3} \text{ hp}$

Example: The factor to convert grams to lb_m is $\left(\frac{2.20462 \text{ lb}_m}{1000 \text{ g}}\right)$.

Examples

✓ If a plane travels at twice the speed of sound (assume that the speed of sound is 1100 ft/s), how fast is it going in miles per hour?

$$\frac{2 \times 1100 \text{ ft}}{\text{s}} \left| \frac{1 \text{ mi}}{5280 \text{ ft}} \right| \frac{60 \text{ s}}{1 \text{ min}} \left| \frac{60 \text{ min}}{1 \text{ hr}} \right|$$

$\frac{\text{ft}}{\text{s}} \quad \frac{\text{mi}}{\text{s}} \quad \frac{\text{mi}}{\text{min}}$

✓ Convert 2 km to miles.

$$\frac{2 \text{ km}}{1} \left| \frac{1 \text{ mile}}{1.61 \text{ km}} \right| = 1.24 \text{ mile}$$

✓ Convert 400 in³/day to cm³/min.

$$\frac{400 \text{ in.}^3}{\text{day}} \left| \left(\frac{2.54 \text{ cm}}{1 \text{ in.}} \right)^3 \right| \frac{1 \text{ day}}{24 \text{ hr}} \left| \frac{1 \text{ hr}}{60 \text{ min}} \right| = 4.55 \frac{\text{cm}^3}{\text{min}}$$

✓ **A semiconductor (ZnS) with a particle diameter of 1.8 nanometers. Convert this value to: (a) dm (decimeters) (b) inches.**

$$(a) \frac{1.8 \text{ nm}}{1 \text{ nm}} \left| \frac{10^{-9} \text{ m}}{1 \text{ m}} \right| \frac{10 \text{ dm}}{1 \text{ m}} = 1.8 \times 10^{-8} \text{ dm}$$

$$(b) \frac{1.8 \text{ nm}}{1 \text{ nm}} \left| \frac{10^{-9} \text{ m}}{1 \text{ m}} \right| \frac{39.37 \text{ in.}}{1 \text{ m}} = 7.09 \times 10^{-8} \text{ in.}$$

✓ **Convert an acceleration of 1 cm/s² to its equivalent in km/yr².**

1 cm	3600 ² s ²	24 ² h ²	365 ² day ²	1 m	1 km
s ²	1 ² h ²	1 ² day ²	1 ² yr ²	10 ² cm	10 ³ m

$$= \frac{(3600 \times 24 \times 365)^2 \text{ km}}{10^2 \times 10^3 \text{ yr}^2} = \boxed{9.95 \times 10^9 \text{ km/yr}^2}$$

A man has a mass of 70 kg_m , what is his mass expressed in lb_m ?

Data: $1 \text{ lb}_m = 0.4536 \text{ kg}_m$. This means that the conversion factor is $\left(\frac{1 \text{ lb}_m}{0.4536 \text{ kg}_m} \right)$ or $\left(\frac{0.4536 \text{ kg}_m}{1 \text{ lb}_m} \right)$

$$\text{Mass of the man} = 70 \text{ kg}_m \times \left(\frac{1 \text{ lb}_m}{0.4536 \text{ kg}_m} \right) = 154.32 \text{ lb}_m$$

A man has a mass of 160 lb_m , what is his mass in kg_m ?

$$\text{Mass of the man} = 160 \text{ lb}_m \times \left(\frac{0.4536 \text{ kg}_m}{1 \text{ lb}_m} \right) = 72.6 \text{ kg}_m$$

Try by yourself:

✓ A man has 1000 Saudi Riyals (SR), how much does he have in US dollar (\$), Euro (€)

✓ What are the factors (numerical values and units) needed to convert:

(a) meters to millimeters?

(b) nanoseconds to seconds?

(c) square centimeters to square meters?

❖ FORCE AND WEIGHT

According to Newton's second law of motion, force is proportional to the product of mass and acceleration (length/time²). **Natural force units are**, therefore, $\text{kg} \cdot \text{m}/\text{s}^2$ (SI), $\text{g} \cdot \text{cm}/\text{s}^2$ (CGS), and $\text{lb}_m \cdot \text{ft}/\text{s}^2$ (American engineering). To avoid having to carry around these complex units in all calculations involving forces, *derived force units have been defined in each system.*

