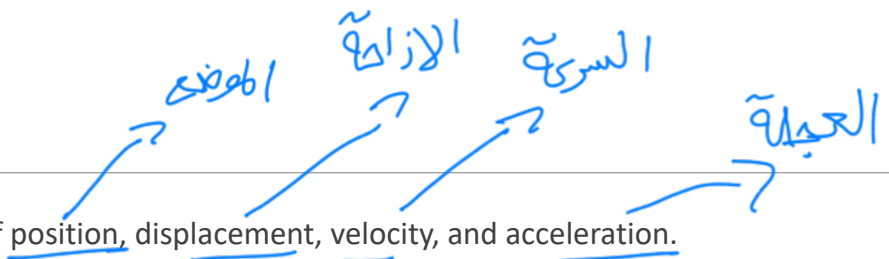


Chapter 12: Kinematics of a Particle



Objectives:

- To introduce the concepts of position, displacement, velocity, and acceleration.

- To study particle motion along a straight line and represent this motion graphically.
- To investigate particle motion along a curved path using different coordinate systems.

- To present an analysis of dependent motion of two particles.
- To examine the principles of relative motion of two particles using translating axes.

Introduction:

- **Mechanics** is a branch of the physical sciences that is concerned with the state of rest or motion of bodies subjected to the action of forces.
- Engineering mechanics is divided into two areas of study:

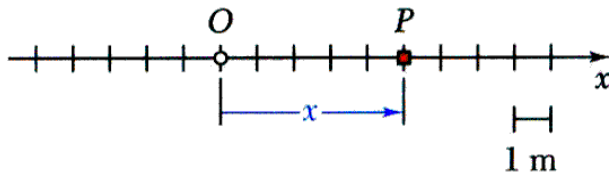
1- Statics 2- Dynamics.

- **Statics** is concerned with the equilibrium of a body that is either at rest or moves with constant velocity.
- **Dynamics**, which deals with the accelerated motion of a body. The subject of dynamics will be presented in two parts:
 - **Kinematics**, which treats only the geometric aspects of the motion, Relates displacement, velocity, acceleration, and time without reference to the cause of motion.
 - **Kinetics**, which is the analysis of the forces causing the motion. study of the relations existing between the forces acting on a body, the mass of the body, and the motion of the body. Kinetics is used to predict the motion caused by given forces or to determine the forces required to produce a given motion.

Rectilinear Kinematics: Continuous Motion

Rectilinear Kinematics. The kinematics of a particle is characterized

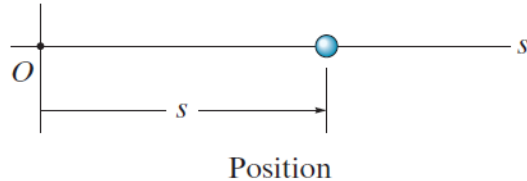
by specifying, at any given instant, the particle's position, velocity, and acceleration. Kinematics of a particle that moves along a rectilinear or **straight-line path**. Recall that a particle has a mass but negligible size and shape.



موقع اور مکان

Position

The straight-line path of a particle will be defined using a single coordinate axis s . The origin O on the path is a fixed point, and from this point the **position coordinate** s is used to specify the location of the particle at any given instant. The magnitude of s is the distance from O to the particle, usually measured in meters (m) or feet (ft), and the sense of direction is defined by the algebraic sign on s .

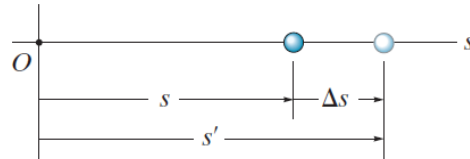


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Displacement

The displacement of the particle is defined as the change in its position. For example, if the particle moves from one point to another, the displacement is

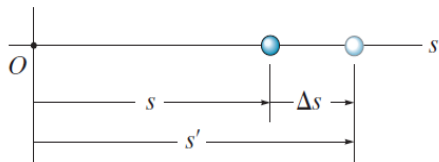
$$\Delta s = s' - s$$



Displacement

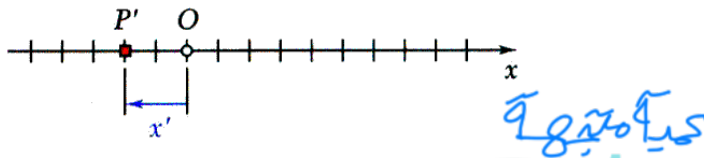
Displacement

$$\Delta s = s' - s$$



Displacement

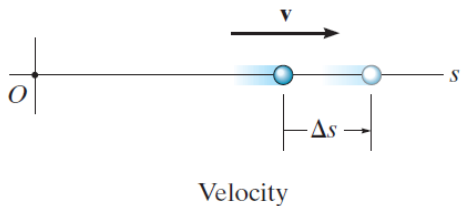
In this case Δs is positive since the particle's final position is to the right of its initial position. Likewise, if the final position were to the left of its initial position, Δs would be negative.



The **displacement** of a particle is also a **vector quantity**, and it should be distinguished from the **distance** the particle travels. Specifically, the distance traveled is a positive scalar that represents the total 'length of path over which the particle travels.

المركبة المتحركة

Velocity



If the particle moves through a displacement Δs during the time interval, the average velocity of the particle during this time interval is:

$$v_{\text{avg}} = \frac{\Delta s}{\Delta t}$$

If we take smaller and smaller values of Δt , the magnitude of Δs becomes smaller and smaller. Consequently, the instantaneous velocity is a vector defined as:

$$v = \frac{ds}{dt}$$

Velocity



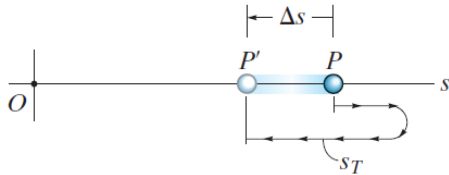
Since Δt or dt is always positive, the sign used to define the sense of the velocity is the same as that of Δs or ds .



The magnitude of the velocity is known as the speed, and it is generally expressed in units of m/s or ft/s.



The average speed is always a positive scalar and is defined as the total distance traveled by a particle, s_T , divided by the elapsed time t ; i.e.,

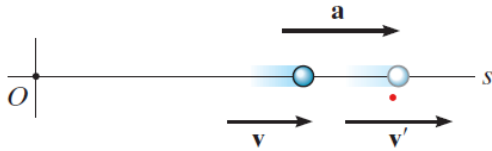


Average velocity and
Average speed

$$(v_{sp})_{avg} = \frac{s_T}{\Delta t} \quad \left\{ \leftarrow \text{speed} \right.$$

$$v_{avg} = -\Delta s / \Delta t.$$

Acceleration



Acceleration

Provided the velocity of the particle is known at two points, the **average acceleration** of the particle during the time interval t is defined as

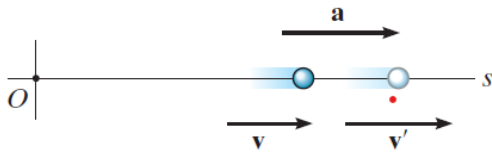
$$a_{\text{avg}} = \frac{\Delta v}{\Delta t}$$

The **instantaneous acceleration** at time t is a *vector* that is found by taking smaller and smaller values of t and corresponding smaller and smaller values of v ,

$$a = \frac{dv}{dt}$$

$$a = \frac{d^2s}{dt^2}$$

Acceleration



Acceleration

Both the average and instantaneous acceleration can be either positive or negative. In particular, when the particle is slowing down, or its speed is **decreasing**, the particle is said to be **decelerating**.

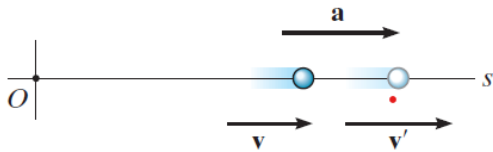
$$a_{\text{avg}} = \frac{\Delta v}{\Delta t}$$

$$a = \frac{dv}{dt}$$

Units commonly used to express the magnitude of acceleration are m/s^2 or ft/s^2 .

a^- } لوالجسم يتباطى = العكس +

Acceleration



Acceleration

Finally, an important differential relation involving the displacement, velocity, and acceleration along the path may be obtained by eliminating the time differential dt between

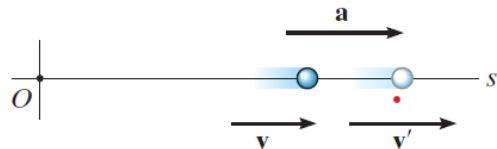
$$\boxed{v = \frac{ds}{dt}} + \boxed{a = \frac{dv}{dt}}$$



$$dt = \frac{ds}{v} = \frac{dv}{a}$$

$$\boxed{a \, ds = v \, dv}$$

Acceleration



Acceleration

اذا كان التسارع ثابتا
Constant Acceleration, $a = a_c$.

When the acceleration is constant, each of the three kinematic equations $a_c = dv/dt$, $v = ds/dt$, and $a_c ds = v dv$ can be integrated to obtain formulas that relate a_c , v , s , and t .

Velocity as a Function of Time. Integrate $a_c = dv/dt$, assuming that initially $v = v_0$ when $t = 0$.

$$\int_{v_0}^v dv = \int_0^t a_c dt$$

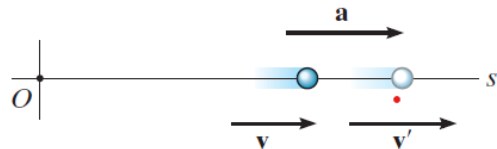
($\pm$)

$$v = v_0 + a_c t$$

Constant Acceleration

(12-4)

Acceleration



Acceleration

Constant Acceleration, $\mathbf{a} = \mathbf{a}_c$.

When the acceleration is constant, each of the three kinematic equations $\mathbf{a}_c = d\mathbf{v}/dt$, $\mathbf{v} = d\mathbf{s}/dt$, and $\mathbf{a}_c d\mathbf{s} = \mathbf{v} d\mathbf{v}$ can be integrated to obtain formulas that relate $\mathbf{a}_c$, $\mathbf{v}$, $\mathbf{s}$, and t .

Position as a Function of Time. Integrate $v = ds/dt = v_0 + a_c t$, assuming that initially $s = s_0$ when $t = 0$.

$$\int_{s_0}^s ds = \int_0^t (v_0 + a_c t) dt$$

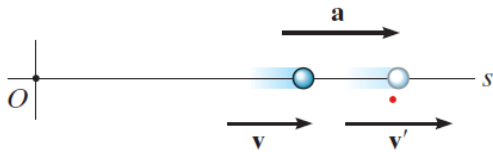
($\pm$)

$$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

Constant Acceleration

(12-5)

Acceleration



Acceleration

Constant Acceleration, $\mathbf{a} = \mathbf{a}_c$.

When the acceleration is constant, each of the three kinematic equations

$\mathbf{a}_c = d\mathbf{v}/dt$, $\mathbf{v} = d\mathbf{s}/dt$,

and $\mathbf{a}_c d\mathbf{s} = \mathbf{v} d\mathbf{v}$ can be integrated to obtain formulas that relate a_c , v , s , and t .

Velocity as a function of position:

$$\int_{v_0}^v v dv = \int_{s_0}^s a_c ds$$

$$v^2 = v_0^2 + 2a_c(s - s_0)$$

Constant Acceleration

($\pm$)

Acceleration

The algebraic signs of s_0 , v_0 , and a_c , used in the above three equations, are determined from the positive direction of the s axis as indicated by the arrow written at the left of each equation. Remember that these equations are useful only when the acceleration is **constant** and when $t = 0$, $s = s_0$, $v = v_0$.

A typical example of constant accelerated motion occurs when a body falls freely toward the earth. If air resistance is neglected and the distance of fall is short, then the downward acceleration of the body when it is close to the earth is constant and approximately 9.81 m/s^2 or 32.2 ft/s^2

**Acceleration is
constant**

$$v = v_0 + a_c t$$

Constant Acceleration

$$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

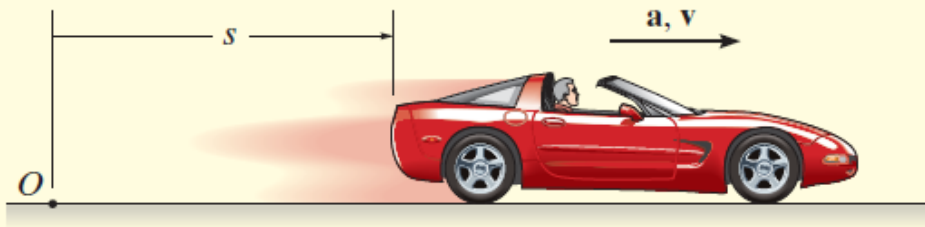
Constant Acceleration

$$v^2 = v_0^2 + 2a_c(s - s_0)$$

Constant Acceleration

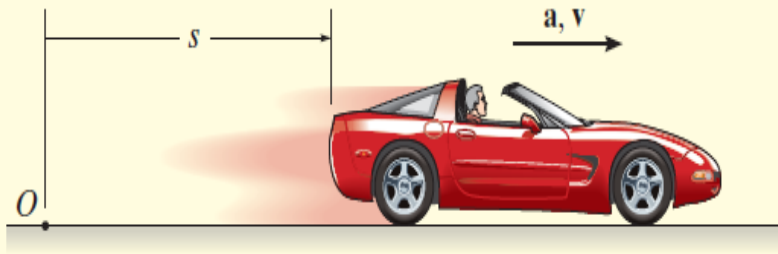
Ex1:

The car on the left in the photo and in Fig. 12–2 moves in a straight line such that for a short time its velocity is defined by $v = (3t^2 + 2t)$ ft/s, where t is in seconds. Determine its position and acceleration when $t = 3$ s. When $t = 0$, $s = 0$.



Ex1:

The car on the left in the photo and in Fig. 12-2 moves in a straight line such that for a short time its velocity is defined by $v = (3t^2 + 2t)$ ft/s, where t is in seconds. Determine its position and acceleration when $t = 3$ s. When $t = 0$, $s = 0$.



$$v = v_0 + a_c t$$

Constant Acceleration

$$v^2 = v_0^2 + 2a_c(s - s_0)$$

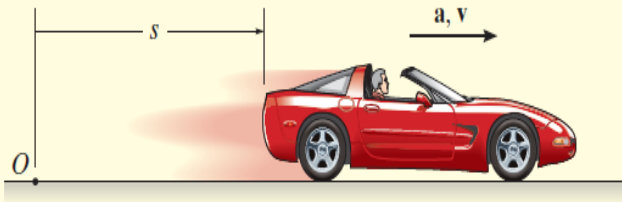
Constant Acceleration

$$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

Constant Acceleration

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The car on the left in the photo and in Fig. 12-2 moves in a straight line such that for a short time its velocity is defined by $v = (3t^2 + 2t)$ ft/s, where t is in seconds. Determine its position and acceleration when $t = 3$ s. When $t = 0$, $s = 0$.



$$v = v_0 + a_c t$$

Constant Acceleration

$$v^2 = v_0^2 + 2a_c(s - s_0)$$

Constant Acceleration

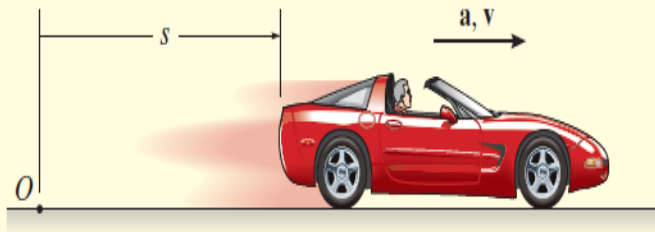
$$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

Constant Acceleration

NOTE: The formulas for constant acceleration cannot be used to solve this problem, because the acceleration is a function of time.

Ex1:

The car on the left in the photo and in Fig. 12-2 moves in a straight line such that for a short time its velocity is defined by $v = (3t^2 + 2t)$ ft/s, where t is in seconds. Determine its position and acceleration when $t = 3$ s. When $t = 0, s = 0$.



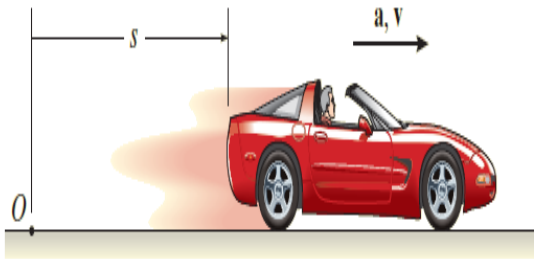
$$v = \frac{ds}{dt}$$

$$a = \frac{dv}{dt}$$

$$a \, ds = v \, dv$$

Ex1:

The car on the left in the photo and in Fig. 12-2 moves in a straight line such that for a short time its velocity is defined by $v = (3t^2 + 2t)$ ft/s, where t is in seconds. Determine its position and acceleration when $t = 3$ s. When $t = 0$, $s = 0$.



SOLUTION

Coordinate System. The position coordinate extends from the fixed origin O to the car, positive to the right.

Position. Since $v = f(t)$, the car's position can be determined from $v = ds/dt$, since this equation relates v , s , and t . Noting that $s = 0$ when $t = 0$, we have*

$$\begin{aligned} (\rightarrow) \quad v &= \frac{ds}{dt} = (3t^2 + 2t) \\ \int_0^s ds &= \int_0^t (3t^2 + 2t) dt \\ s \Big|_0^s &= t^3 + t^2 \Big|_0^t \\ s &= t^3 + t^2 \end{aligned}$$

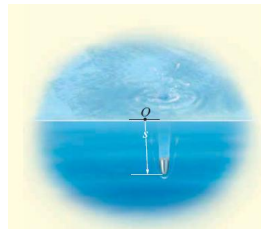
When $t = 3$ s,

$$s = (3)^3 + (3)^2 = 36 \text{ ft}$$

Ans.

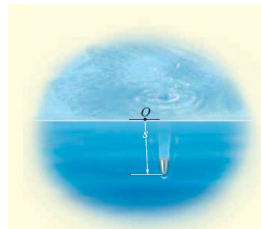
Ex2:

A small projectile is fired vertically *downward* into a fluid medium with an initial velocity of 60 m/s. Due to the drag resistance of the fluid the projectile experiences a deceleration of $a = (-0.4v^3) \text{ m/s}^2$, where v is in m/s. Determine the projectile's velocity and position 4 s after it is fired.



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A small projectile is fired vertically *downward* into a fluid medium with an initial velocity of 60 m/s. Due to the drag resistance of the fluid the projectile experiences a deceleration of $a = (-0.4v^3) \text{ m/s}^2$, where v is in m/s. Determine the projectile's velocity and position 4 s after it is fired.



$$v = v_0 + a_c t$$

Constant Acceleration

$$v^2 = v_0^2 + 2a_c(s - s_0)$$

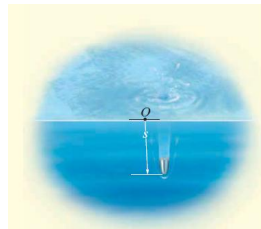
Constant Acceleration

$$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

Constant Acceleration

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Coordinate System. Since the motion is downward, the position coordinate is positive downward, with origin located at O , Fig. 12–3.

Velocity. Here $a = f(v)$ and so we must determine the velocity as a function of time using $a = dv/dt$, since this equation relates v , a , and t . (Why not use $v = v_0 + a_c t$?) Separating the variables and integrating, with $v_0 = 60 \text{ m/s}$ when $t = 0$, yields

Ex2:

A small projectile is fired vertically *downward* into a fluid medium with an initial velocity of 60 m/s. Due to the drag resistance of the fluid the projectile experiences a deceleration of $a = (-0.4v^3)$ m/s², where v is in m/s. Determine the projectile's velocity and position 4 s after it is fired.

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Velocity. Here $a = f(v)$ and so we must determine the velocity as a function of time using $a = dv/dt$, since this equation relates v , a , and t . (Why not use $v = v_0 + a_c t$?) Separating the variables and integrating, with $v_0 = 60$ m/s when $t = 0$, yields

(+↓)

$$\begin{aligned} a &= \frac{dv}{dt} = -0.4v^3 \\ \int_{60 \text{ m/s}}^v \frac{dv}{-0.4v^3} &= \int_0^t dt \\ \frac{1}{-0.4} \left(\frac{1}{-2} \right) \frac{1}{v^2} \bigg|_{60}^v &= t - 0 \\ \frac{1}{0.8} \left[\frac{1}{v^2} - \frac{1}{(60)^2} \right] &= t \\ v &= \left\{ \left[\frac{1}{(60)^2} + 0.8t \right]^{-1/2} \right\} \text{ m/s} \end{aligned}$$

Here the positive root is taken, since the projectile will continue to move downward. When $t = 4$ s,

$$v = 0.559 \text{ m/s} \downarrow$$

Ans.

Ex2:

A small projectile is fired vertically *downward* into a fluid medium with an initial velocity of 60 m/s. Due to the drag resistance of the fluid the projectile experiences a deceleration of $a = (-0.4v^3)$ m/s², where v is in m/s. Determine the projectile's velocity and position 4 s after it is fired.

Coordinate System. Since the motion is downward, the position coordinate is positive downward, with origin located at O , Fig. 12–3.