In the metric systems, the derived force units (the Newton in SI, the dyne in the CGS system) are defined to equal the natural units:

$$\begin{aligned} 1 \text{ newton (N)} &\equiv 1 \text{ kg} \cdot \text{m}/\text{s}^2 \\ 1 \text{ dyne} &\equiv 1 \text{ g} \cdot \text{cm}/\text{s}^2 \end{aligned}$$

$$1 \text{ lb}_f \equiv 32.17 \left(\frac{\text{lb}_m \cdot \text{ft}}{\text{s}^2} \right)$$

**In the American engineering system, the derived force unit - called a pound-force (lb_f).*

For example, the force needed to accelerate a mass of 5.00 kg at a rate of $9.00 \text{ m}/\text{s}^2$ is:

$$F = m \cdot a = 5 \text{ kg} \cdot 9 \frac{\text{m}}{\text{s}^2} \times \left(\frac{1 \text{ N}}{\left(\frac{\text{kg}_m \cdot \text{m}}{\text{s}^2} \right)} \right) = 45 \text{ N}$$

The force in lb_f required to accelerate a mass of 4.00 lb_m at a rate of $9.00 \text{ ft}/\text{s}^2$ is

$$F = \frac{4.00 \text{ lb}_m}{1} \times \frac{9.00 \text{ ft}}{\text{s}^2} \times \frac{1 \text{ lb}_f}{32.174 \text{ lb}_m \cdot \text{ft}/\text{s}^2} = 1.12 \text{ lb}_f$$

What is the difference between Weight & Mass

✓The weight of an object is the force exerted on the object by gravitational attraction.

✓Suppose that an object of mass m is subjected to a gravitational force W (W is by definition the weight of the object) and that if this object were falling freely its acceleration would be g . The weight, mass, and free-fall acceleration of the object are related by:

$$W = mg$$

The value of g at sea level and 45° latitude is



$$\begin{aligned} g &= 9.8066 \frac{m}{s^2} \\ g &= 980.66 \frac{cm}{s^2} \\ g &= 32.174 \frac{ft}{s^2} \end{aligned}$$

TEST YOURSELF!

1. What is a force of $2 \text{ kg} \cdot \text{m}/\text{s}^2$ equivalent to in newtons? What is a force of $2 \text{ lbm} \cdot \text{ft}/\text{s}^2$ equivalent to in lbf?
2. If the acceleration of gravity at a point is $g = 9.8 \text{ m}/\text{s}^2$ and an object is resting on the ground at this point, is this object accelerating at a rate of $9.8 \text{ m}/\text{s}^2$?
3. Suppose an object weighs 9.8 N at sea level. What is its mass? Would its mass be greater, less, or the same on the moon? How about its weight?
4. Suppose an object weighs 2 lbf at sea level. What is its mass? Would its mass be greater, less, or the same at the center of the earth? How about its weight? (Careful!).

Example:

Water has a density of $62.4 \text{ lb}_m/\text{ft}^3$. How much does 2.0 ft^3 of water weigh;

(1) at sea level and 45° latitude?

(2) in Denver, Colorado, where the altitude is 5374 ft and the gravitational acceleration is 32.139 ft/s^2 ?

(3) at the surface of the moon where the gravitational acceleration is 6.0 ft/s^2 ?

(4) at the top end of the atmosphere ?

$$\text{The mass of water: } M = \rho \cdot V = 62.4 \frac{\text{lb}_m}{\text{ft}^3} \cdot 2.0 \text{ ft}^3 = 124.8 \text{ lb}_m$$

The weight of the water is

$$W = (124.8 \text{ lb}_m) g \left(\frac{\text{ft}}{\text{s}^2} \right) \left(\frac{1 \text{ lb}_f}{32.174 \text{ lb}_m \cdot \text{ft/s}^2} \right)$$

1. At sea level, $g = 32.174 \text{ ft/s}^2$, so that $W = 124.8 \text{ lb}_f$.

2. In Denver, $g = 32.139 \text{ ft/s}^2$, and $W = 124.7 \text{ lb}_f$.

$$\text{@moon: } W = M \cdot g = 124.8 \text{ lb}_m \cdot g = 124.8 \text{ lb}_m \cdot 6.0 \frac{\text{ft}}{\text{s}^2} \times \left(\frac{1 \text{ lb}_f}{32.17 \left(\frac{\text{lb}_m \cdot \text{ft}}{\text{s}^2} \right)} \right) = 23.3 \text{ lb}_f$$

@top of the atmosphere:

$$W = M \cdot g = 124.8 \text{ lb}_m \cdot g = 124.8 \text{ lb}_m \cdot 0.0 \frac{\text{ft}}{\text{s}^2} \times \left(\frac{1 \text{ lb}_f}{32.17 \left(\frac{\text{lb}_m \cdot \text{ft}}{\text{s}^2} \right)} \right) = 0.0 \text{ lb}_f$$

$$1 \text{ lb}_f \equiv 32.17 \left(\frac{\text{lb}_m \cdot \text{ft}}{\text{s}^2} \right)$$

Exercise : Calculate the density (lb_m/ft^3) of a wooden material given that a cube of 1000 cm^3 has a mass of 800 g.

❖ NUMERICAL CALCULATION AND ESTIMATION

❖ Scientific Notation, Significant Figures, and Precision

✓ Both very large and very small numbers are commonly encountered in process calculations. A convenient way to represent such numbers is to use scientific notation, in which a number is expressed as the product of another number (usually between 0.1 and 10) and a power of 10

$$123,000,000 = 1.23 \times 10^8 \text{ (or } 0.123 \times 10^9 \text{)}$$

$$0.000028 = 2.8 \times 10^{-5} \text{ (or } 0.28 \times 10^{-4} \text{)}$$

Scientific Notation

The significant figures of a number are the digits from the first nonzero digit on the left to either (a) the last digit (zero or nonzero) on the right if there is a decimal point, or (b) the last nonzero digit of the number if there is no decimal point.

$2300 = 2.3 \times 10^3$ has two significant figures.

$2300. = 2.300 \times 10^3$ has four significant figures.

$2300.0 = 2.3000 \times 10^3$ has five significant figures.

$23,040 = 2.304 \times 10^4$ has four significant figures.

$0.035 = 3.5 \times 10^{-2}$ has two significant figures.

$0.03500 = 3.500 \times 10^{-2}$ has four significant figures.

Significant Figures

✓ Five rules govern significant figures:

- Non-zero digits are always significant; **1.121** has four significant digits.
- Any zeros between two significant digits are significant; **1.08701** has six significant digits.
- Zeros before the decimal point are placeholders and *not* significant; **in the number .00254, only the 2, 5 and 4 are significant**, meaning the number has 3 significant figures.
- Zeros after the decimal point and after figures are significant; **in the number 0.2540, the 2, 4, 5 and last 0 are significant.**
- Exponential digits in scientific notation are not significant; **1.12×10^6 has three significant digits, 1, 1, and 2.**

✓ These rules ensure accurate representation and interpretation of data. If, for example, you were to read of an experimental reaction in which the resulting chemical weighed **0.0254 g**, you would know that the measurement is accurate to **0.0001 g** and contains 3 significant figures.

Thus, if you report a mass as **8.3 g** (two significant figures), you indicate that the mass lies somewhere between **8.25 and 8.35 g**, whereas if you give the value as **8.300 g** (four significant figures) you indicate that the mass lies between **8.2995 and 8.3005 g**

✓ Most exact numbers (**complete certainty**) are integers: exactly 12 inches are in a foot, there might be exactly 23 students in a class. Exact numbers are often found as conversion factors or as counts of objects. **Exact numbers can be considered to have an infinite number of significant figures.**

✓ Addition and Subtraction of Significant Figures

- *The final value must have only as many decimals as the original value with the least number of decimal places.*

For example:

$$1.121 + 48.00679392 + 6.3457 = 55.47349392$$

*But, because 1.121 has only three decimal places, the answer must actually be: **55.473**, since it is at the third decimal place that uncertainty begins to enter the picture.*

$$\begin{array}{r}
 \downarrow \\
 1530 \quad \downarrow \\
 - 2.56 \\
 \hline
 1527.44 \implies 1530 \\
 \uparrow
 \end{array}$$

$$\begin{array}{r}
 \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\
 1.0000 + 0.036 + 0.22 = 1.2560 \implies 1.26 \\
 \downarrow
 \end{array}$$

$$\begin{array}{r}
 2.75 \times 10^6 + 3.400 \times 10^4 = (2.75 + 0.03400) \times 10^6 \\
 \downarrow \\
 = 2.784000 \times 10^6 \implies 2.78 \times 10^6
 \end{array}$$

✓ Significant Figures in Multiplication and Division

The rule governing multiplication and division of significant figures is slightly different than that for addition and subtraction “the final value can only have as many significant figures as the original value with the least significant figures”

For example:

A scientist needs to calculate a constant value, K, based on the following equation:

$$K = (D \times E) / B$$

where B, D and E are measured values that the scientist observed (weight, volume, temperature, pressure). B=6.00 g D= 22 C E= 22.457 mL