Velocity. Here $a = f(v)$ and so we must determine the velocity as a function of time using $a = dv/dt$, since this equation relates v , a , and t . (Why not use $v = v_0 + a_c t$?) Separating the variables and integrating, with $v_0 = 60$ m/s when $t = 0$, yields

Position. Knowing $v = f(t)$, we can obtain the projectile's position from $v = ds/dt$, since this equation relates s , v , and t . Using the initial condition $s = 0$, when $t = 0$, we have

$$\begin{aligned} (+\downarrow) \quad v &= \frac{ds}{dt} = \left[\frac{1}{(60)^2} + 0.8t \right]^{-1/2} \\ \int_0^s ds &= \int_0^t \left[\frac{1}{(60)^2} + 0.8t \right]^{-1/2} dt \\ s &= \frac{2}{0.8} \left[\frac{1}{(60)^2} + 0.8t \right]^{1/2} \bigg|_0^t \\ s &= \frac{1}{0.4} \left\{ \left[\frac{1}{(60)^2} + 0.8t \right]^{1/2} - \frac{1}{60} \right\} \text{ m} \end{aligned}$$

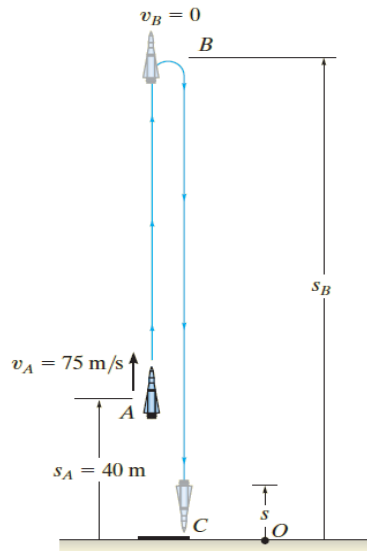
When $t = 4$ s,

$$s = 4.43 \text{ m}$$

Ans.

Ex3:

During a test a rocket travels upward at 75 m/s , and when it is 40 m from the ground its engine fails. Determine the maximum height s_B reached by the rocket and its speed just before it hits the ground. While in motion the rocket is subjected to a constant downward acceleration of 9.81 m/s^2 due to gravity. Neglect the effect of air resistance.



Ex3:

During a test a rocket travels upward at 75 m/s, and when it is 40 m from the ground its engine fails. Determine the maximum height s_B reached by the rocket and its speed just before it hits the ground. While in motion the rocket is subjected to a constant downward acceleration of 9.81 m/s^2 due to gravity. Neglect the effect of air resistance.

$$v = v_0 + a_c t$$

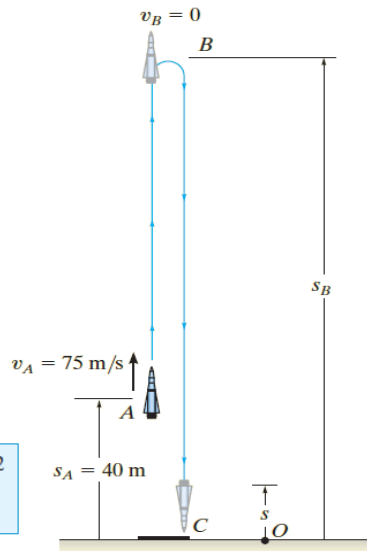
Constant Acceleration

$$v^2 = v_0^2 + 2a_c(s - s_0)$$

Constant Acceleration

$$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

Constant Acceleration



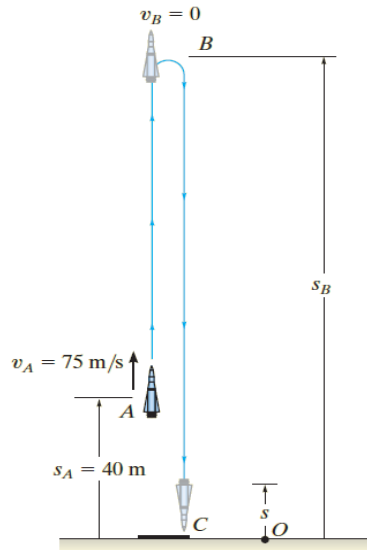
Ex3:

During a test a rocket travels upward at 75 m/s, and when it is 40 m from the ground its engine fails. Determine the maximum height s_B reached by the rocket and its speed just before it hits the ground. While in motion the rocket is subjected to a constant downward acceleration of 9.81 m/s^2 due to gravity. Neglect the effect of air resistance.

Velocity. To obtain the velocity of the rocket just before it hits the ground, we can apply Eq. 12–6 between points B and C , Fig. 12–4.

$$\begin{aligned} (+\uparrow) \quad v_C^2 &= v_B^2 + 2a_c(s_C - s_B) \\ &= 0 + 2(-9.81 \text{ m/s}^2)(0 - 327 \text{ m}) \\ v_C &= -80.1 \text{ m/s} = 80.1 \text{ m/s} \downarrow \end{aligned} \quad \text{Ans.}$$

The negative root was chosen since the rocket is moving downward.



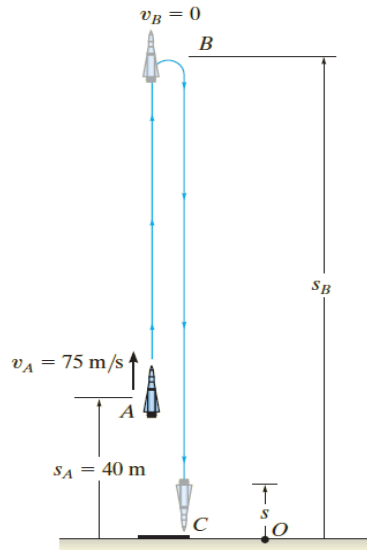
Ex3:

During a test a rocket travels upward at 75 m/s, and when it is 40 m from the ground its engine fails. Determine the maximum height s_B reached by the rocket and its speed just before it hits the ground. While in motion the rocket is subjected to a constant downward acceleration of 9.81 m/s^2 due to gravity. Neglect the effect of air resistance.

Similarly, Eq. 12-6 may also be applied between points A and C, i.e.,

$$\begin{aligned} (+\uparrow) \quad v_C^2 &= v_A^2 + 2a_c(s_C - s_A) \\ &= (75 \text{ m/s})^2 + 2(-9.81 \text{ m/s}^2)(0 - 40 \text{ m}) \\ v_C &= -80.1 \text{ m/s} = 80.1 \text{ m/s} \downarrow \end{aligned}$$

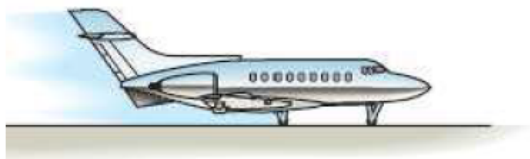
Ans.



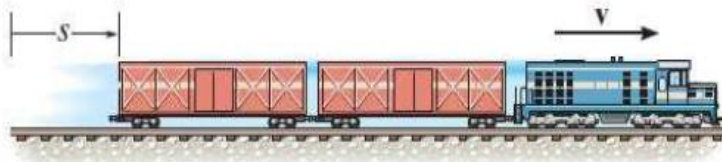
Example 1: the acceleration of a particle moving along a straight line is given by $a = 2t - 10$ (m/s^2). Determine the velocity v and displacement s as functions of time. The initial displacement at $t = 0$ is $s_o = -4$ (m), and the initial velocity is $v_o = 3$ (m/s).

Example 1: the acceleration of a particle moving along a straight line is given by $a = 2t - 10 \text{ (m/s}^2\text{)}$. Determine the velocity v and displacement s as functions of time. The initial displacement at $t = 0$ is $s_o = -4 \text{ (m)}$, and the initial velocity is $v_o = 3 \text{ (m/s)}$.

Example 2: Starting from rest, an airplane moves in a straight line has an acceleration of $a = (2t - 6) \text{ m/s}^2$, where t is in seconds. What is the particle's velocity v when $t = 6 \text{ (s)}$, and what is its position s when $t = 11 \text{ (s)}$?



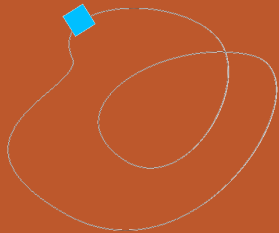
Example 3: a freight train travels at $v = 60(1 + t)$ (m/s), where t is the elapsed time in seconds. Determine the distance traveled in three seconds, and the acceleration at this time.



Example 4: a particle is moving along a straight line with an acceleration of $a = 1.5v$ (m^2/s). It starts with an initial velocity of $v_0 = 4$ (m/s). Determine when the particle stops the motion.

Example 5: a particle starts moving with a velocity of 3 (m/s) in a straight line with a constant acceleration. After 5 seconds, it has gained a velocity 5.5 (m/s). Find its acceleration and the distance traveled in 10 seconds.

Chapter 12.4: Curvilinear Motion

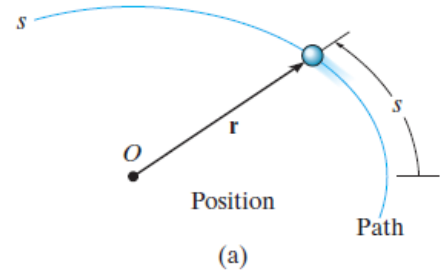


Objectives:

- To introduce the concepts of position, displacement, velocity, and acceleration.
- To study particle motion along a straight line.
- To investigate particle motion along a curved path using different coordinate systems.
- To present an analysis of dependent motion of two particles.
- To examine the principles of relative motion of two particles using translating axes.

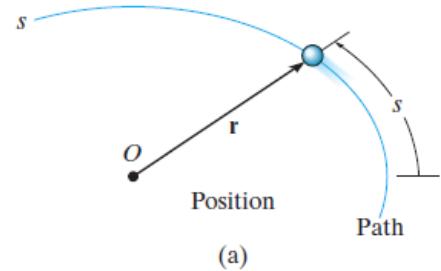
Curvilinear Motion

Curvilinear motion occurs when a particle moves along a curved path. Since this path is often described in three dimensions, vector analysis will be used to formulate the particle's position, velocity, and acceleration. In this section the general aspects of curvilinear motion are discussed, and in subsequent sections we will consider three types of coordinate systems often used to analyze this motion.



Curvilinear Motion

Position: Consider a particle located at a point on a space curve defined by the path function $s(t)$. The position of the particle, measured from a fixed-point O , will be designated by the position vector $\mathbf{r} = \mathbf{r}(t)$. Notice that both the magnitude and direction of this vector will change as the particle moves along the curve.



Curvilinear Motion

Displacement: Suppose that during a small time interval Δt the particle moves a distance Δs along the curve to a new position defined by : $\mathbf{r}' = \mathbf{r} + \Delta \mathbf{r}$,

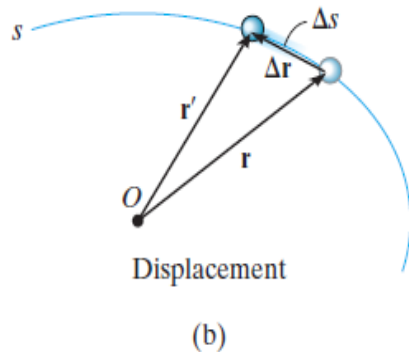
The displacement $\Delta \mathbf{r}$ represents the change in the particle's position and is determined by vector subtraction: $\Delta \mathbf{r} = \mathbf{r}' - \mathbf{r}$

Velocity: During the time Δt , the average velocity of the particle is:

$$\mathbf{v}_{\text{avg}} = \frac{\Delta \mathbf{r}}{\Delta t}$$

The instantaneous velocity is:

$$\vec{v} = \frac{d(\vec{r})}{dt}$$



12.5 Curvilinear Motion: Rectangular Components

Occasionally the motion of a particle can best be described along a path that can be expressed in terms of its x , y , z coordinates.

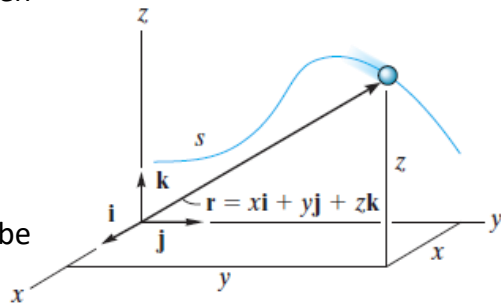
If the particle is at point (x, y, z) on the curved path s shown in, then its location is defined by the position vector:

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$

When the particle moves, the x , y , z components of $\mathbf{r}$ will be functions of time; i.e., $x = x(t)$, $y = y(t)$, $z = z(t)$, so that $\mathbf{r} = \mathbf{r}(t)$.

At any instant the magnitude of $\mathbf{r}$ is defined as:

$$r = \sqrt{x^2 + y^2 + z^2}$$



Position

12.5 Curvilinear Motion: Rectangular Components

Velocity: The first-time derivative of $\mathbf{r}$ yields the velocity of the particle.

Hence:

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \frac{d}{dt}(x\mathbf{i}) + \frac{d}{dt}(y\mathbf{j}) + \frac{d}{dt}(z\mathbf{k})$$

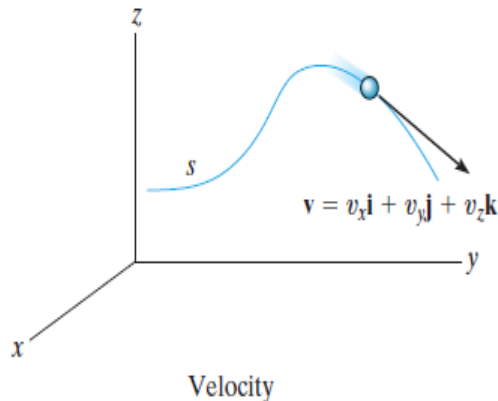
$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = v_x\mathbf{i} + v_y\mathbf{j} + v_z\mathbf{k}$$

where

$$v_x = \dot{x} \quad v_y = \dot{y} \quad v_z = \dot{z}$$

The velocity has a **magnitude** that is found from

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2}$$



12.5 Curvilinear Motion: Rectangular Components

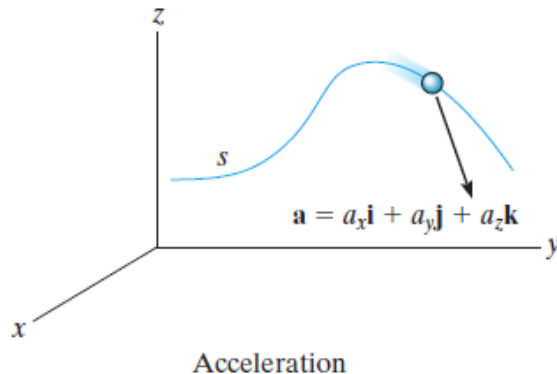
Velocity: Acceleration. The acceleration of the particle is obtained by taking the first-time derivative of velocity(or the second time derivative of position:

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k}$$

$$\begin{aligned} a_x &= \dot{v}_x = \ddot{x} \\ a_y &= \dot{v}_y = \ddot{y} \\ a_z &= \dot{v}_z = \ddot{z} \end{aligned}$$

The **acceleration** has a *magnitude*.

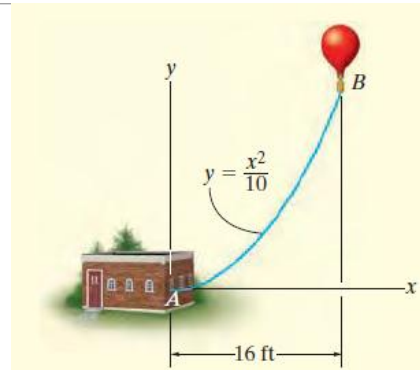
$$a = \sqrt{a_x^2 + a_y^2 + a_z^2}$$



EXAMPLE

At any instant the horizontal position of the weather balloon in is defined by $x = (8t)$ ft, where t is in seconds. If the equation of the path is $y = x^2/10$, determine the magnitude and direction of the velocity and the acceleration when $t = 2$ s.

Solution:



EXAMPLE

At any instant the horizontal position of the weather balloon in is defined by $x = (8t)$ ft, where t is in seconds. If the equation of the path is $y = x^2/10$, determine the magnitude and direction of the velocity and the acceleration when $t = 2$ s.

Solution:

$$v_x = \dot{x} = \frac{d}{dt}(8t) = 8 \text{ ft/s} \rightarrow$$

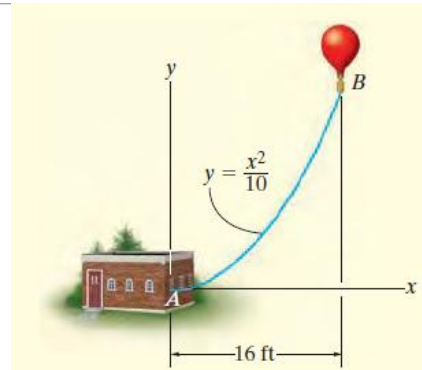
$$v_y = \dot{y} = \frac{d}{dt}(x^2/10) = 2x\dot{x}/10 = 2(16)(8)/10 = 25.6 \text{ ft/s} \uparrow$$

When $t = 2$ s, the magnitude of velocity is therefore

$$v = \sqrt{(8 \text{ ft/s})^2 + (25.6 \text{ ft/s})^2} = 26.8 \text{ ft/s} \quad \text{Ans.}$$

The direction is tangent to the path, Fig. 12-18b, where

$$\theta_v = \tan^{-1} \frac{v_y}{v_x} = \tan^{-1} \frac{25.6}{8} = 72.6^\circ \quad \text{Ans.}$$



EXAMPLE

At any instant the horizontal position of the weather balloon in is defined by $x = (8t)$ ft, where t is in seconds. If the equation of the path is $y = x^2/10$, determine the magnitude and direction of the velocity and the acceleration when $t = 2$ s.

Solution: **Acceleration.** The relationship between the acceleration components is determined using the chain rule. (See Appendix C.) We have

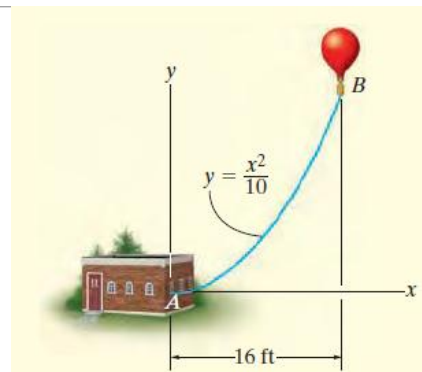
$$\begin{aligned}a_x &= \dot{v}_x = \frac{d}{dt}(8) = 0 \\a_y &= \dot{v}_y = \frac{d}{dt}(2x\dot{x}/10) = 2(\dot{x})\dot{x}/10 + 2x(\ddot{x})/10 \\&= 2(8)^2/10 + 2(16)(0)/10 = 12.8 \text{ ft/s}^2 \uparrow\end{aligned}$$

Thus,

$$a = \sqrt{(0)^2 + (12.8)^2} = 12.8 \text{ ft/s}^2 \quad \text{Ans.}$$

The direction of $\mathbf{a}$, as shown in Fig. 12–18c, is

$$\theta_a = \tan^{-1} \frac{12.8}{0} = 90^\circ \quad \text{Ans.}$$



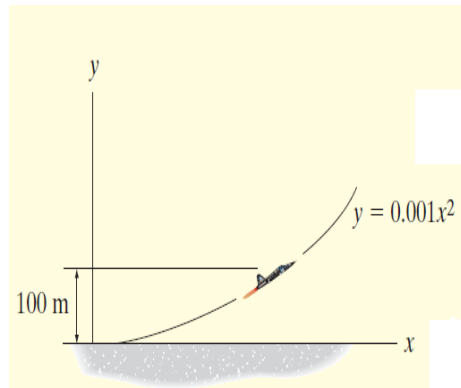
EXAMPLE

For a short time, the path of the plane is described by $y = (0.001x^2)$ m. If the plane is rising with a constant upward velocity of 10 m/s, determine the magnitudes of the velocity and acceleration of the plane when it reaches an altitude of $y = 100$ m.

SOLUTION

When $y = 100$ m, then $100 = 0.001x^2$ or $x = 316.2$ m. Also, due to constant velocity $v_y = 10$ m/s, so

$$y = v_y t; \quad 100 \text{ m} = (10 \text{ m/s}) t \quad t = 10 \text{ s}$$



EXAMPLE

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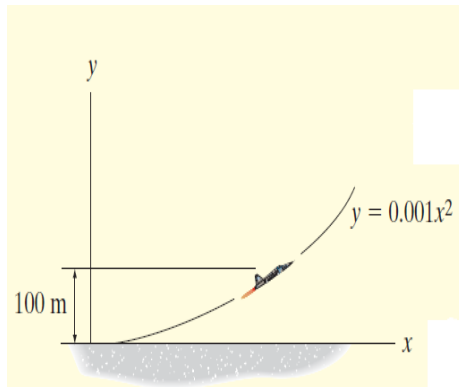
$$y = v_y t; \quad 100 \text{ m} = (10 \text{ m/s}) t \quad t = 10 \text{ s}$$

Velocity. Using the chain rule (see Appendix C) to find the relationship between the velocity components, we have

$$\begin{aligned} y &= 0.001x^2 \\ v_y = \dot{y} &= \frac{d}{dt}(0.001x^2) = (0.002x)\dot{x} = 0.002xv_x \quad (1) \end{aligned}$$

Thus

$$\begin{aligned} 10 \text{ m/s} &= 0.002(316.2 \text{ m})(v_x) \\ v_x &= 15.81 \text{ m/s} \end{aligned}$$



EXAMPLE

The magnitude of the velocity is therefore

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(15.81 \text{ m/s})^2 + (10 \text{ m/s})^2} = 18.7 \text{ m/s} \quad \text{Ans.}$$

Acceleration. Using the chain rule, the time derivative of Eq. (1) gives the relation between the acceleration components.

$$a_y = \dot{v}_y = (0.002\dot{x})\dot{x} + 0.002x(\ddot{x}) = 0.002(v_x^2 + xa_x)$$

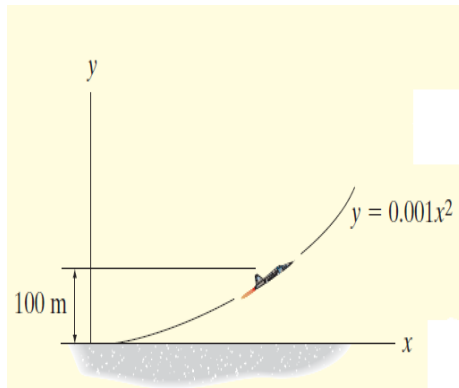
When $x = 316.2 \text{ m}$, $v_x = 15.81 \text{ m/s}$, $\dot{v}_y = a_y = 0$,

$$\begin{aligned} 0 &= 0.002[(15.81 \text{ m/s})^2 + 316.2 \text{ m}(a_x)] \\ a_x &= -0.791 \text{ m/s}^2 \end{aligned}$$

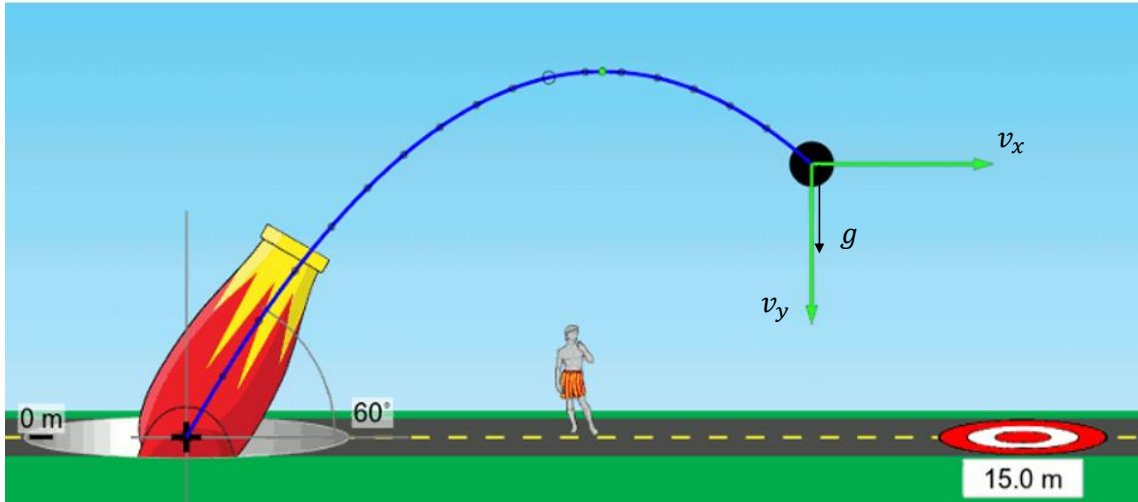
The magnitude of the plane's acceleration is therefore

$$\begin{aligned} a &= \sqrt{a_x^2 + a_y^2} = \sqrt{(-0.791 \text{ m/s}^2)^2 + (0 \text{ m/s}^2)^2} \\ &= 0.791 \text{ m/s}^2 \quad \text{Ans.} \end{aligned}$$

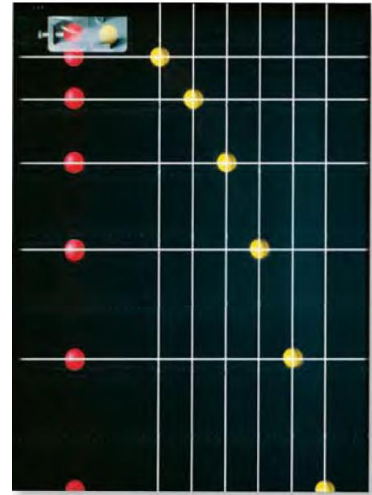
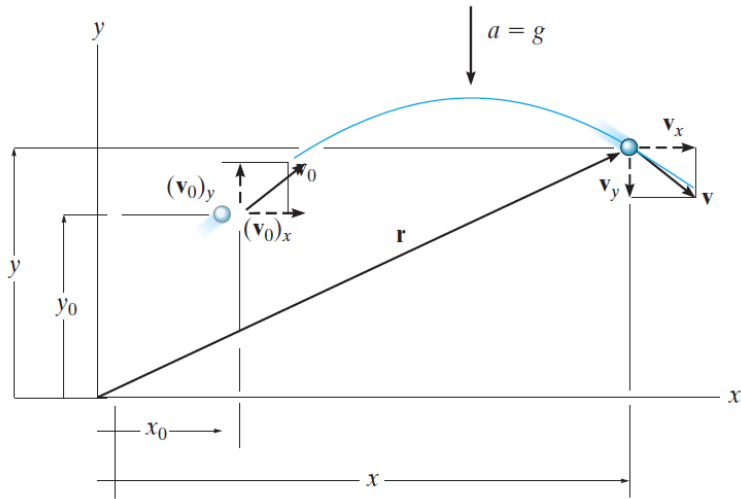
These results are shown in Fig. 12–19*b*.



12.6 Motion of a Projectile



12.6 Motion of a Projectile



12.6 Motion of a Projectile

Horizontal Motion. Since $a_x = 0$, application of the constant acceleration equations, yields

$(\pm \rightarrow)$	$v = v_0 + a_c t$	$v_x = (v_0)_x$
$(\pm \rightarrow)$	$x = x_0 + v_0 t + \frac{1}{2} a_c t^2;$	$x = x_0 + (v_0)_x t$
$(\pm \rightarrow)$	$v^2 = v_0^2 + 2a_c(x - x_0);$	$v_x = (v_0)_x$

12.6 Motion of a Projectile

Vertical Motion. Since the positive y axis is directed upward, then $a_y = -g$. we get

$(+\uparrow)$	$v = v_0 + a_c t;$	$v_y = (v_0)_y - gt$
$(+\uparrow)$	$y = y_0 + v_0 t + \frac{1}{2} a_c t^2;$	$y = y_0 + (v_0)_y t - \frac{1}{2} g t^2$
$(+\uparrow)$	$v^2 = v_0^2 + 2a_c(y - y_0);$	$v_y^2 = (v_0)_y^2 - 2g(y - y_0)$

12.6 Motion of a Projectile

($\pm \rightarrow$)

$$v = v_0 + a_c t$$

$$v_x = (v_0)_x$$

($\pm \rightarrow$)

$$x = x_0 + v_0 t + \frac{1}{2} a_c t^2;$$

$$x = x_0 + (v_0)_x t$$

($\pm \rightarrow$)

$$v^2 = v_0^2 + 2a_c(x - x_0);$$

$$v_x = (v_0)_x$$

($+\uparrow$)

$$v = v_0 + a_c t;$$

$$v_y = (v_0)_y - gt$$

($+\uparrow$)

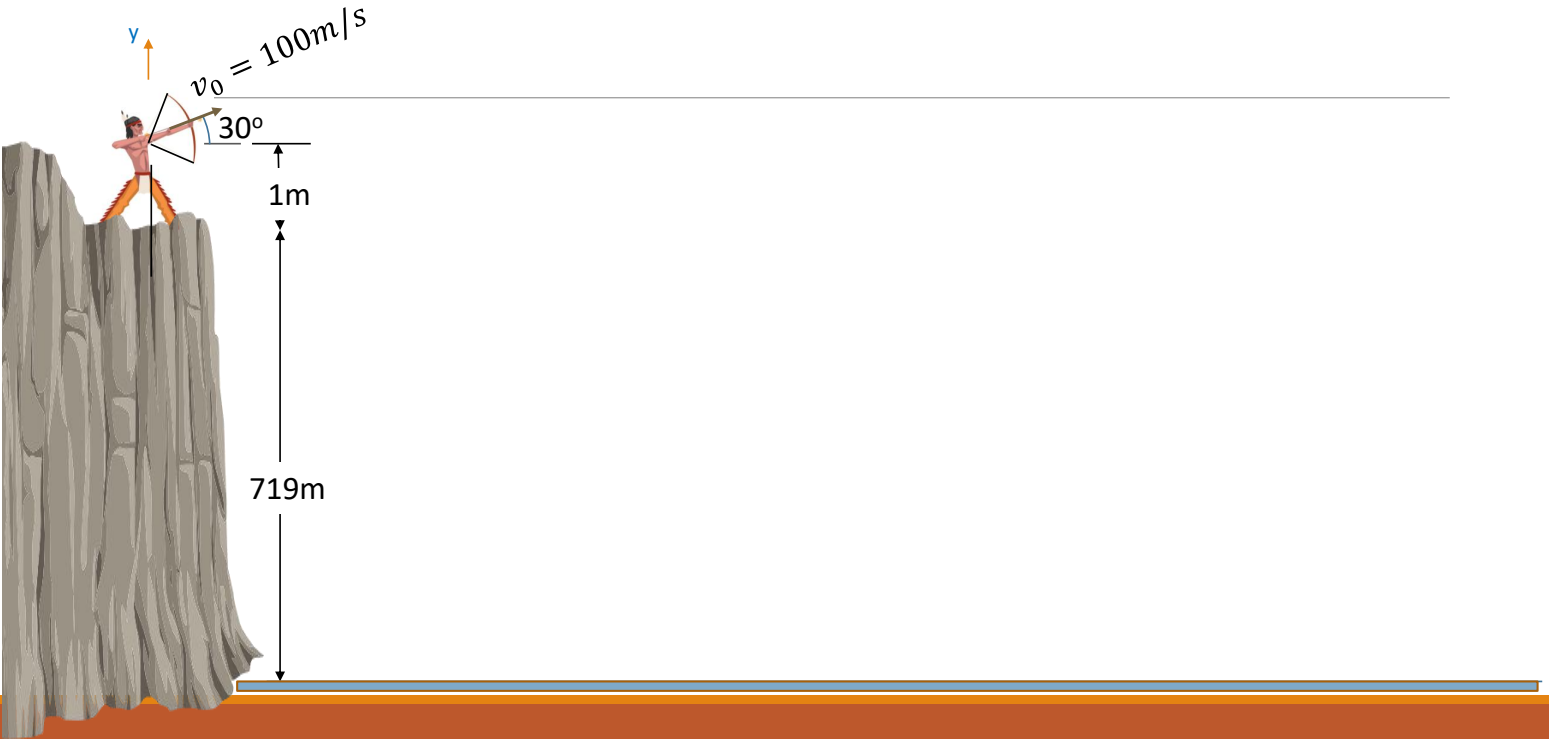
$$y = y_0 + v_0 t + \frac{1}{2} a_c t^2;$$

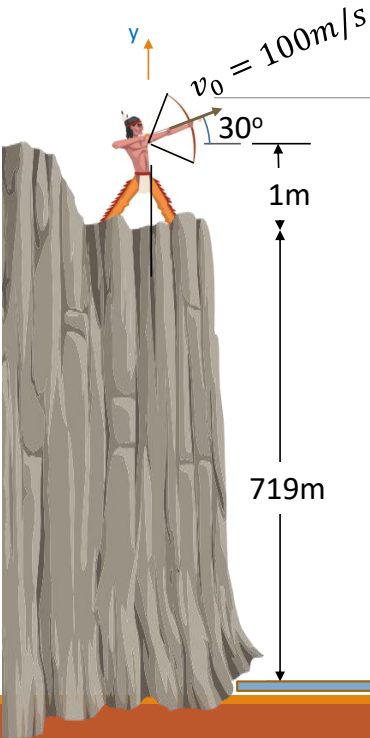
$$y = y_0 + (v_0)_y t - \frac{1}{2} g t^2$$

($+\uparrow$)

$$v^2 = v_0^2 + 2a_c(y - y_0);$$

$$v_y^2 = (v_0)_y^2 - 2g(y - y_0)$$

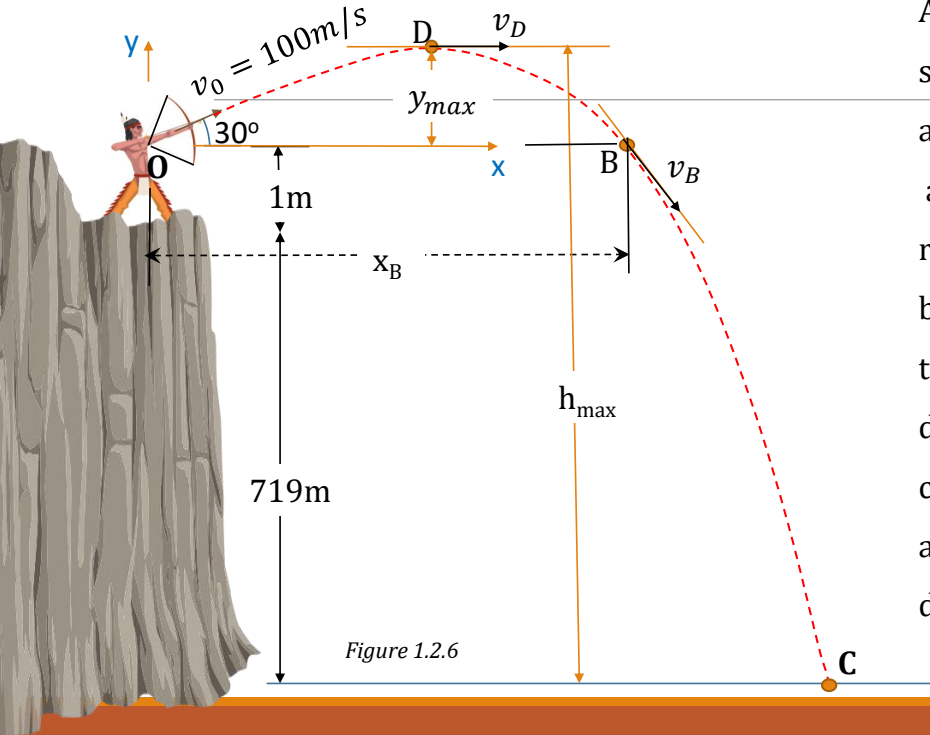




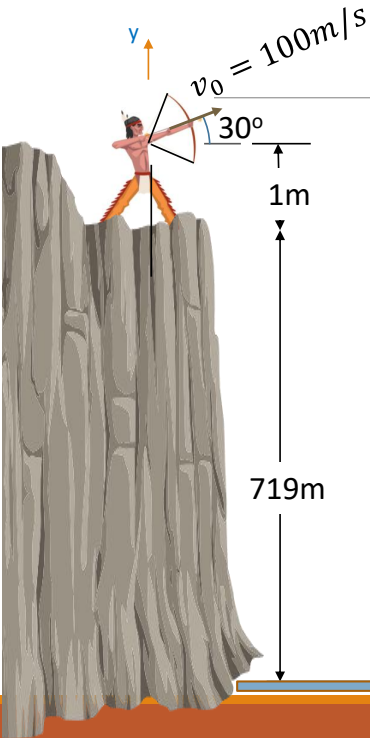
A man on the edge of a 719m slope shoots an arrow at a velocity of 100m/s at an angle of 30° . Calculate,

- the maximum height the arrow will reach,
- the velocity of the arrow as it passes the line of where it was first shot and the distance between the arrow and the man,
- Calculate the velocity at which the arrow hits the ground and the linear distance between it and the man.

Solution:



- A man on the edge of a 719m slope shoots an arrow at a velocity of 100m/s at an angle of 30° . Calculate,
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$$(\pm)$$

$$v = v_0 + a_c t$$

$$v_x = (v_0)_x$$

$$(\pm)$$

$$x = x_0 + v_0 t + \frac{1}{2} a_c t^2;$$

$$x = x_0 + (v_0)_x t$$

$$(\pm)$$

$$v^2 = v_0^2 + 2a_c(x - x_0);$$

$$v_x = (v_0)_x$$

$$(+\uparrow)$$

$$v = v_0 + a_c t;$$

$$v_y = (v_0)_y - gt$$

$$(+\uparrow)$$

$$y = y_0 + v_0 t + \frac{1}{2} a_c t^2;$$

$$y = y_0 + (v_0)_y t - \frac{1}{2} g t^2$$

$$(+\uparrow)$$

$$v^2 = v_0^2 + 2a_c(y - y_0);$$

$$v_y^2 = (v_0)_y^2 - 2g(y - y_0)$$

Solution:

a-) $h_{max} = ?$

At peak D, the velocity is in the horizontal direction : $v_y = 0 = v_{0y} - gt$

$$v_{0y} = v_0 \cdot \sin 30^\circ = 100 \sin 30^\circ = 50 \text{ m/s}$$

$$v_y = 0 = 50 - 10t \rightarrow t = 5 \text{ s} = t_D$$

$$y_{max} = y_D = y_0 + v_{0y}t_D - \frac{1}{2}gt_D^2 = 50 \times 5 - \frac{1}{2}10 \times 5^2 = 125 \text{ m}$$

$$h_{max} = 719 + 1 + 125 \rightarrow h_{max} = 845 \text{ m}$$

b-) $v_B = ?$, $x_B = ?$

$$y_B = 0 = y_0 + v_{0y}t - \frac{1}{2}gt^2 = 50 \cdot t - \frac{1}{2}10t^2$$

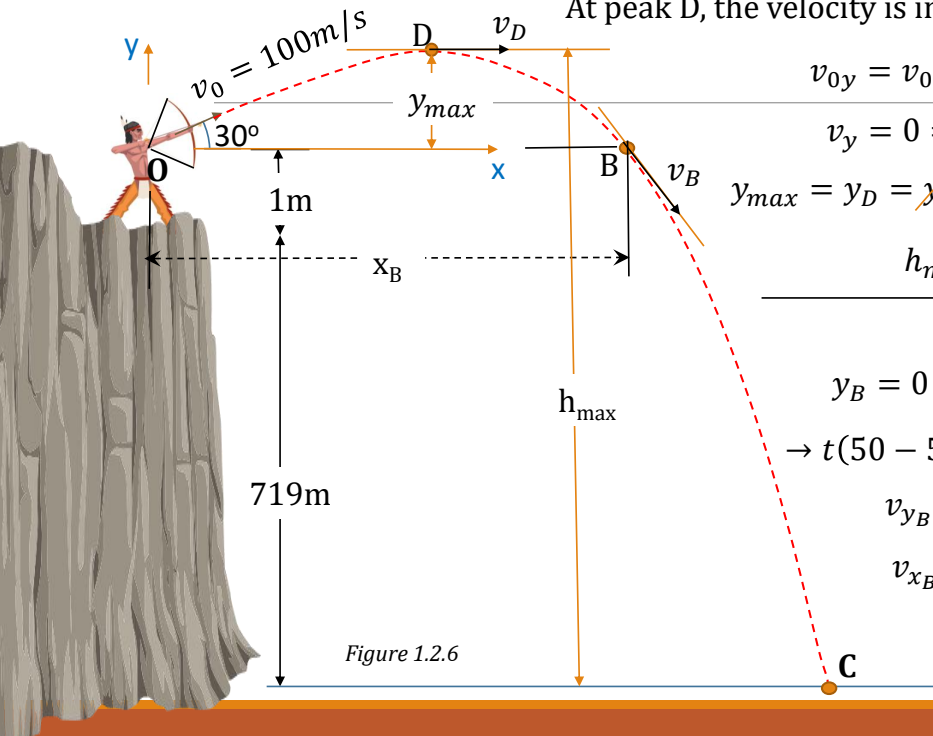
$$\rightarrow t(50 - 5t) = 0 \rightarrow (50 - 5t) = 0 \rightarrow t = t_B = 10 \text{ s}$$

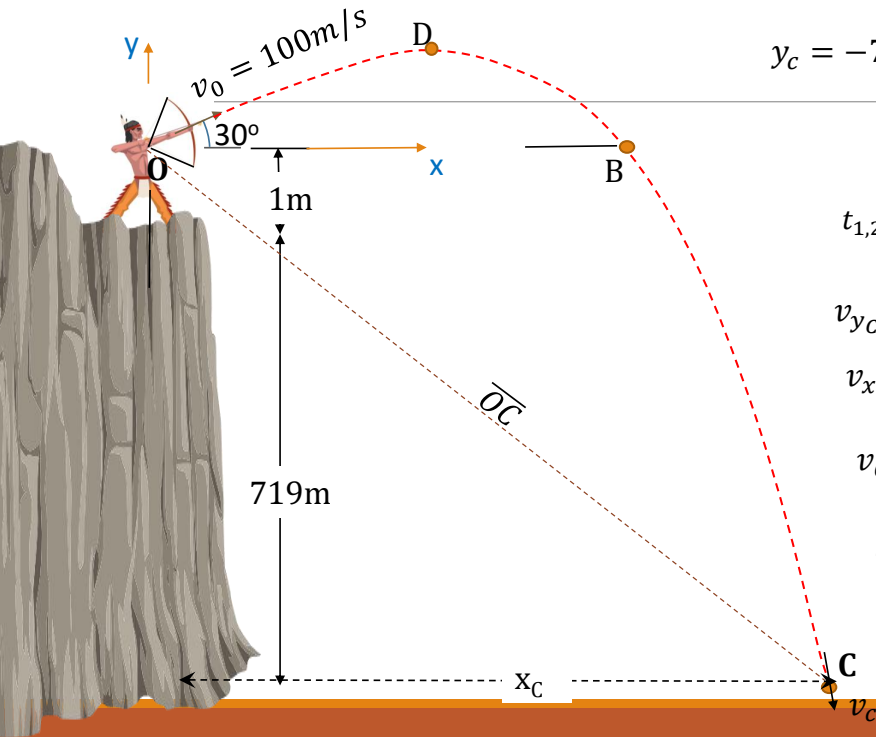
$$v_{yB} = v_{0y} - gt_B = 50 - 10 \times 10 = -50 \text{ m/s}$$

$$v_{xB} = v_{0x} = 100 \cos 30^\circ = 86.6 \text{ m/s}$$

$$v_B = \sqrt{v_{yB}^2 + v_{xB}^2} \rightarrow v_B = 100 \text{ m/s}$$

$$x_B = x_0 + v_{0x}t_B = 86.6 \times 10 \rightarrow x_B = 866 \text{ m}$$





c-) $v_C = ?$, $\overline{OC} = ?$

$$y_C = -720 = y_0 + v_{0y}t - \frac{1}{2}gt^2 = 0 + 50t_C - \frac{1}{2}10t_C^2$$

$$5t^2 - 50t - 720 = 0$$

$$t_1 < 0$$

$$t_{1,2} = \frac{50 \pm \sqrt{(-50)^2 - 4 \times 5 \times (-720)}}{2 \times 5} \quad \swarrow \quad t_2 = 18 = t_C$$

$$v_{y_C} = v_{0y} - gt_C = 50 - 10 \times 18 = -130 \text{ m/s}$$

$$v_{x_C} = v_{0x} = 86.6 \text{ m/s}$$

$$v_C = \sqrt{v_{y_C}^2 + v_{x_C}^2} \rightarrow v_C = 156.2 \text{ m/s}$$

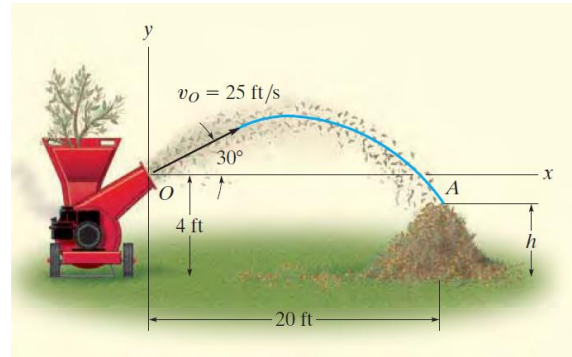
$$x_C = x_0 + v_{0x}t_C = 0 + 86.6 \times 18 = 1558.8 \text{ m}$$

$$\overline{OC} = \sqrt{x_C^2 + 720^2} = \sqrt{1558.8^2 + 720^2}$$

$$\rightarrow \overline{OC} \cong 1717 \text{ m}$$

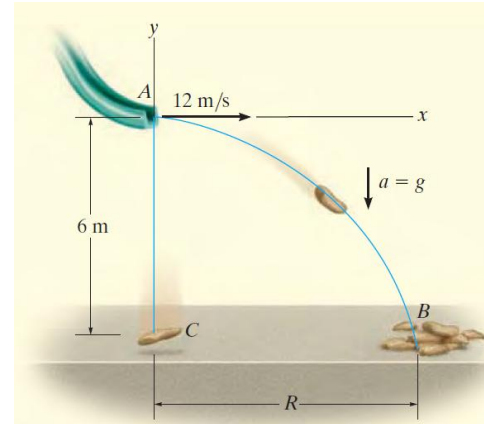
EX1:

The chipping machine is designed to eject wood chips at $v_O = 25 \text{ ft/s}$ as shown in Fig. 12–22. If the tube is oriented at 30° from the horizontal, determine how high, h , the chips strike the pile if at this instant they land on the pile 20 ft from the tube.