22.457 mL has 5 significant figures and 22 C has 2 significant figures. The number that the calculator gives is 82.34233... However this has 7 significant figures and none of the measurements were this accurate. In fact, we must reduce the answer to only two significant figures, since that's how many 22 has. The answer, K, must be truncated to 82 ml C/g to reflect the 2 significant figures in the D value

$$\begin{array}{ccccccc} & (3) & (4) & & (7) & & (3) \\ & (3.57)(4.286) = 15.30102 \Rightarrow 15.3 \\ (2) & & (4) & & (3) & & (9) & & (2) & & (2) \\ (5.2 \times 10^{-4})(0.1635 \times 10^7)/(2.67) = 318.426966 \Rightarrow 3.2 \times 10^2 = 320 \end{array}$$

✓ Finally, a rule of thumb for rounding off numbers in which the digit to be dropped is a 5 is always **to make the last digit of the rounded-off number even**:

12.6 is rounded to 13

12.4 is rounded to 12

$$1.35 \Rightarrow 1.4$$

$$1.25 \Rightarrow 1.2$$

12.51 is rounded to 13

11.50 is rounded to 12

12.50 is rounded to 12

✓ **Precision** is the degree to which the results of multiple repeat experiments agree with one another. For instance if an experiment is repeated 3 times and the same result is obtained all three times, then the result is considered to be very precise.

✓ **Accuracy** is the degree to which the results of an experiment agree with the true or known value. an experiment or set of experiments may be very precise but not accurate. Less commonly, the results could be accurate but not precise.

❖ DIMENSIONAL HOMOGENEITY AND DIMENSIONLESS QUANTITIES

✓ *Every valid equation must be dimensionally homogeneous: that is, all additive terms on both sides of the equation must have the same dimensions.*

$$Distance(m) = Speed\left(\frac{m}{s}\right) \times Time(s)$$

$$u(m/s) = u_0(m/s) + g(m/s^2)t(s)$$

This equation is dimensionally homogeneous, since each of the terms u , u_0 and gt has the same dimensions (length/time).

$$u = u_0 + g$$

is not dimensionally homogeneous (why not?) and therefore cannot possibly be valid.

If it is desired to express the time (t) in minutes, the equation can be written as:

$$\begin{aligned} u(m/s) &= u_0(m/s) + g(m/s^2)t(min)(60\text{ s/min}) \\ &= u_0 + 60gt \end{aligned}$$

For the following equation, what are the units of 3 and 4?

$$D(ft) = 3t(s) - 4$$

For the equation to be valid, it must be dimensionally homogeneous, so that each term must have the dimension of length. The constant 3 must therefore have the dimension length/time, and 4 must have the dimension length. **For consistency, the constants must be 3 ft/s and 4ft.**

General Problems on chapter 2

Question 1

Using the table of conversion factors on the inside front cover, convert

- (a) 760 miles/h to m/s. (c) 5.37×10^3 kJ/min to hp.
(b) 921 kg/m^3 to lb_m/ft^3 .

Question 2

Calculate

- (a) the weight in lb_f of a 25.0-lb_m object.
(b) the mass in kg of an object that weighs 25 newtons.
(c) the weight in dynes of a 10-ton object (not metric tons).

Question 3

A supersonic aircraft consumes 5320 imperial gallons of kerosene per hour of flight and flies an average of 14 hours per day. It takes roughly seven tons of crude oil to produce one ton of kerosene. The density of kerosene is 0.965 g/cm^3 . How many planes would it take to consume the entire annual world production of 4.02×10^9 metric tons of crude oil?

Question 4

A waste treatment pond is 50 m long and 15 m wide, and has an average depth of 2 m. The density of the waste is $85.3 \text{ lb}_m/\text{ft}^3$. Calculate the weight of the pond contents in lb_f , using a single dimensional equation for your calculation.

Question 5

Perform the following calculations. In each case, first estimate the solution without using a calculator, following the procedure outlined in Section 2.5b, and then do the calculation, paying attention to significant figures.

(a) $(2.7)(8.632)$

(c) $2.365 + 125.2$

(b) $(3.600 \times 10^{-4})/45$

(d) $(4.753 \times 10^4) - (9 \times 10^2)$

Question 6

A quantity k depends on the temperature T in the following manner:

$$k \left(\frac{\text{mol}}{\text{cm}^3 \cdot \text{s}} \right) = 1.2 \times 10^5 \exp \left(-\frac{20,000}{1.987T} \right)$$

The units of the quantity 20,000 are cal/mol, and T is in K (kelvin). What are the units of 1.2×10^5 and 1.987?