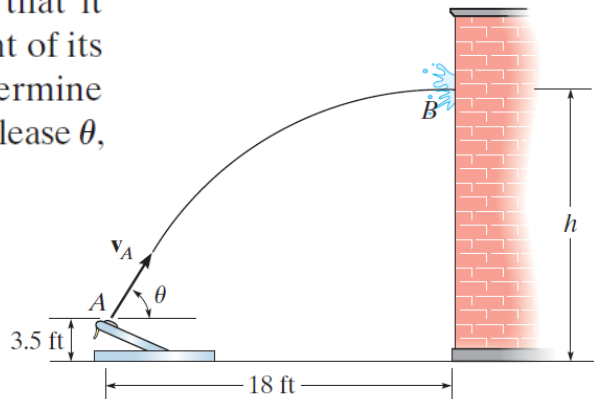
EX2:

A sack slides off the ramp, shown in Fig. 12–21, with a horizontal velocity of 12 m/s. If the height of the ramp is 6 m from the floor, determine the time needed for the sack to strike the floor and the range R where sacks begin to pile up.



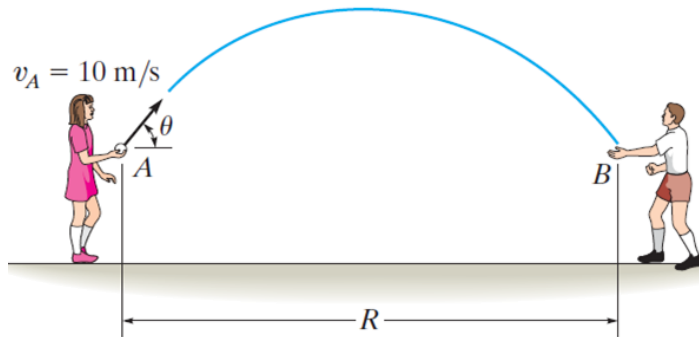
EX3:

12–87. The catapult is used to launch a ball such that it strikes the wall of the building at the maximum height of its trajectory. If it takes 1.5 s to travel from A to B , determine the velocity $\mathbf{v}_A$ at which it was launched, the angle of release θ , and the height h .



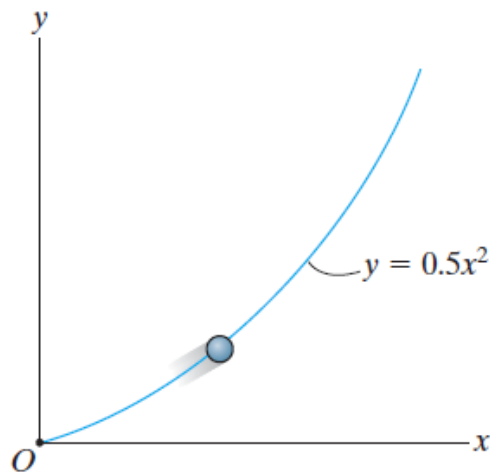
EX4:

12–89. The girl at A can throw a ball at $v_A = 10 \text{ m/s}$. Calculate the maximum possible range $R = R_{\max}$ and the associated angle θ at which it should be thrown. Assume the ball is caught at B at the same elevation from which it is thrown.



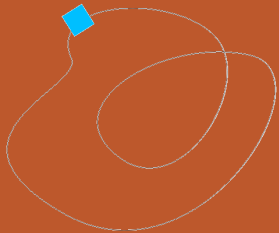
EX5:

12–79. The particle travels along the path defined by the parabola $y = 0.5x^2$. If the component of velocity along the x axis is $v_x = (5t)$ ft/s, where t is in seconds, determine the particle's distance from the origin O and the magnitude of its acceleration when $t = 1$ s. When $t = 0$, $x = 0$, $y = 0$.



Chapter 12.7: Curvilinear Motion_

Nornal and Tangential Components

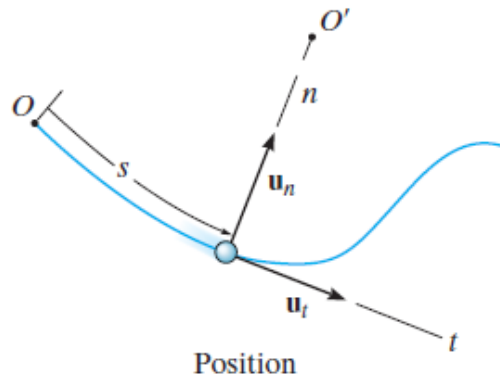


Objectives:

- To introduce the concepts of position, displacement, velocity, and acceleration.
- To study particle motion along a straight line.
- To investigate particle motion along a curved path using different coordinate systems.
- To present an analysis of dependent motion of two particles.
- To examine the principles of relative motion of two particles using translating axes.

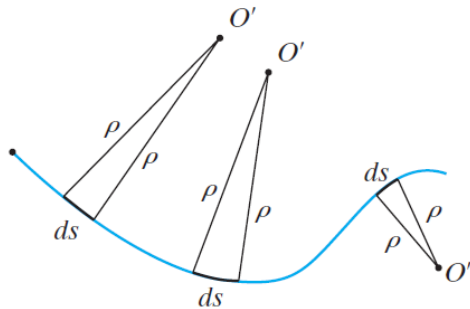
Curvilinear Motion: Normal and Tangential Components

When the path along which a particle travels is **known**, then it is often convenient to describe the motion using **n** and **t** coordinate axes which act normal and tangent to the path, respectively, and at the instant considered have their origin located at the particle.



Normal and Tangential Components

The **normal axis** can be made by noting that geometrically the curve is constructed from a series of differential arc segments ds . Each segment ds is formed from the arc of an associated circle having a radius of curvature ρ (rho) and center of curvature O' . The normal axis **n** is perpendicular to the **t** axis with its positive sense directed toward the center of curvature O' . This positive direction, which is always on the concave side of the curve, will be designated by the unit vector $\mathbf{u}_n$.



Radius of curvature

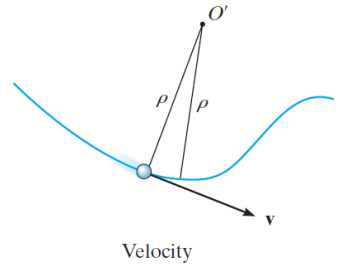
Normal and Tangential Components

Velocity: Since the particle moves, s is a function of time. The particle's velocity $\mathbf{v}$ has a direction that is always **tangent** to the path, and a magnitude that is determined by taking the time derivative of the path function $s = s(t)$, i.e., $v = ds/dt$

$$v = \dot{s} = \frac{ds}{dt}$$

Hence:

$$\mathbf{V} = v \mathbf{u}_t$$



Normal and Tangential Components

Acceleration: The acceleration of the particle is the time rate of change of the velocity. Thus,

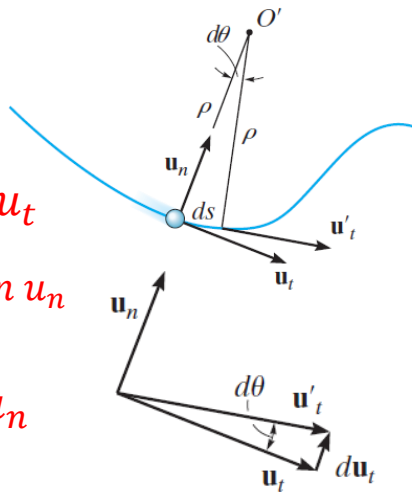
$$\mathbf{a} = \frac{dv}{dt} = \dot{v} \mathbf{u}_t + v \dot{\mathbf{u}}_t$$

In order to determine the time derivative $\mathbf{u}_t$

$$s = \rho\theta \rightarrow ds = \rho d\theta \rightarrow \dot{s} = \rho\dot{\theta} \quad \mathbf{u}'_t = \mathbf{u}_t + d\mathbf{u}_t$$

infinitesimal arc of radius $u_t = 1$. $d\mathbf{u}_t = (1)d\theta \rightarrow \text{direction } \mathbf{u}_n$

$$d\mathbf{u}_t = d\theta \mathbf{u}_n \rightarrow \dot{\mathbf{u}}_t = \dot{\theta} \mathbf{u}_n \rightarrow \dot{\mathbf{u}}_t = \frac{\dot{s}}{\rho} \mathbf{u}_n \rightarrow \dot{\mathbf{u}}_t = \frac{v}{\rho} \mathbf{u}_n$$



Normal and Tangential Components

Acceleration: The acceleration of the particle is the time rate of change of the velocity. Thus,

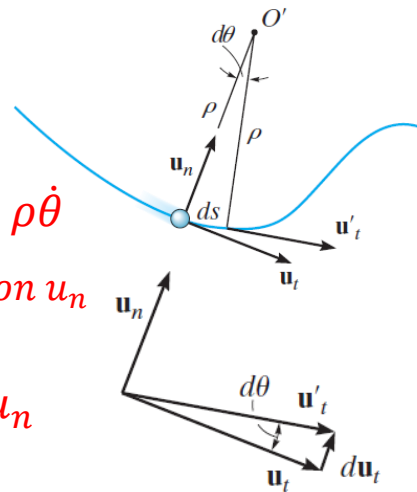
$$a = \frac{dv}{dt} = \dot{v} u_t + v \dot{u}_t$$

In order to determine the time derivative u_t

$$u'_t = u_t + du_t \quad s = \rho\theta \rightarrow ds = \rho d\theta \rightarrow \dot{s} = \rho\dot{\theta}$$

infinitesimal arc of radius $u_t = 1$. $du_t = (1)d\theta \rightarrow \text{direction } u_n$

$$du_t = d\theta u_n \rightarrow \dot{u}_t = \dot{\theta} u_n \rightarrow \ddot{u}_t = \frac{\dot{s}}{\rho} u_n \rightarrow \ddot{u}_t = \frac{v}{\rho} u_n$$



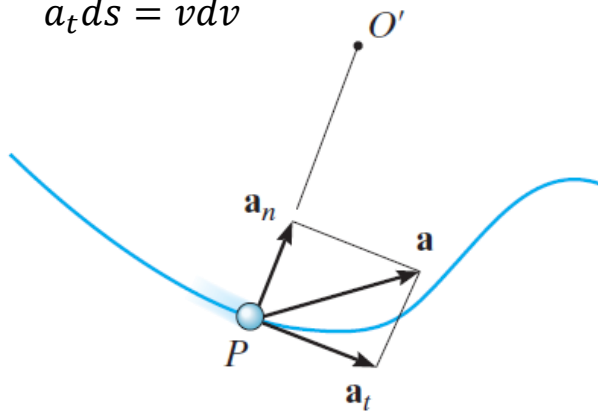
Normal and Tangential Components

$$\mathbf{a} = \frac{dv}{dt} = \dot{v} \mathbf{u}_t + v \dot{\mathbf{u}}_t \quad \text{where } a_t = \dot{v} \quad \text{or} \quad a_t ds = v dv$$

$$\mathbf{a} = \frac{dv}{dt} = a_t \mathbf{u}_t + a_n \mathbf{u}_n \quad \text{where } a_n = \frac{v^2}{\rho}$$

$$\mathbf{a} = \frac{dv}{dt} = \dot{v} \mathbf{u}_t + \frac{v^2}{\rho} \mathbf{u}_n$$

$$a = \sqrt{a_t^2 + a_n^2}$$



Normal and Tangential Components

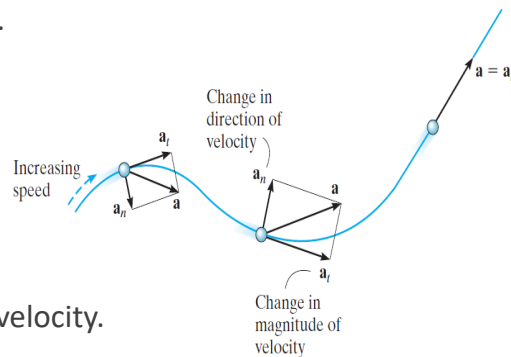
So:

1- If the particle moves along a straight line, then $\rho \rightarrow \infty$ and from $\mathbf{a}_n = \mathbf{0}$. Thus $\mathbf{a} = \mathbf{a}_t = \dot{v}$, and we can conclude that the tangential component of acceleration represents the time rate of change in the magnitude of the velocity.

2- If the particle moves along a curve with a constant speed, then

$\mathbf{a}_t = \dot{v} = 0$ and $\mathbf{a}_n = \frac{v^2}{\rho}$. Therefore, the normal component of

acceleration represents the time rate of change in the direction of the velocity.



Normal and Tangential Components

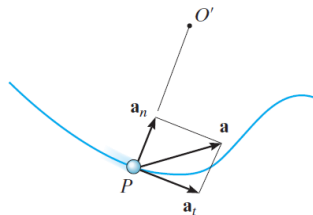
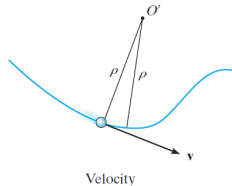
$$v = \dot{s} = \frac{ds}{dt}$$

$$\mathbf{V} = v \mathbf{u}_t$$

$$\mathbf{a} = \frac{dv}{dt} \mathbf{u}_t + v \dot{\mathbf{u}}_t \quad \text{where } a_t = \dot{v} \quad \text{or} \quad a_t ds = v dv$$

$$\mathbf{a} = \frac{dv}{dt} \mathbf{u}_t + \frac{v^2}{\rho} \mathbf{u}_n$$

$$a = \sqrt{a_t^2 + a_n^2}$$



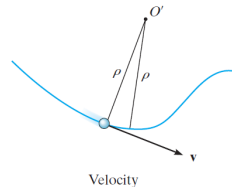
Normal and Tangential Components

If a_t is constant, $a_t = a_{tc}$, using integration like rectilinear motion, yield:

$$s = s_0 + v_0 t + \frac{1}{2}(a_t)_c t^2$$

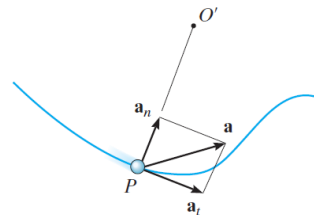
$$v = v_0 + (a_t)_c t$$

$$v^2 = v_0^2 + 2(a_t)_c(s - s_0)$$



If the path is expressed as $y = f(x)$, the radius of curvature r, ρ at any point on the path is determined from the equation:

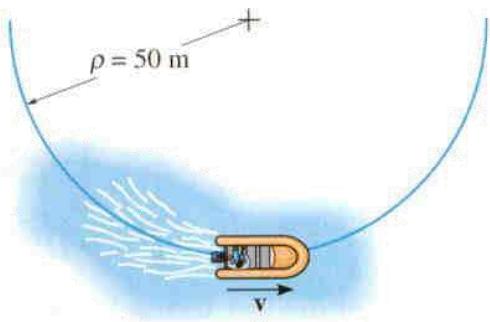
$$\rho = \frac{[1 + (dy/dx)^2]^{3/2}}{|d^2y/dx^2|}$$



EX1

Starting from rest, a motorboat travels around a circular path of $\rho = 50$ m at a speed that increases with time, $v = (0.2 t^2)$ m/s. Find the magnitudes of the boat's velocity and acceleration at the instant $t = 3$ s.

Solution:

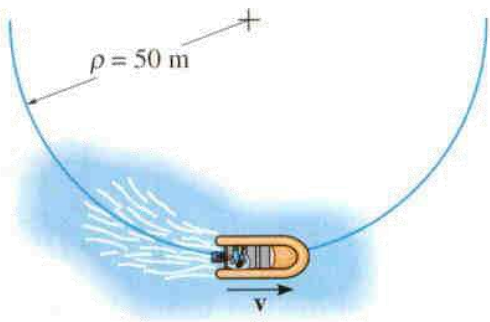


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Solution:

The velocity vector is $\mathbf{v} = v \mathbf{u}_t$, where the magnitude is given by $v = (0.2 t^2)$ m/s. At $t = 3$ s: $v = 0.2 t^2 = 0.2(3)^2 = 1.8$ m/s



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The acceleration vector is $\mathbf{a} = a_t \mathbf{u}_t + a_n \mathbf{u}_n = \dot{v} \mathbf{u}_t + \frac{v^2}{\rho} \mathbf{u}_n$

Tangential component: $a_t = \dot{v} = d(v)/dt = 0.4t$ m/s²

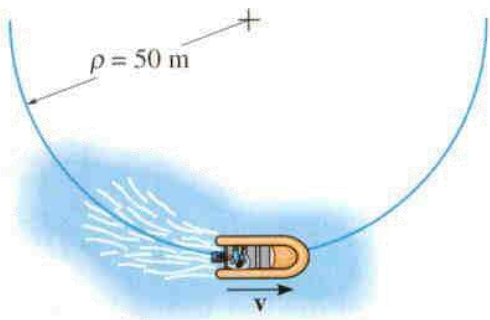
At $t = 3$ s: $a_t = 0.4t = 0.4(3) = 1.2$ m/s²

Normal component: $a_n = v^2 / \rho = (0.2t^2)^2 / (\rho)$ m/s²

At $t = 3$ s: $a_n = [(0.2)(9)]^2 / (50) = 0.0648$ m/s²

The magnitude of the acceleration is

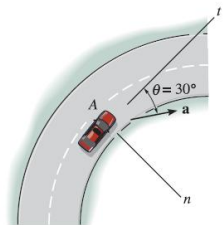
$$a = [(a_t^2 + (a_n)^2)]^{0.5} = [(1.2)^2 + (0.0648)^2]^{0.5} = 1.20 \text{ m/s}^2$$



EX2

The automobile has a speed of 80 ft/s at point A and an acceleration a having a magnitude of 10 ft/s², acting in the direction shown. Determine the radius of curvature of the path at point A and the tangential component of acceleration.

Solution:



EX2

The automobile has a speed of 80 ft/s at point A and an acceleration a having a magnitude of 10 ft/s², acting in the direction shown. Determine the radius of curvature of the path at point A and the tangential component of acceleration.

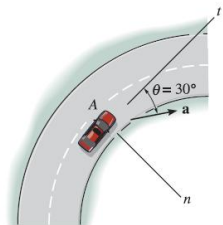
Solution:

Acceleration: The tangential acceleration is:

$$a_t = a \cos(30) = 10 \cos(30) = 8.66 \frac{ft}{s^2}$$

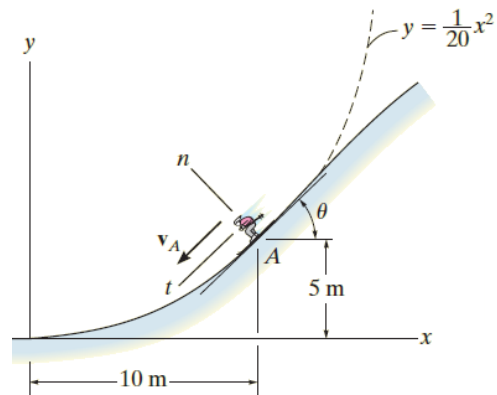
$$a_n = a \sin(30) = 10 \sin(30) = 5 \frac{ft}{s^2}$$

$$\rho = \frac{v^2}{a_n} = \frac{80^2}{5} = 1280 ft$$



EX3

When the skier reaches point A along the parabolic path he has a speed of 6 m/s which is increasing at 2 m/s^2 . Determine the direction of his velocity and the direction and magnitude of his acceleration at this instant. Neglect the size of the skier in the calculation.



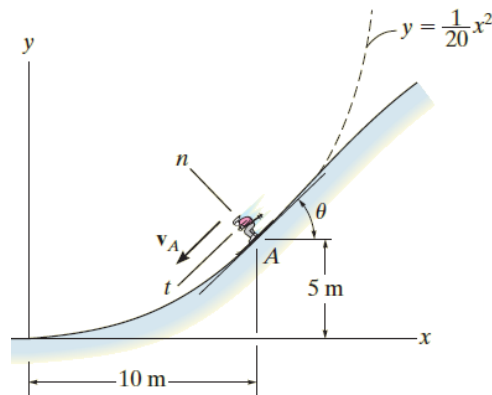
EX3

When the skier reaches point A along the parabolic path he has a speed of 6 m/s which is increasing at 2 m/s². Determine the direction of his velocity and the direction and magnitude of his acceleration at this instant. Neglect the size of the skier in the calculation.

Velocity. By definition, the velocity is always directed tangent to the path. Since $y = \frac{1}{20}x^2$, $dy/dx = \frac{1}{10}x$, then at $x = 10$ m, $dy/dx = 1$. Hence, at A , $\mathbf{v}$ makes an angle of $\theta = \tan^{-1}1 = 45^\circ$ with the x axis, Fig. 12–27*b*. Therefore,

$$\mathbf{v}_A = 6 \text{ m/s } 45^\circ \nearrow$$

Ans.

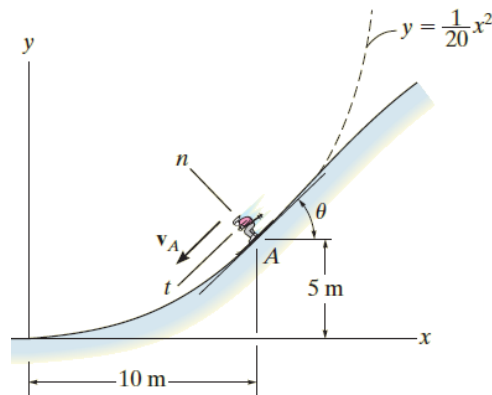


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The acceleration is determined from $\mathbf{a} = \dot{v}\mathbf{u}_t + (v^2/\rho)\mathbf{u}_n$. However, it is first necessary to determine the radius of curvature of the path at A (10 m, 5 m). Since $d^2y/dx^2 = \frac{1}{10}$, then

$$\rho = \frac{[1 + (dy/dx)^2]^{3/2}}{|d^2y/dx^2|} = \frac{[1 + (\frac{1}{10}x)^2]^{3/2}}{|\frac{1}{10}|} \bigg|_{x=10 \text{ m}} = 28.28 \text{ m}$$



EX3

When the skier reaches point A along the parabolic path he has a speed of 6 m/s which is increasing at 2 m/s². Determine the direction of his velocity and the direction and magnitude of his acceleration at this instant. Neglect the size of the skier in the calculation.

The acceleration becomes

$$\begin{aligned}\mathbf{a}_A &= \dot{v}\mathbf{u}_t + \frac{v^2}{\rho}\mathbf{u}_n \\ &= 2\mathbf{u}_t + \frac{(6 \text{ m/s})^2}{28.28 \text{ m}}\mathbf{u}_n \\ &= \{2\mathbf{u}_t + 1.273\mathbf{u}_n\} \text{ m/s}^2\end{aligned}$$

As shown in Fig. 12–27*b*,

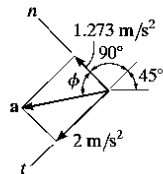
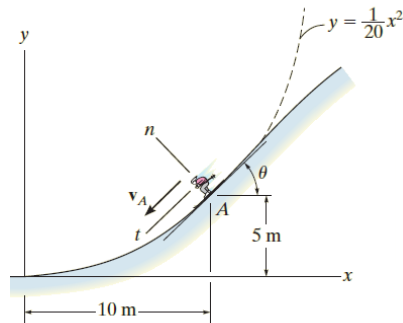
$$a = \sqrt{(2 \text{ m/s}^2)^2 + (1.273 \text{ m/s}^2)^2} = 2.37 \text{ m/s}^2$$

$$\phi = \tan^{-1} \frac{2}{1.273} = 57.5^\circ$$

Thus, $45^\circ + 90^\circ + 57.5^\circ - 180^\circ = 12.5^\circ$ so that,

$$a = 2.37 \text{ m/s}^2 \quad 12.5^\circ \nearrow$$

Ans.



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When the skier reaches point A along the parabolic path he has a speed of 6 m/s which is increasing at 2 m/s². Determine the direction of his velocity and the direction and magnitude of his acceleration at this instant. Neglect the size of the skier in the calculation.

The acceleration becomes

$$\begin{aligned}\mathbf{a}_A &= \dot{v}\mathbf{u}_t + \frac{v^2}{\rho}\mathbf{u}_n \\ &= 2\mathbf{u}_t + \frac{(6 \text{ m/s})^2}{28.28 \text{ m}}\mathbf{u}_n \\ &= \{2\mathbf{u}_t + 1.273\mathbf{u}_n\} \text{ m/s}^2\end{aligned}$$

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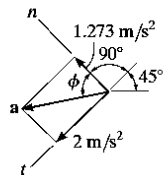
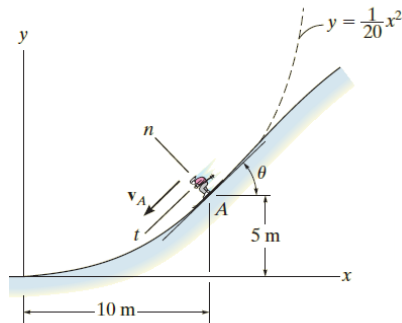
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Thus, $45^\circ + 90^\circ + 57.5^\circ - 180^\circ = 12.5^\circ$ so that,

$$a = 2.37 \text{ m/s}^2 \quad 12.5^\circ \nearrow$$

Ans.



EX4

An automobile is traveling on a curve having a radius of 800 ft. If the acceleration of the automobile is $5 \frac{ft}{s^2}$, determine the constant speed at which the automobile is traveling.

Solution:

EX4

An automobile is traveling on a curve having a radius of 800 ft. If the acceleration of the automobile is $5 \frac{\text{ft}}{\text{s}^2}$, determine the constant speed at which the automobile is traveling.

Solution:

Acceleration: Since the automobile is traveling at a constant speed, $a_t = 0$.

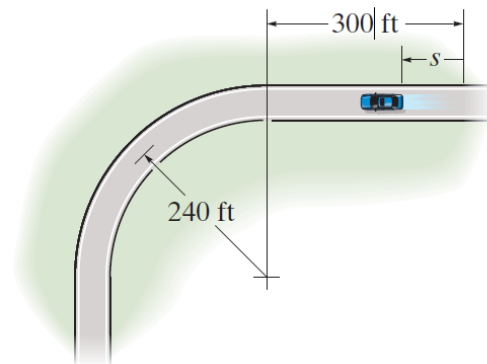
Thus, $a_n = a = 5 \text{ ft/s}^2$. Applying Eq. 12-20, $a_n = \frac{v^2}{\rho}$, we have

$$v = \sqrt{\rho a_n} = \sqrt{800(5)} = 63.2 \text{ ft/s} \quad \textbf{Ans.}$$

EX5

The automobile is originally at rest $s = 0$. If it then starts to increase its speed at $\dot{v} = 0.05t^2 \text{ ft/s}^2$ where t is in seconds, determine the magnitudes of its velocity and acceleration at $s = 550 \text{ ft}$.

Solution:



EX5

The automobile is originally at rest $s = 0$. If it then starts to increase its speed at $\dot{v} = 0.05t^2 \text{ ft/s}^2$ where t is in seconds, determine the magnitudes of its velocity and acceleration at $s = 550 \text{ ft}$.

Solution:

The car is on the curved path.

$$a_t = 0.05 t^2$$

$$\int_0^v dv = \int_0^t 0.05 t^2 dt$$

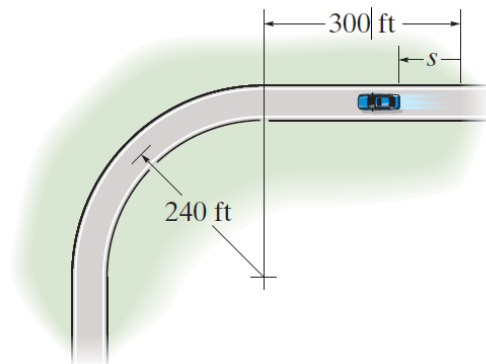
$$v = 0.0167 t^3$$

$$\int_0^s ds = \int_0^t 0.0167 t^3 dt$$

$$s = 4.167(10^{-3}) t^4$$

$$550 = 4.167(10^{-3}) t^4$$

$$t = 19.06 \text{ s}$$



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$$a_t = 0.05 t^2$$

$$\int_0^v dv = \int_0^t 0.05 t^2 dt$$

$$v = 0.0167 t^3$$

$$\int_0^s ds = \int_0^t 0.0167 t^3 dt$$

$$s = 4.167(10^{-3}) t^4$$

$$550 = 4.167(10^{-3}) t^4$$

$$t = 19.06 \text{ s}$$

So that

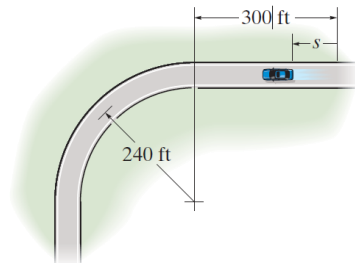
$$v = 0.0167(19.06)^3 = 115.4$$

$$v = 115 \text{ ft/s}$$

$$a_n = \frac{(115.4)^2}{240} = 55.51 \text{ ft/s}^2$$

$$a_t = 0.05(19.06)^2 = 18.17 \text{ ft/s}^2$$

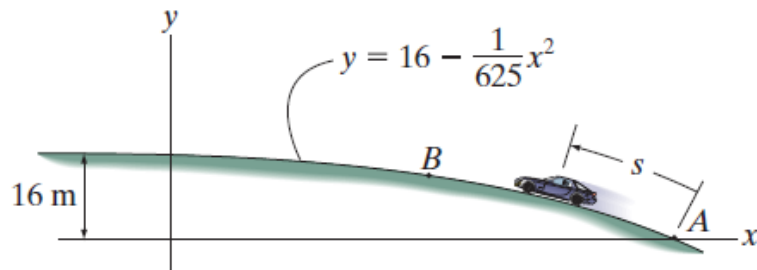
$$a = \sqrt{(55.51)^2 + (18.17)^2} = 58.4 \text{ ft/s}^2$$



EX5

If the car passes point A with a speed of 20 m/s and begins to increase its speed at a constant rate of $a_t = 0.5 \text{ m/s}^2$, determine the magnitude of the car's acceleration when $s = 100 \text{ m}$ and $x = 0$.

Solution:



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If the car passes point A with a speed of 20 m/s and begins to increase its speed at a constant rate of $a_t = 0.5 \text{ m/s}^2$, determine the magnitude of the car's acceleration when $s = 100 \text{ m}$ and $x = 0$.

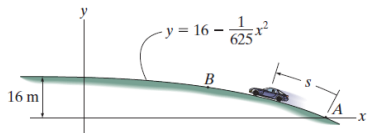
Solution:

Velocity: The speed of the car at C is

$$v_C^2 = v_A^2 + 2a_t(s_C - s_A)$$

$$v_C^2 = 20^2 + 2(0.5)(100 - 0)$$

$$v_C = 22.361 \text{ m/s}$$



EX5

If the car passes point A with a speed of 20 m/s and begins to increase its speed at a constant rate of $a_t = 0.5 \text{ m/s}^2$, determine the magnitude of the car's acceleration when $s = 100 \text{ m}$ and $x = 0$.

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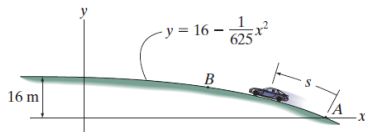
Radius of Curvature:

$$y = 16 - \frac{1}{625}x^2$$

$$\frac{dy}{dx} = -3.2(10^{-3})x$$

$$\frac{d^2y}{dx^2} = -3.2(10^{-3})$$

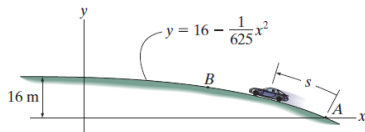
$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1 + \left(-3.2(10^{-3})x\right)^2\right]^{3/2}}{\left|-3.2(10^{-3})\right|} \bigg|_{x=0} = 312.5 \text{ m}$$



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If the car passes point A with a speed of 20 m/s and begins to increase its speed at a constant rate of $a_t = 0.5 \text{ m/s}^2$, determine the magnitude of the car's acceleration when $s = 100 \text{ m}$ and $x = 0$.

Solution:



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$$v_C^2 = 20^2 + 2(0.5)(100 - 0)$$

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Acceleration:

$$a_t = \dot{v} = 0.5 \text{ m/s}^2$$

$$a_n = \frac{v_C^2}{\rho} = \frac{22.361^2}{312.5} = 1.60 \text{ m/s}^2$$

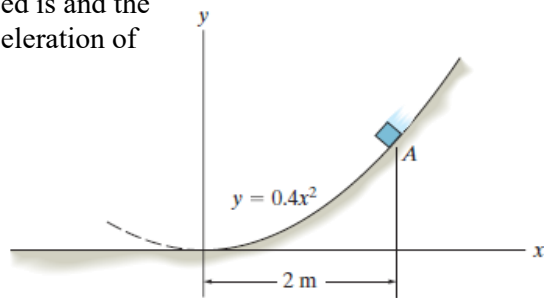
The magnitude of the car's acceleration at C is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{0.5^2 + 1.60^2} = 1.68 \text{ m/s}^2$$

EX5

The box of negligible size is sliding down along a curved path defined by the parabola $y = 0.4x^2$. When it is at A($x_A = 2\text{ m}$, $y_A = 1.5\text{ m}$), the speed is and the increase in speed is $v = 8\text{ m/s}$. Determine the magnitude of the acceleration of the box at this instant.

Solution:

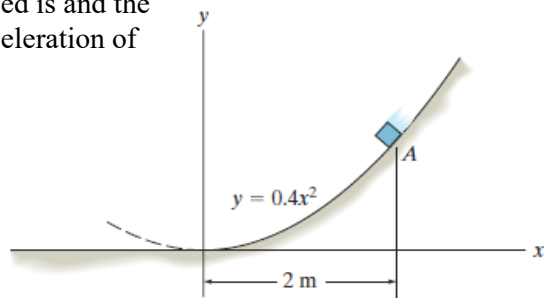


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Solution:

$$\rho = \frac{[1 + (dy/dx)^2]^{3/2}}{|d^2y/dx^2|}$$



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Solution:

$$y = 0.4x^2$$

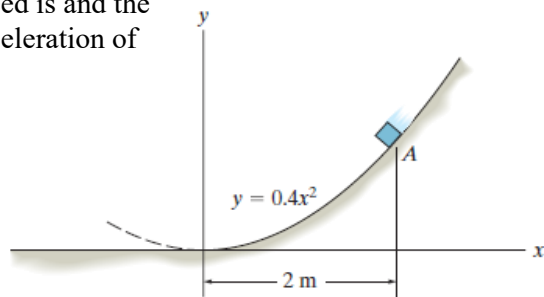
$$\left. \frac{dy}{dx} \right|_{x=2\text{ m}} = 0.8x \Big|_{x=2\text{ m}} = 1.6$$

$$\left. \frac{d^2y}{dx^2} \right|_{x=2\text{ m}} = 0.8$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{3/2}}{\left| \frac{d^2y}{dx^2} \right|} \Bigg|_{x=2\text{ m}} = \frac{\left[1 + (1.6)^2 \right]^{3/2}}{|0.8|} = 8.396\text{ m}$$

$$a_n = \frac{v^2}{\rho} = \frac{8^2}{8.396} = 7.622\text{ m/s}^2$$

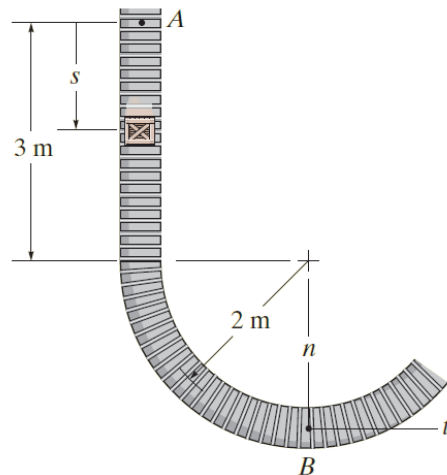
$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(4)^2 + (7.622)^2} = 8.61\text{ m/s}^2$$



EX5

The boxes travel along the industrial conveyor. If a box as in Fig. starts from rest at A and increases its speed such that $a_t = (0.2t) \text{ m/s}^2$, where t is in seconds, determine the magnitude of its acceleration when it arrives at point B.

Solution:



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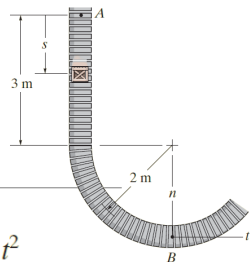
Solution:

$$a_t = \dot{v} = 0.2t$$

$$\int_0^v dv = \int_0^t 0.2t \, dt$$

$$v = 0.1t^2$$

The time needed for the box to reach point B can be determined by realizing that the position of B is $s_B = 3 + 2\pi(2)/4 = 6.142 \text{ m}$,



$$v = \frac{ds}{dt} = 0.1t^2$$

$$\int_0^{6.142 \text{ m}} ds = \int_0^{t_B} 0.1t^2 dt$$

$$6.142 \text{ m} = 0.0333t_B^3$$

$$t_B = 5.690 \text{ s}$$

$$(a_B)_t = \dot{v}_B = 0.2(5.690) = 1.138 \text{ m/s}^2$$

$$v_B = 0.1(5.69)^2 = 3.238 \text{ m/s}$$

At B, $\rho_B = 2 \text{ m}$, so that

$$(a_B)_n = \frac{v_B^2}{\rho_B} = \frac{(3.238 \text{ m/s})^2}{2 \text{ m}} = 5.242 \text{ m/s}^2$$

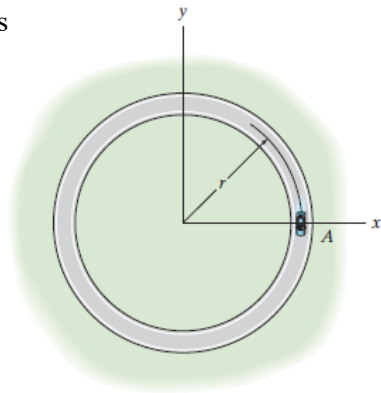
The magnitude of $\mathbf{a}_B$, Fig. 12–29c, is therefore

$$a_B = \sqrt{(1.138 \text{ m/s}^2)^2 + (5.242 \text{ m/s}^2)^2} = 5.36 \text{ m/s}^2$$

EX5

The car travels around the circular track having a radius of $r = 300$ m such that when it is at point A it has a velocity of 5 m/s, which is increasing at the rate of $v = (0.06t)$ m/s², where t is in seconds. Determine the magnitudes of its velocity and acceleration when it has traveled one-third the way around the track.

Solution:



EX5

The car travels around the circular track having a radius of $r = 300$ m such that when it is at point A it has a velocity of 5 m/s, which is increasing at the rate of $v = (0.06t)$ m/s², where t is in seconds. Determine the magnitudes of its velocity and acceleration when it has traveled one-third the way around the track.

Solution:

$$a_t = v = 0.06t$$

$$dv = a_t dt$$

$$\int_s^v dv = \int_0^t 0.06t dt$$

$$v = 0.03t^2 + 5$$

$$ds = v dt$$

$$\int_0^s ds = \int_0^t (0.03t^2 + 5) dt$$

$$s = 0.01t^3 + 5t$$

$$s = \frac{1}{3} (2\pi(300)) = 628.3185$$

$$0.01t^3 + 5t - 628.3185 = 0$$

Solve for the positive root,

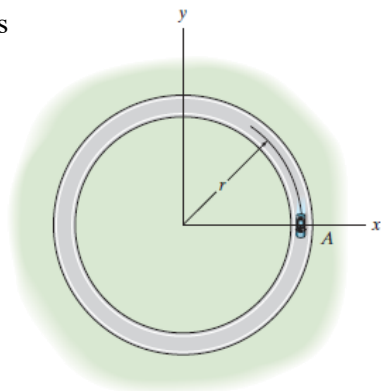
$$t = 35.58 \text{ s}$$

$$v = 0.03(35.58)^2 + 5 = 42.978 \text{ m/s} = 43.0 \text{ m/s}$$

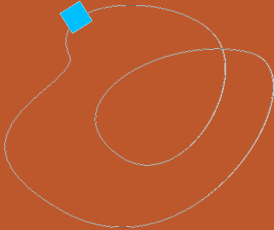
$$a_n = \frac{v^2}{\rho} = \frac{(42.978)^2}{300} = 6.157 \text{ m/s}^2$$

$$a_t = 0.06(35.58) = 2.135 \text{ m/s}^2$$

$$a = \sqrt{(6.157)^2 + (2.135)^2} = 6.52 \text{ m/s}^2$$



Chapter 12.10: Relative Motion of two Particles

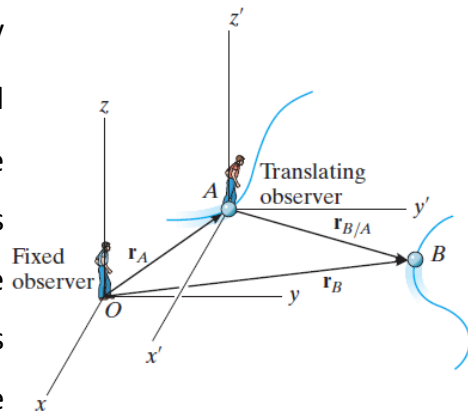


Objectives:

- To introduce the concepts of position, displacement, velocity, and acceleration.
- To study particle motion along a straight line.
- To investigate particle motion along a curved path using different coordinate systems.
- **To examine the principles of relative motion of two particles using translating axes.**

Relative-Motion of Two Particles Using Translating Axes

Position. Consider particles A and B, which move along the arbitrary paths. The absolute position of each particle, $\mathbf{r}_A$ and $\mathbf{r}_B$, is measured from the common origin O of the fixed x, y, z reference frame. The origin of a second frame of reference x', y', z' is attached to and moves with particle A. The axes of this frame are only permitted to translate relative to the fixed frame. The position of B measured relative to A is denoted by the relative-position vector $\mathbf{r}_{B/A}$. Using vector addition, the three vectors shown in Fig. 12–42 can be related by the equation



Relative-Motion of Two Particles Using Translating Axes

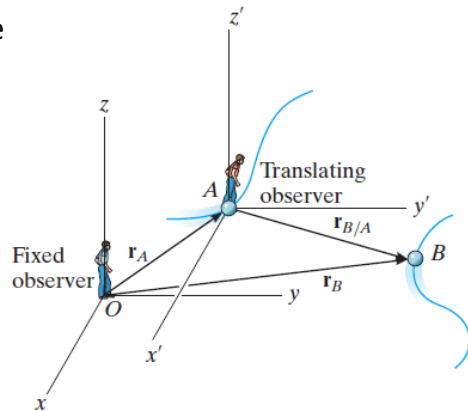
Using vector addition, the three vectors shown in Fig. 12–42 can be related by the equation:

$$\mathbf{r}_B = \mathbf{r}_A + \mathbf{r}_{B/A}$$

Velocity. An equation that relates the velocities of the particles is determined by taking the time derivative of the above equation; i

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

Here $\mathbf{v}_B = d\mathbf{r}_B/dt$ and $\mathbf{v}_A = d\mathbf{r}_A/dt$ refer to absolute velocities



Relative-Motion of Two Particles Using Translating Axes

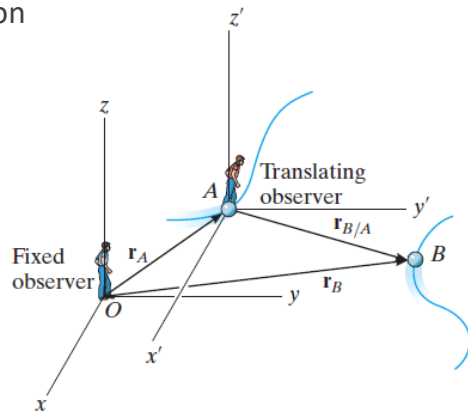
Acceleration. The time derivative velocity yields a similar vector relation between the *absolute* and *relative accelerations* of

particles A and B.

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

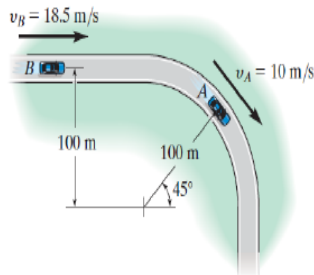
Here $\mathbf{a}_{B/A}$ is the acceleration of B as seen by the observer located at A

And translating with the x, y, z reference frame.



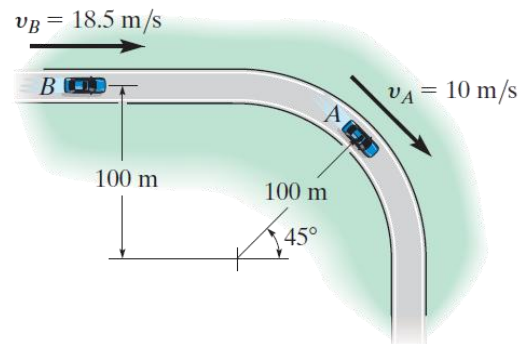
EX1

- 1) at the instant shown, the car at **A** is traveling at 10 (m/s) around the curve while increasing its speed at $5 \text{ (m/s}^2\text{)}$. The car at **B** is traveling at 18.5 (m/s) along the straightaway and increasing its speed at $2 \text{ (m/s}^2\text{)}$. Determine the relative velocity and relative acceleration of **A** with respect to **B** at this instant.



EX1

- given data: car A traveling at $v_A = 10 \text{ (m/s)}$ around a curve with acceleration of $a = 10 \text{ (m/s}^2\text{)}$. Car B traveling at $v_B = 18.5 \text{ (m/s)}$ around a curve with acceleration of $a = 2 \text{ (m/s}^2\text{)}$. $\rho = 100 \text{ (m)}$ and the angle $\theta = 45^\circ$.
- required: $v_{A/B}$ and $a_{A/B}$



EX1

- 1) velocity of car A is moving along a curve so it has two components along **i** and **j**

$$v_A = 10 \cos 45^\circ \mathbf{i} - 10 \sin 45^\circ \mathbf{j} = (7.071 \mathbf{i} - 7.071 \mathbf{j}) \text{ (m/s)}$$

- 2) velocity of car B is moving along a line so it has one components along **i**

$$v_B = 18.5 \mathbf{i} \text{ (m/s)}$$

- 3) the relative velocity $v_{A/B}$ is

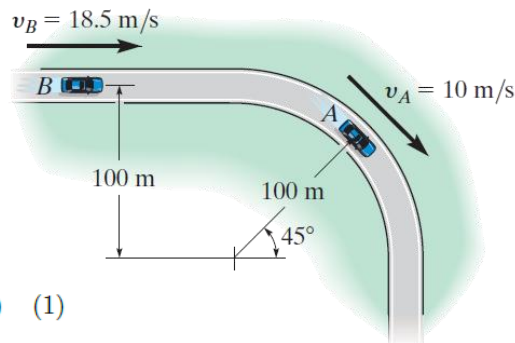
$$v_A = v_B + v_{A/B}$$

so

$$v_{A/B} = v_A - v_B = (7.071 \mathbf{i} - 7.071 \mathbf{j}) - 18.5 \mathbf{i} = (-11.429 \mathbf{i} - 7.071 \mathbf{j}) \text{ (m/s)} \quad (1)$$

and its magnitude is

$$v_{A/B} = \sqrt{v_{A/B}^2 \mathbf{i} + v_{A/B}^2 \mathbf{j}} = \sqrt{(-11.429)^2 + (-7.071)^2} = 13.4 \text{ (m/s)}$$



EX1

- 1) acceleration of car **A** is moving along a curve so it has two components along n and t coordinates
the tangential acceleration

$$a_{A_t} = 5 \text{ (m/s}^2\text{)}$$

the normal acceleration

$$a_{A_n} = \frac{v_A^2}{\rho} = \frac{10^2}{100} = 1 \text{ (m/s}^2\text{)}$$

- 2) note that to find the total acceleration of a_A , we have to find the a_{A_t} and a_{A_n} components along $\mathbf{i}$ and $\mathbf{j}$
for the tangential acceleration

$$a_{A_t} = (5 \cos 45^\circ \mathbf{i} - 5 \sin 45^\circ \mathbf{j}) \text{ (m/s}^2\text{)}$$

for the normal acceleration

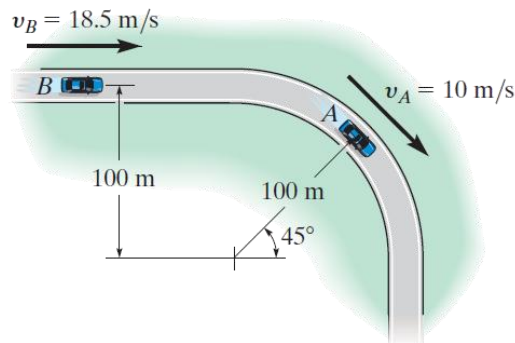
$$a_{A_n} = (-1 \cos 45^\circ \mathbf{i} - 1 \sin 45^\circ \mathbf{j}) \text{ (m/s}^2\text{)}$$

so the total is the sum along $\mathbf{i}$ and $\mathbf{j}$

$$\mathbf{a}_A = (5 \cos 45^\circ - 1 \cos 45^\circ)\mathbf{i} + (-1 \sin 45^\circ - 5 \sin 45^\circ)\mathbf{j} = (2.828 \mathbf{i} - 4.243 \mathbf{j}) \text{ (m/s}^2\text{)}$$

- 3) acceleration of car **B** is moving along a line so it has one components along $\mathbf{i}$

$$\mathbf{a}_B = 2 \mathbf{i} \text{ (m/s}^2\text{)}$$



EX1

3) acceleration of car B is moving along a line so it has one components along $\mathbf{i}$

$$a_B = 2 \mathbf{i} \text{ (m/s}^2\text{)}$$

4) the relative acceleration $a_{A/B}$ is

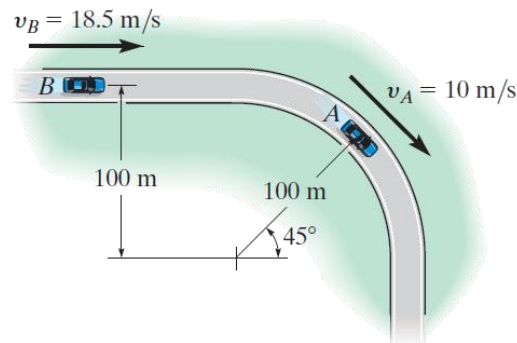
$$a_A = a_B + a_{A/B}$$

so

$$a_{A/B} = a_A - a_B = (2.828 \mathbf{i} - 4.243 \mathbf{j}) - 2 \mathbf{i} = (0.828 \mathbf{i} - 4.243 \mathbf{j}) \text{ (m/s}^2\text{)} \quad (2)$$

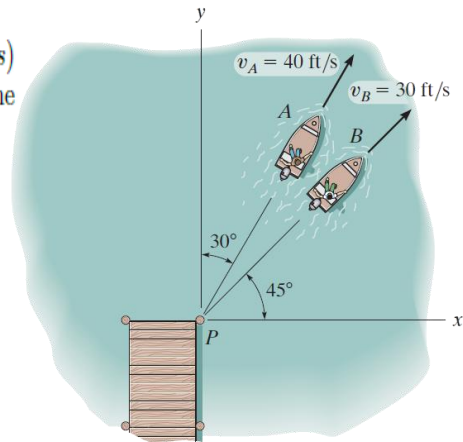
and its magnitude is

$$a_{A/B} = \sqrt{a_{A/B}^2 \mathbf{i} + a_{A/B}^2 \mathbf{j}} = \sqrt{(0.828)^2 + (-4.243)^2} = 4.32 \text{ (m/s}^2\text{)}$$



EX2

Two boats leave the pier P at the same time and travel in the directions shown. If $v_A = 40 \text{ (ft/s)}$ and $v_B = 30 \text{ (ft/s)}$, determine the velocity of boat A relative to boat B ($v_{A/B}$). Find also the direction of the relative velocity ($v_{A/B}$).

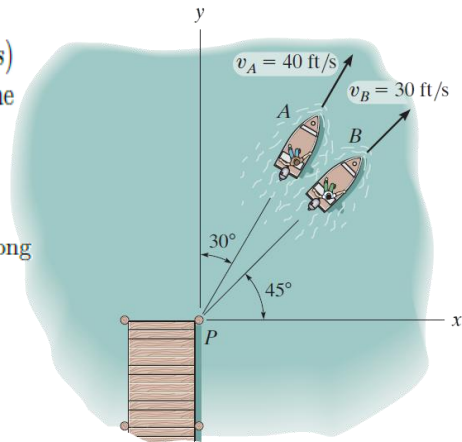


EX2

Two boats leave the pier P at the same time and travel in the directions shown. If $v_A = 40$ (ft/s) and $v_B = 30$ (ft/s), determine the velocity of boat A relative to boat B ($v_{A/B}$). Find also the direction of the relative velocity ($v_{A/B}$).

- given data: boat A traveling at $v_A = 40$ (ft/s) and boat B traveling at $v_B = 30$ (ft/s) along a line.
- required: $v_{A/B}$ and the direction θ .

* Solution:



EX2

* Solution:

a) components of v_A

$$v_A = 40 \sin 30^\circ \vec{i} + 40 \cos 30^\circ \vec{j} \Rightarrow v_A = 20\vec{i} + 34.64\vec{j}$$

b) components of v_B

$$v_B = 30 \cos 45^\circ \vec{i} + 30 \sin 45^\circ \vec{j} \Rightarrow v_B = 21.21\vec{i} + 21.21\vec{j}$$

c) relative velocity of $v_{A/B}$

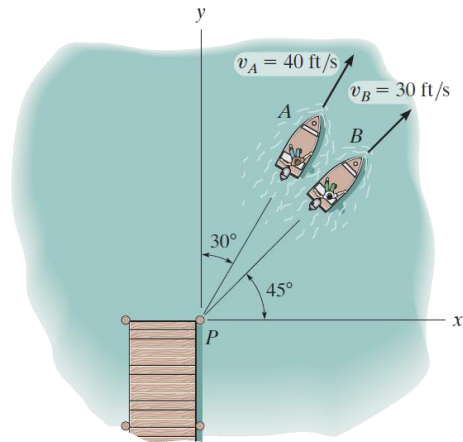
$$v_{A/B} = v_A - v_B \Rightarrow v_{A/B} = 20\vec{i} - 21.21\vec{i} + 34.64\vec{j} - 21.21\vec{j} = -1.21\vec{i} + 13.43\vec{j}$$

d) magnitude of $v_{A/B}$

$$|v_{A/B}| = \sqrt{(-1.21)^2 + (13.43)^2} = \sqrt{1.464 + 180.36} = \sqrt{181.82} = 13.48 \text{ (ft/s)}$$

e) direction of $v_{A/B}$

$$\theta = \tan^{-1} \left(\frac{13.43}{1.21} \right) = 84.85^\circ$$



Chapter 13: Kinetics of a Particle (Friction)



Objectives:

- To state Newton's Second Law of Motion and to define mass and weight.
- To analyze the accelerated motion of a particle using the equation of motion with different coordinate systems.
- To investigate central-force motion and apply it to problems in space mechanics.

13.1: Newton's Second Law of Motion

- **Kinetics** is a branch of dynamics that deals with the relationship between the change in motion of a body and the forces that cause this change.
- The basis for kinetics is **Newton's second law**, which states that when an unbalanced force acts on a particle, the particle will accelerate in the direction of the force with a magnitude that is proportional to the force.
- If the mass of the particle is m , Newton's second law of motion may be written in mathematical form as:

$$\mathbf{F} = m\mathbf{a}$$

13.1: Newton's Second Law of Motion

- The forces typically acting on a particle or body are

1) Weight:

$$W = mg$$

- acts at the center of gravity towards Earth's center.

2) Spring force:

$$F_s = kx$$

- acts along the spring to oppose spring deformation.

3) Normal force N : Newton's Third Law

- a force pair normal to the contact surface/point.

13.1: Newton's Second Law of Motion

- The forces typically acting on a particle or body are

4) Tension force T : Newton's Third Law

- a pair acting along the cable/rope/chain line of action.
- constant throughout the cable/rope/chain.

4) Friction:

μ_r
 μ_d

$$F_f = \mu F_{nor}$$

(4)

- acts parallel to the contact surface to retard ($\mu_k F_{nor}$) or prevent ($\mu_s F_{nor}$) motion. μ_k kinetic friction coefficient and μ_s static friction coefficient.

6) Damping:

$$F_d = c\dot{x}$$

(5)

- acts along the damper shaft to retard motion.

13.1: Newton's Second Law of Motion

- The forces typically acting on a particle or body are

4) Tension force T : Newton's Third Law

- a pair acting along the cable/rope/chain line of action.
- constant throughout the cable/rope/chain.

4) Friction:

$$F_f = \mu F_{nor}$$

(4)

- acts parallel to the contact surface to retard ($\mu_k F_{nor}$) or prevent ($\mu_s F_{nor}$) motion. μ_k kinetic friction coefficient and μ_s static friction coefficient.

6) Damping:

$$F_d = c\dot{x}$$

(5)

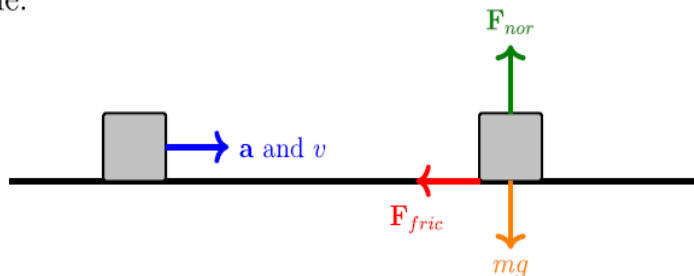
- acts along the damper shaft to retard motion.

13.1: Newton's Second Law of Motion

Definition

Friction is a force between two surfaces that are sliding, or trying to slide, across each other.

- Let's assume:



- friction opposes to the motion
- friction occurs in translation motion
- it is tangent to the surface of the object and

13.1: Newton's Second Law of Motion

- A surface has two coefficient of friction:

- static coefficient friction (μ_s)
- kinetic coefficient friction (μ_k)

- static friction: when object is at rest

- kinetic friction: when object is at motion

When an object can move?

13.1: Newton's Second Law of Motion

When an object can move?

- for an object to start moving it must overcome the maximum static force of the surface ($\mathbf{F}_{smax} = \mu_s F_{nor}$)
 - $\mathbf{F}_{fric} > \mathbf{F}_{smax}$: in this case the object moves with a kinetic friction ($\mathbf{F}_{fric} = \mu_k F_{nor}$)
 - $\mathbf{F}_{fric} < \mathbf{F}_{smax}$: in this case the object is not moving and still at equilibrium ($\mathbf{F}_{fric} \neq \mu_k F_{nor}$)

$$\begin{cases} \mathbf{F}_{fric} > \mathbf{F}_{smax} & \text{motion} \\ \mathbf{F}_{fric} < \mathbf{F}_{smax} & \text{at equilibrium} \end{cases}$$

(1)

Example1:

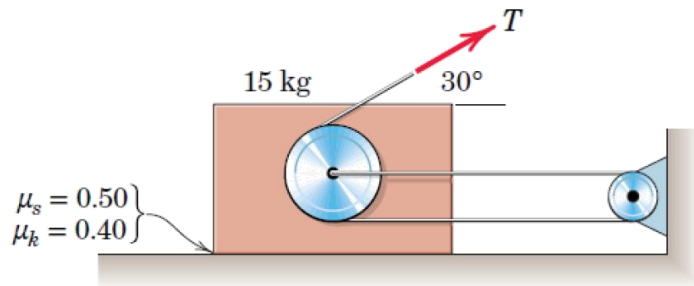
Determine the initial acceleration of the 15 (kg) block if:

(a) $T = 23$ (N)

(b) $T = 26$ (N).

The system is initially at rest with no slack in the cable, and the mass and friction of the pulleys are negligible.

Solution:



Example1:

Determine the initial acceleration of the 15 (kg) block if:

(a) $T = 23$ (N)

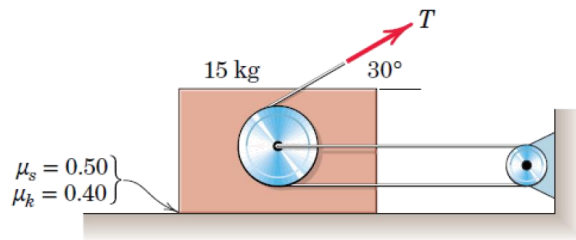
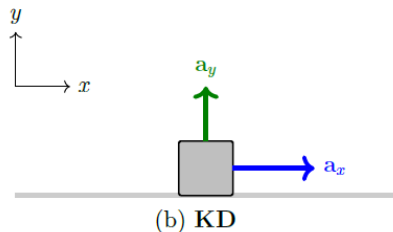
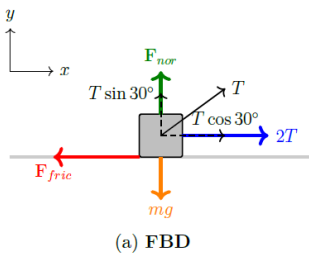
(b) $T = 26$ (N).

The system is initially at rest with no slack in the cable, and the mass and friction of the pulleys are negligible.

Solution:

-Given data: $m = 15$ (kg), $\mu_s = 0.5$ and $\mu_k = 0.4$

– required: **a** at (a) $T = 23$ (N) and (b) $T = 26$ (N)



Example1:

Determine the initial acceleration of the 15 (kg) block if:

(a) $T = 23$ (N)

(b) $T = 26$ (N).

The system is initially at rest with no slack in the cable, and the mass and friction of the pulleys are negligible.

Solution:

-Given data: $m = 15$ (kg), $\mu_s = 0.5$ and $\mu_k = 0.4$

- required: **a** at (a) $T = 23$ (N) and (b) $T = 26$ (N)

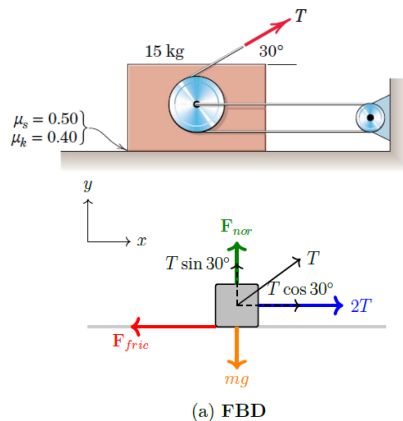
$$\Sigma F_x = m a_x \quad \Rightarrow \quad -F_{fric} + T \cos 30^\circ + 2T = m a_x$$

$$\Sigma F_y = m a_y \quad \Rightarrow \quad F_{nor} + T \sin 30^\circ - mg = m a_y \quad \begin{matrix} \nearrow 0 \end{matrix}$$

$$F_{nor} = -T \sin 30^\circ + mg$$

for $T = 23$ (N), we get

$$F_{nor} = -23 \sin 30^\circ + 15 \times 9.81 = 135.65 \text{ (N)}$$



Example1:

Determine the initial acceleration of the 15 (kg) block if:

(a) $T = 23$ (N)

(b) $T = 26$ (N).

The system is initially at rest with no slack in the cable, and the mass and friction of the pulleys are negligible.

Solution:

-Given data: $m = 15$ (kg), $\mu_s = 0.5$ and $\mu_k = 0.4$

- required: **a** at (a) $T = 23$ (N) and (b) $T = 26$ (N)

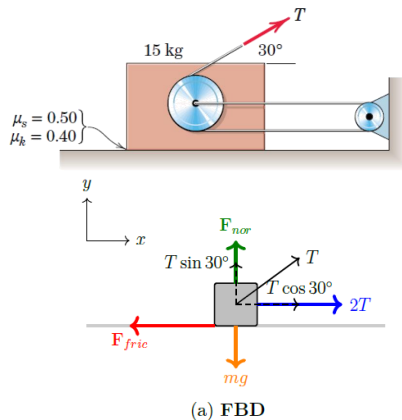
for $T = 23$ (N), we get

$$F_{nor} = -23 \sin 30^\circ + 15 \times 9.81 = 135.65 \text{ (N)}$$

$$\Sigma F_x = m a_x \quad \Rightarrow \quad -F_{fric} + T \cos 30^\circ + 2T = m a_x$$

$$F_{smax} = \mu_s F_{nor} = 0.5 \times 135.65 = 67.825 \text{ (N)}$$

$$\Sigma F_{max} = 2T + T \cos 30^\circ = 2 \times 23 + 23 \cos 30^\circ = 65.92 \text{ (N)}$$



Example1:

Determine the initial acceleration of the 15 (kg) block if:

(a) $T = 23$ (N)

(b) $T = 26$ (N).

The system is initially at rest with no slack in the cable, and the mass and friction of the pulleys are negligible.

Solution:

-Given data: $m = 15$ (kg), $\mu_s = 0.5$ and $\mu_k = 0.4$

- required: **a** at (a) $T = 23$ (N) and (b) $T = 26$ (N)

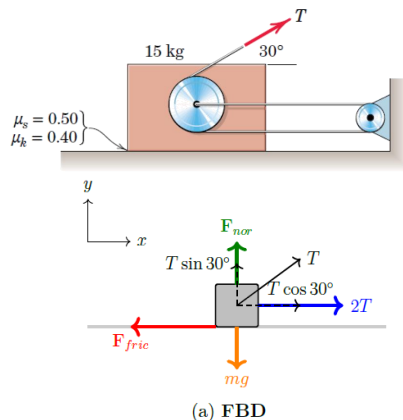
$$\Sigma F_x = m a_x \quad \Rightarrow \quad -F_{fric} + T \cos 30^\circ + 2T = m a_x$$

$$F_{smax} = \mu_s F_{nor} = 0.5 \times 135.65 = 67.825 \text{ (N)}$$

$$\Sigma F_{max} = 2T + T \cos 30^\circ = 2 \times 23 + 23 \cos 30^\circ = 65.92 \text{ (N)}$$

$$\Sigma F_{max} < F_{smax}$$

(no motion so no acceleration)



Example1:

Determine the initial acceleration of the 15 (kg) block if:

(a) $T = 23$ (N)

(b) $T = 26$ (N).

The system is initially at rest with no slack in the cable, and the mass and friction of the pulleys are negligible.

Solution:

-Given data: $m = 15$ (kg), $\mu_s = 0.5$ and $\mu_k = 0.4$

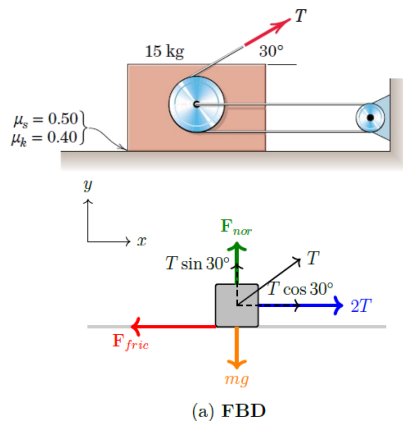
- required: **a** at (a) $T = 23$ (N) and (b) $T = 26$ (N)

$$\Sigma F_x = m a_x \quad \Rightarrow \quad -F_{fric} + T \cos 30^\circ + 2T = m a_x$$

$$F_{smax} = \mu_s F_{nor} = 0.5 \times 135.65 = 67.825 \text{ (N)}$$

If we continue the solution:

$$a_x = (-F_{fric} + T \cos(30) + 2T) / m = (-67.825 + 65.92) / 15 = -0.127 \text{ m/s}^2$$



Example1:

Determine the initial acceleration of the 15 (kg) block if:

(a) $T = 23$ (N)

(b) $T = 26$ (N).

The system is initially at rest with no slack in the cable, and the mass and friction of the pulleys are negligible.

Solution:

-Given data: $m = 15$ (kg), $\mu_s = 0.5$ and $\mu_k = 0.4$

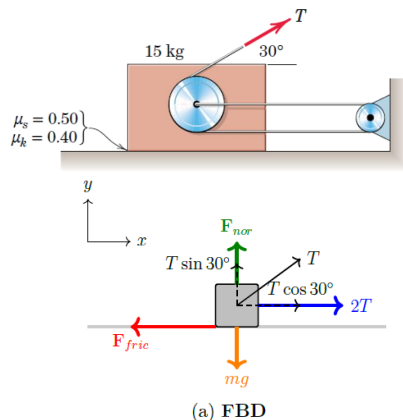
– required: **a** at (a) $T = 23$ (N) and (b) $T = 26$ (N)

for $T = 26$ (N), we get

$$F_{nor} = -26 \sin 30^\circ + 15 \times 9.81 = 134.15 \text{ (N)}$$

$$F_{smax} = \mu_s F_{nor} = 0.5 \times 135.65 = 67.075 \text{ (N)}$$

$$\Sigma F_{max} = 2T + T \cos 30^\circ = 2 \times 23 + 23 \cos 30^\circ = 74.51 \text{ (N)}$$



Example1:

Determine the initial acceleration of the 15 (kg) block if:

(a) $T = 23$ (N)

(b) $T = 26$ (N).

The system is initially at rest with no slack in the cable, and the mass and friction of the pulleys are negligible.

Solution:

-Given data: $m = 15$ (kg), $\mu_s = 0.5$ and $\mu_k = 0.4$

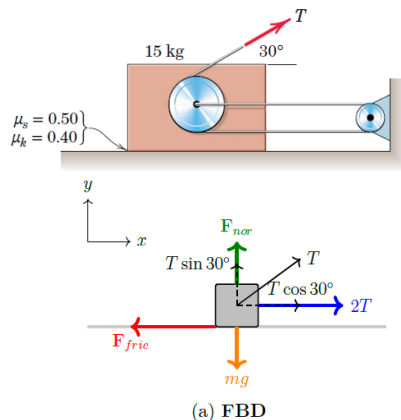
– required: **a** at (a) $T = 23$ (N) and (b) $T = 26$ (N)

$$F_{s_{max}} = \mu_s F_{nor} = 0.5 \times 135.65 = 67.075 \text{ (N)}$$

$$\Sigma F_{max} = 2T + T \cos 30^\circ = 2 \times 23 + 23 \cos 30^\circ = 74.51 \text{ (N)}$$

$$\Sigma F_{max} > F_{s_{max}}$$

$$\Sigma F_x = m a_x \Rightarrow -F_{fric} + T \cos 30^\circ + 2T = m a_x$$



Example1:

Determine the initial acceleration of the 15 (kg) block if:

(a) $T = 23$ (N)

(b) $T = 26$ (N).

The system is initially at rest with no slack in the cable, and the mass and friction of the pulleys are negligible.

Solution:

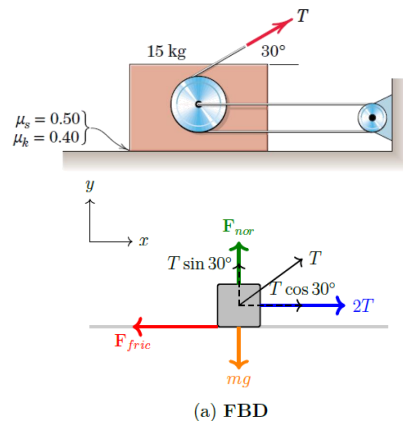
-Given data: $m = 15$ (kg), $\mu_s = 0.5$ and $\mu_k = 0.4$

- required: **a** at (a) $T = 23$ (N) and (b) $T = 26$ (N)

$$\Sigma F_x = m a_x \Rightarrow -F_{fric} + T \cos 30^\circ + 2T = m a_x$$

$$a_x = \frac{-\mu_k F_{nor} + T \cos 30^\circ + 2T}{m}$$

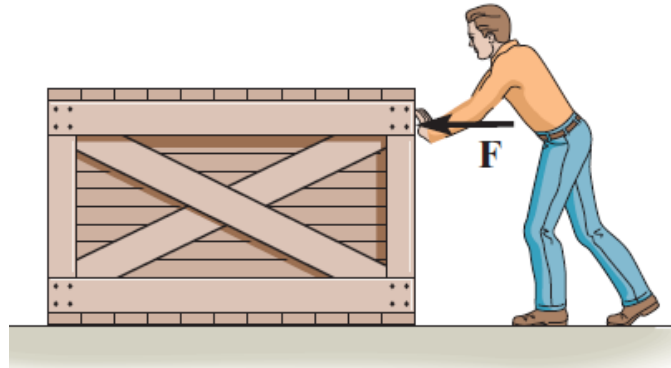
$$a_x = 1.390 \text{ (m/s}^2\text{)} \rightarrow$$



Example2:

The 75-kg man pushes on the 150-kg crate with a horizontal force $\mathbf{F}$. If the coefficients of static and kinetic friction between the crate and the surface are $\mu_s = 0.3$ and $\mu_k = 0.2$, and the coefficient of static friction between the man's shoes and the surface is $\mu_s = 0.8$,

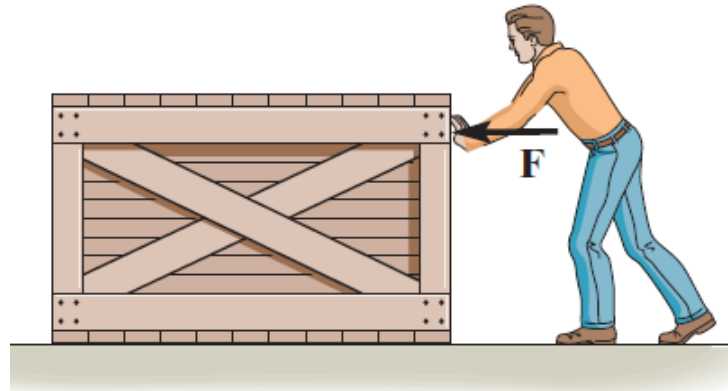
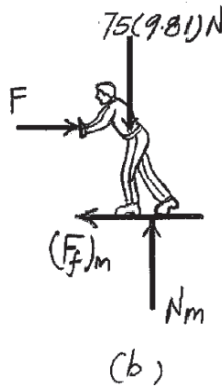
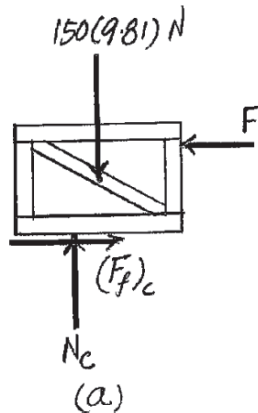
show that the man can move the crate. What is the greatest acceleration the man can give the crate?



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Show that the man can move the crate. What is the greatest acceleration the man can give the crate?

Solution: Assuming the crate is about to move:

$$F_{fc} = \mu_s N_c = 0.3 N_c$$

FBD for the crate

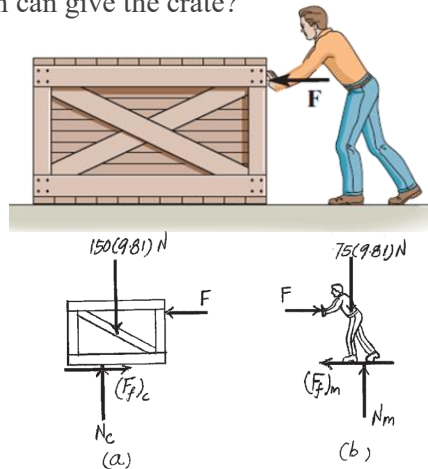
$$+\uparrow \Sigma F_y = 0; \quad N_C - 150(9.81) = 0 \quad N_C = 1471.5 \text{ N}$$

$$\rightarrow \Sigma F_x = 0; \quad 0.3(1471.5) - F = 0 \quad F = 441.45 \text{ N}$$

FBD for the man

$$+\uparrow \Sigma F_y = 0; \quad N_m - 75(9.81) = 0 \quad N_m = 735.75 \text{ N}$$

$$\rightarrow \Sigma F_x = 0; \quad 441.45 - (F_f)_m = 0 \quad (F_f)_m = 441.45 \text{ N}$$



Example2:

The 75-kg man pushes on the 150-kg crate with a horizontal force $\mathbf{F}$. If the coefficients of static and kinetic friction between the crate and the surface are $\mu_s = 0.3$ and $\mu_k = 0.2$, and the coefficient of static friction between the man's shoes and the surface is $\mu_s = 0.8$.

Show that the man can move the crate. **What is the greatest acceleration the man can give the crate?**

Solution: Assuming the crate is about to move:

$$F_{fc} = \mu_s N_c = 0.3 N_c$$

FBD for the crate

$$+\uparrow \Sigma F_y = 0; \quad N_c - 150(9.81) = 0 \quad N_c = 1471.5 \text{ N}$$

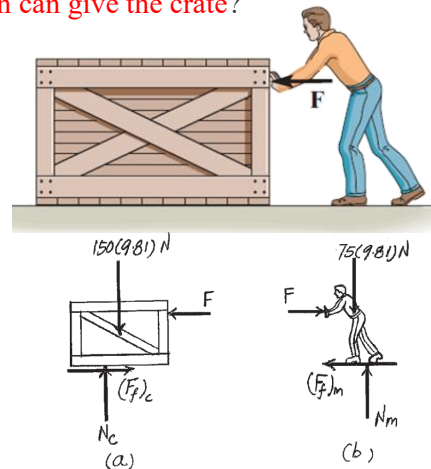
$$\rightarrow \Sigma F_x = 0; \quad 0.3(1471.5) - F = 0 \quad F = 441.45 \text{ N}$$

FBD for the man

$$+\uparrow \Sigma F_y = 0; \quad N_m - 75(9.81) = 0 \quad N_m = 735.75 \text{ N}$$

$$\rightarrow \Sigma F_x = 0; \quad 441.45 - (F_f)_m = 0 \quad (F_f)_m = 441.45 \text{ N}$$

Since $(F_f)_m < \mu'_s N_m = 0.8(735.75) = 588.6 \text{ N}$, **the man is able to move the crate.**



Example2:

The 75-kg man pushes on the 150-kg crate with a horizontal force $\mathbf{F}$. If the coefficients of static and kinetic friction between the crate and the surface are $\mu_s = 0.3$ and $\mu_k = 0.2$, and the coefficient of static friction between the man's shoes and the surface is $\mu_s = 0.8$.

Show that the man can move the crate. **What is the greatest acceleration the man can give the crate?**

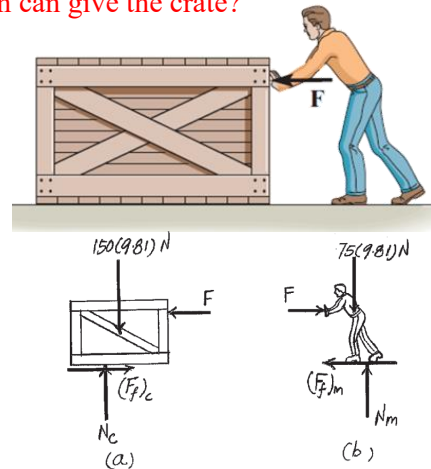
Solution: Assuming the crate is about start moving:

Equation of Motion. The greatest acceleration of the crate can be produced when the man is on the verge of slipping. Thus, $(F_f)_m = \mu'_s N_m = 0.8(735.75) = 588.6 \text{ N}$.

$$\rightarrow \Sigma F_x = 0; \quad F - 588.6 = 0 \quad F = 588.6 \text{ N}$$

$$(F_f)_C = \mu_k N_C = 0.2(1471.5) = 294.3 \text{ N. Thus,}$$

$$\begin{aligned} \leftarrow \Sigma F_x &= ma_x; & 588.6 - 294.3 &= 150 a \\ & & a &= 1.962 \text{ m/s}^2 = 1.96 \text{ m/s}^2 \end{aligned}$$

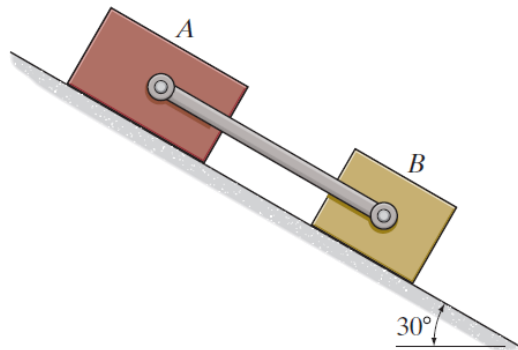


Example3:

If blocks A and B of mass 10 kg and 6 kg respectively, are placed on the inclined plane and released. The coefficients of kinetic friction between the blocks and the inclined plane are $\mu_A = 0.1$ and $\mu_B = 0.3$. Neglect the mass of the link.

Determine the force developed in the link.

Solution:

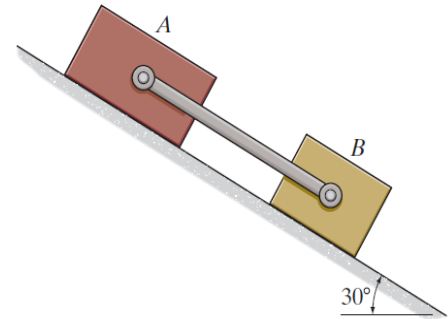
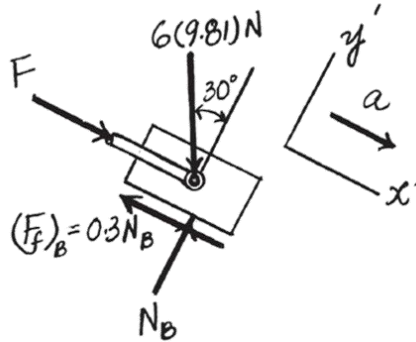
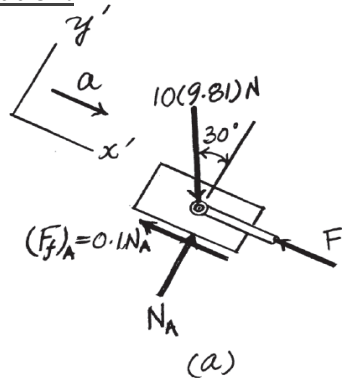


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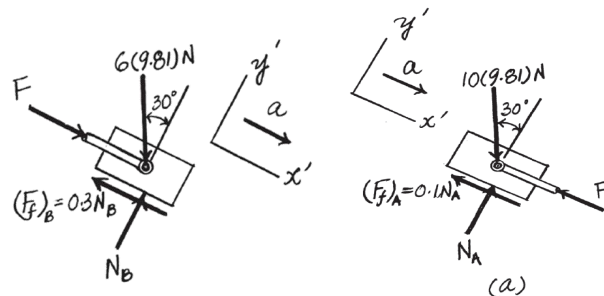
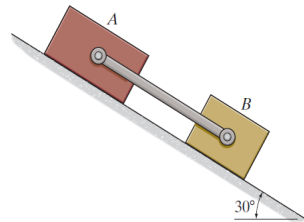
Solution:

$$+\nearrow \Sigma F_{y'} = ma_{y'}; \quad N_A - 10(9.81) \cos 30^\circ = 10(0)$$

$$N_A = 84.96 \text{ N}$$

$$\searrow + \Sigma F_{x'} = ma_{x'}; \quad 10(9.81) \sin 30^\circ - 0.1(84.96) - F = 10a$$

$$40.55 - F = 10a$$



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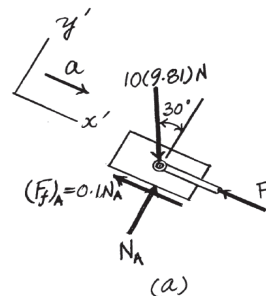
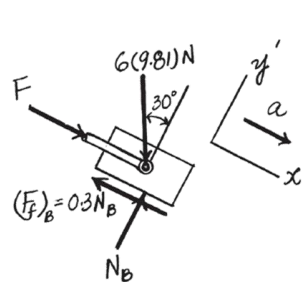
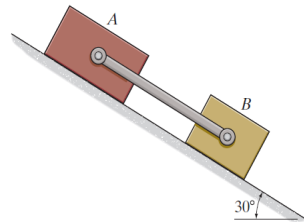
Solution:

$$+\nearrow \Sigma F_{y'} = ma_{y'}; \quad N_B - 6(9.81) \cos 30^\circ = 6(0)$$

$$N_B = 50.97 \text{ N}$$

$$\searrow + \Sigma F_{x'} = ma_{x'}; \quad F + 6(9.81) \sin 30^\circ - 0.3(50.97) = 6a$$

$$F + 14.14 = 6a$$



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Determine the force developed in the link.

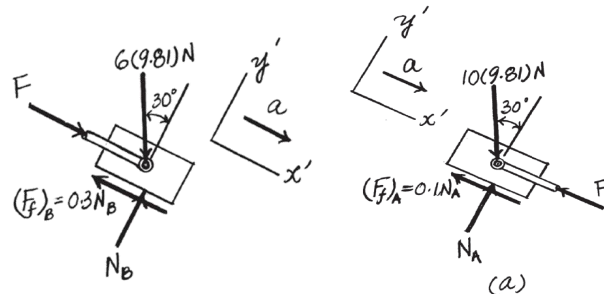
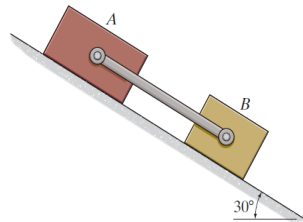
Solution:

$$40.55 - F = 10a$$

$$F + 14.14 = 6a$$

$$a = 3.42 \text{ m/s}^2$$

$$F = 6.37 \text{ N}$$



Example4:

If the force exerted on cable AB by the motor is $F = (100t^{3/2})$ N, where t is in seconds. Determine the 50-kg crate's velocity when $t = 5$ s. The coefficients of static and kinetic friction between the crate and the ground are $\mu_s = 0.4$ and $\mu_k = 0.3$, respectively. Initially the crate is at rest.

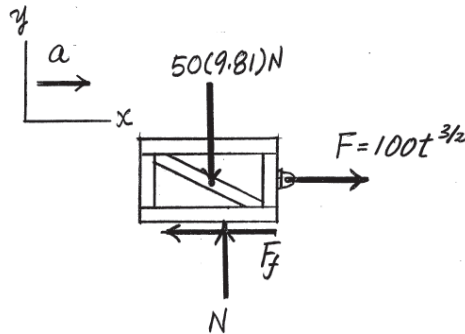
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Solution:

Equations of Motion: Here, $a_y = 0$. Thus,

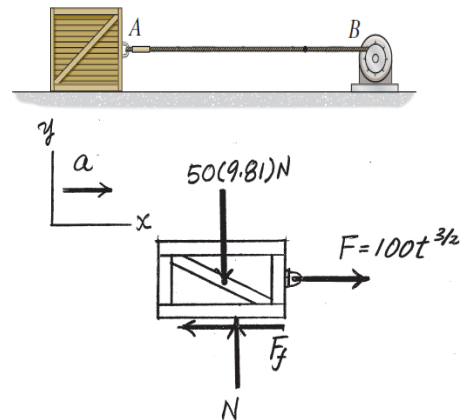
$$+\uparrow \Sigma F_y = ma_y; \quad N - 50(9.81) = 50(0)$$

$$N = 490.5 \text{ N}$$

Realizing that $F_f = \mu_k N = 0.3(490.5) = 147.15 \text{ N}$,

$$+\uparrow \Sigma F_x = ma_x; \quad 100t^{3/2} - 147.15 = 50a$$

$$a = (2t^{3/2} - 2.943) \text{ m/s}^2$$



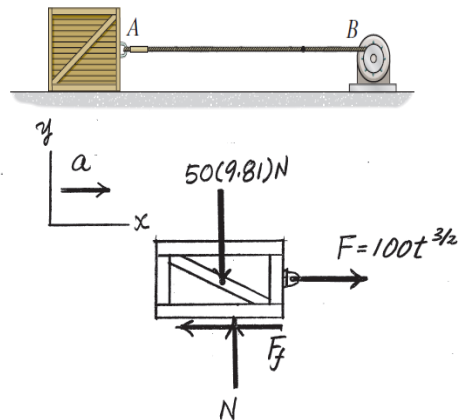
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Solution:

Equilibrium: For the crate to move, force $\mathbf{F}$ must overcome the static friction of $F_f = \mu_s N = 0.4(490.5) = 196.2$ N. Thus, the time required to cause the crate to be on the verge of moving can be obtained from.

$$\begin{aligned}\rightarrow \Sigma F_x &= 0; & 100t^{3/2} - 196.2 &= 0 \\ t &= 1.567 \text{ s}\end{aligned}$$



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Solution:

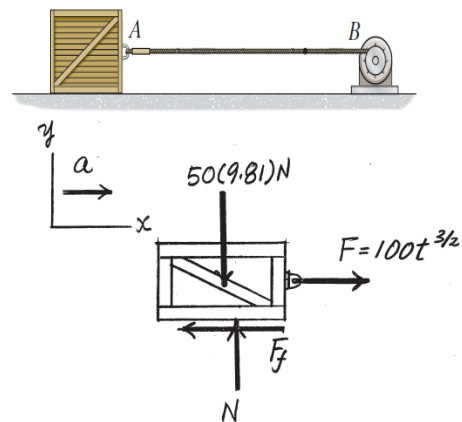
Kinematics: Using the result of **a** and integrating the kinematic equation $dv = a dt$ with the initial condition $v = 0$ at $t = 1.567$ as the lower integration limit,

$$\begin{aligned}(\rightarrow) \quad \int dv &= \int a dt \\ \int_0^v dv &= \int_{1.567 \text{ s}}^t (2t^{3/2} - 2.943) dt \\ v &= \left(0.8t^{5/2} - 2.943t \right) \Big|_{1.567 \text{ s}}^t \\ v &= (0.8t^{5/2} - 2.943t + 2.152) \text{ m/s}\end{aligned}$$

When $t = 5$ s,

$$v = 0.8(5)^{5/2} - 2.943(5) + 2.152 = 32.16 \text{ ft/s} = 32.2 \text{ ft/s}$$

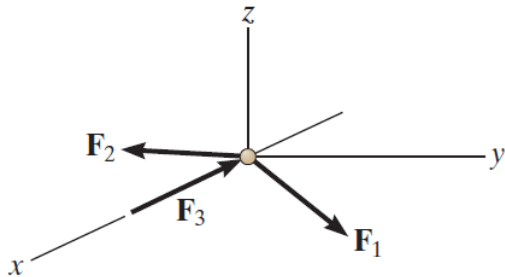
Ans.



Example5:

The 6-lb particle is subjected to the action of its weight and forces $\mathbf{F}_1 = \{2\mathbf{i} + 6\mathbf{j} - 2t\mathbf{k}\}$ lb, $\mathbf{F}_2 = \{t^2\mathbf{i} - 4t\mathbf{j} - 1\mathbf{k}\}$ lb, and $\mathbf{F}_3 = \{-2t\mathbf{i}\}$ lb, where t is in seconds. Determine the distance the ball is from the origin 2 s after being released from rest.

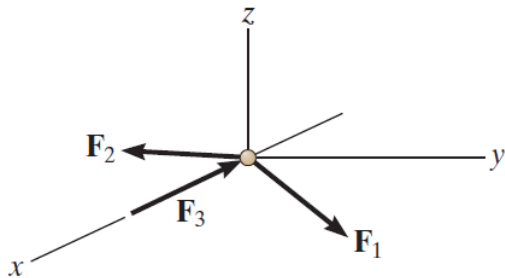
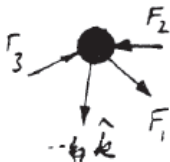
Solution:



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Solution:

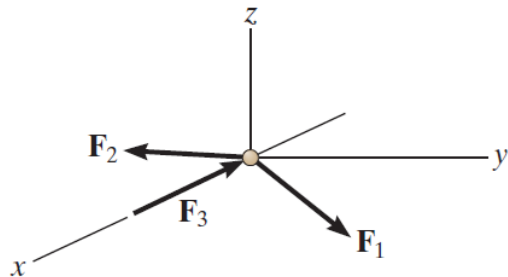


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Solution:

$$\Sigma \mathbf{F} = m\mathbf{a}; \quad (2\mathbf{i} + 6\mathbf{j} - 2\mathbf{k}) + (t^2\mathbf{i} - 4t\mathbf{j} - 1\mathbf{k}) - 2t\mathbf{i} - 6\mathbf{k} = \left(\frac{6}{32.2}\right)(a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k})$$



Example5:

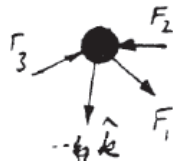
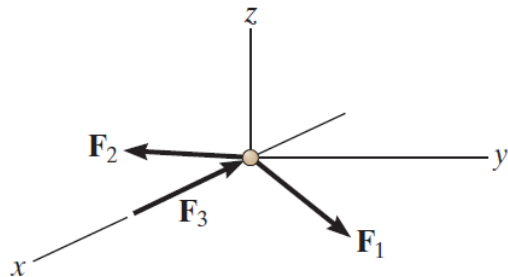
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Solution:

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Equating components:

$$\left(\frac{6}{32.2}\right)a_x = t^2 - 2t + 2 \quad \left(\frac{6}{32.2}\right)a_y = -4t + 6 \quad \left(\frac{6}{32.2}\right)a_z = -2t - 7$$



Example5:

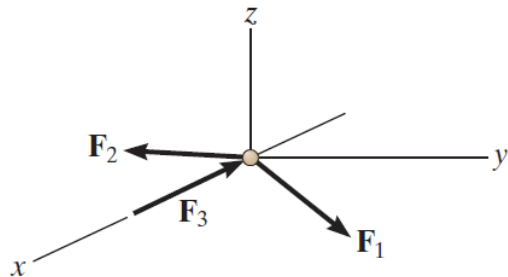
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Solution:

$$\Sigma \mathbf{F} = m\mathbf{a}; \quad (2\mathbf{i} + 6\mathbf{j} - 2t\mathbf{k}) + (t^2\mathbf{i} - 4t\mathbf{j} - 1\mathbf{k}) - 2t\mathbf{i} - 6\mathbf{k} = \left(\frac{6}{32.2}\right)(a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k})$$

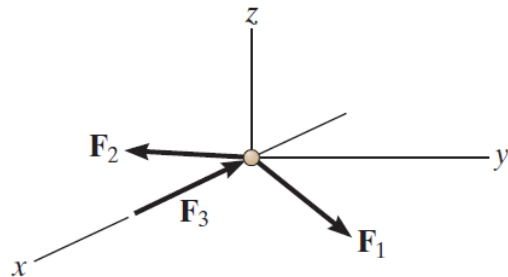
Equating components:

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The 6-lb particle is subjected to the action of its weight and forces $\mathbf{F}_1 = \{2\mathbf{i} + 6\mathbf{j} - 2t\mathbf{k}\}$ lb, $\mathbf{F}_2 = \{t^2\mathbf{i} - 4t\mathbf{j} - 1\mathbf{k}\}$ lb, and $\mathbf{F}_3 = \{-2t\mathbf{i}\}$ lb, where t is in seconds. Determine the distance the ball is from the origin 2 s after being released from rest.



Solution:

Since $dv = a dt$, integrating from $v = 0, t = 0$, yields

$$\left(\frac{6}{32.2}\right)v_x = \frac{t^3}{3} - t^2 + 2t \quad \left(\frac{6}{32.2}\right)v_y = -2t^2 + 6t \quad \left(\frac{6}{32.2}\right)v_z = -t^2 - 7t$$

Since $ds = v dt$, integrating from $s = 0, t = 0$ yields

$$\left(\frac{6}{32.2}\right)s_x = \frac{t^4}{12} - \frac{t^3}{3} + t^2 \quad \left(\frac{6}{32.2}\right)s_y = -\frac{2t^3}{3} + 3t^2 \quad \left(\frac{6}{32.2}\right)s_z = -\frac{t^3}{3} - \frac{7t^2}{2}$$



Example5:

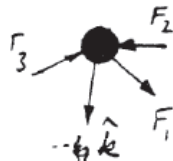
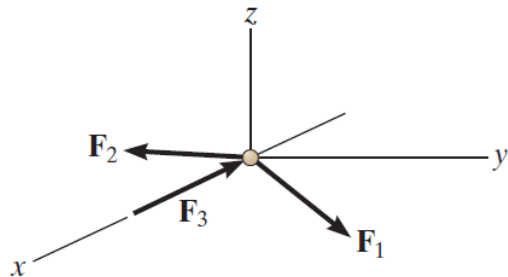
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Solution:

When $t = 2$ s then, $s_x = 14.31$ ft, $s_y = 35.78$ ft, $s_z = -89.44$ ft

Thus,

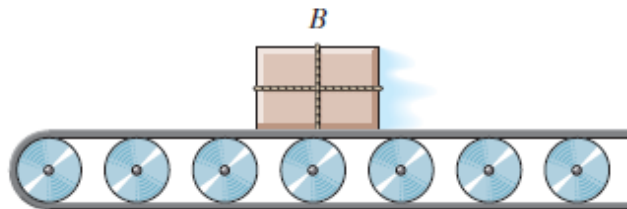
$$s = \sqrt{(14.31)^2 + (35.78)^2 + (-89.44)^2} = 97.4 \text{ ft}$$



Example6:

The conveyor belt is designed to transport packages of various weights. Each 10-kg package has a coefficient of kinetic friction $\mu_k = 0.15$. If the speed of the conveyor is 5 m/s, and then it suddenly stops, determine the distance the package will slide on the belt before coming to rest.

Solution:



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Solution:

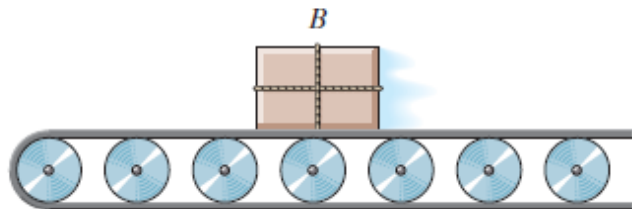
$$\rightarrow \Sigma F_x = ma_x; \quad 0.15 m(9.81) = ma$$

$$a = 1.4715 \text{ m/s}^2$$

$$(\rightarrow) v^2 = v_0^2 + 2a_c(s - s_0)$$

$$0 = (5)^2 + 2(-1.4715)(s - 0)$$

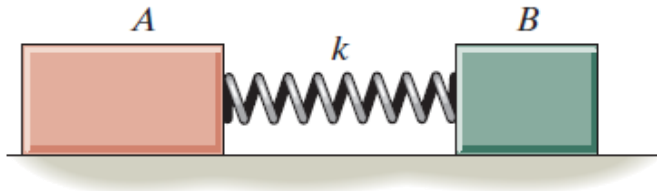
$$s = 8.49 \text{ m}$$



Example6:

Block A has a weight of 8 lb and block B has a weight of 6 lb. They rest on a surface for which the coefficient of kinetic friction is $\mu_k = 0.2$. If the spring has a stiffness of $k = 20 \text{ lb/ft}$, and it is compressed 0.2 ft, determine the acceleration of each block just after they are released.

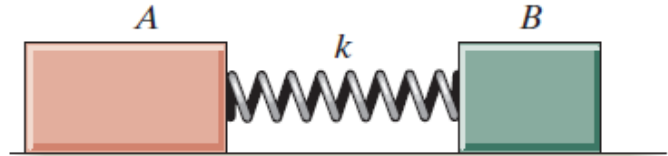
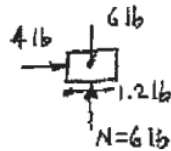
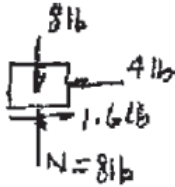
Solution:



Example6:

Block A has a weight of 8 lb and block B has a weight of 6 lb. They rest on a surface for which the coefficient of kinetic friction is $\mu_K = 0.2$. If the spring has a stiffness of $k = 20 \text{ lb/ft}$, and it is compressed 0.2 ft, determine the acceleration of each block just after they are released.

Solution:



Example6:

Block A has a weight of 8 lb and block B has a weight of 6 lb. They rest on a surface for which the coefficient of kinetic friction is $\mu_K = 0.2$. If the spring has a stiffness of $k = 20 \text{ lb/ft}$, and it is compressed 0.2 ft, determine the acceleration of each block just after they are released.

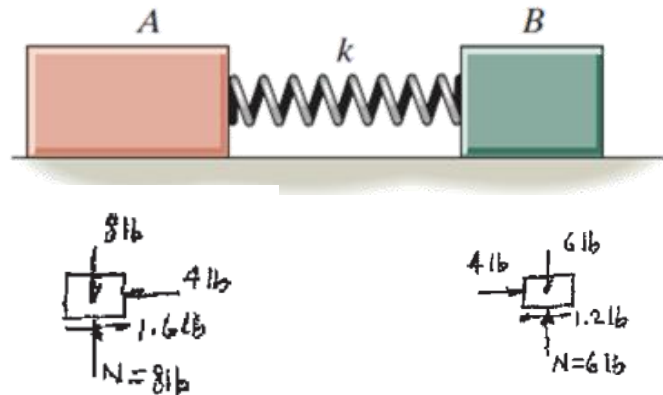
Solution:

Block A:

$$\begin{aligned} \leftarrow \Sigma F_x = ma_x; \quad 4 - 1.6 &= \frac{8}{32.2} a_A \\ a_A &= 9.66 \text{ ft/s}^2 \leftarrow \end{aligned}$$

Block B:

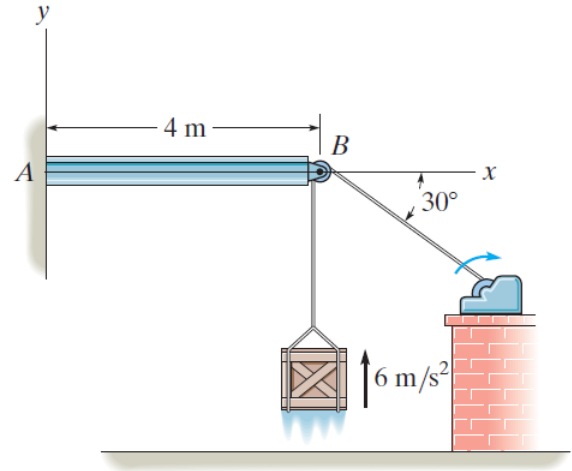
$$\begin{aligned} \rightarrow \Sigma F_x = ma_x; \quad 4 - 12 &= \frac{6}{32.2} a_B \\ a_B &= 15.0 \text{ ft/s}^2 \rightarrow \end{aligned}$$



Example7:

The motor lifts the 50-kg crate with an acceleration of 6 m/s^2 . Determine the components of force reaction and the couple moment at the fixed support A.

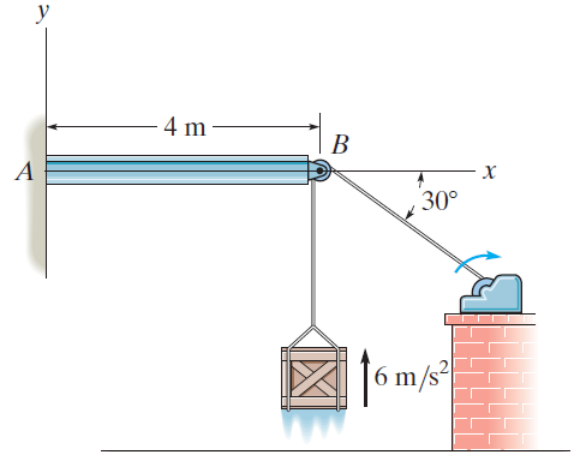
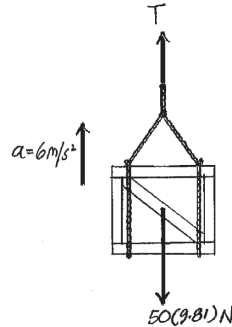
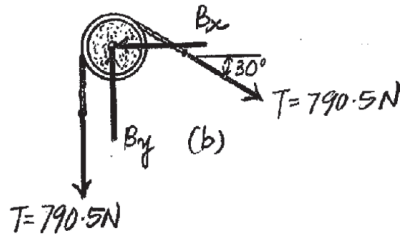
Solution:



Example7:

The motor lifts the 50-kg crate with an acceleration of 6 m/s^2 . Determine the components of force reaction and the couple moment at the fixed support A.

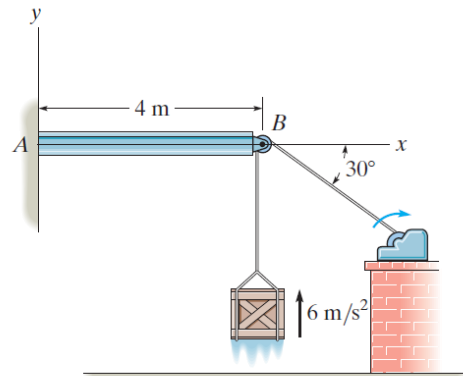
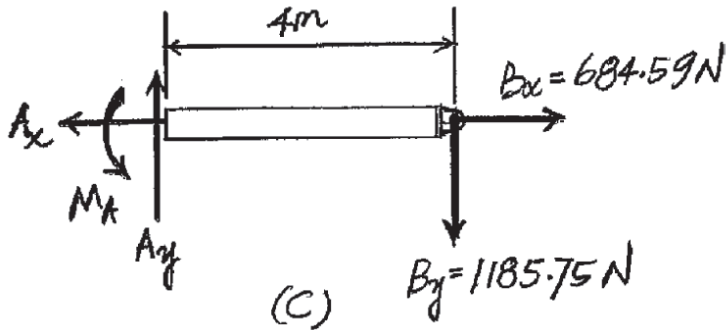
Solution:



Example7:

The motor lifts the 50-kg crate with an acceleration of 6 m/s^2 . Determine the components of force reaction and the couple moment at the fixed support A.

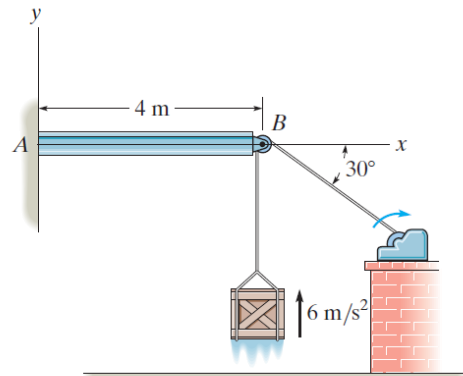
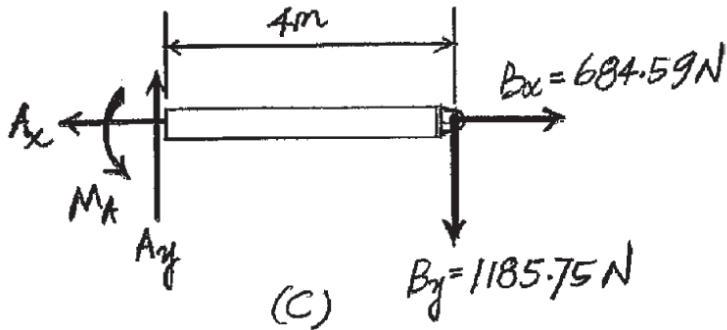
Solution:



Example7:

The motor lifts the 50-kg crate with an acceleration of 6 m/s^2 . Determine the components of force reaction and the couple moment at the fixed support A.

Solution:



Example7:

The motor lifts the 50-kg crate with an acceleration of 6 m/s^2 . Determine the components of force reaction and the couple moment at the fixed support A.

Solution:

Equation of Motion. Referring to the FBD of the crate shown in Fig. *a*,

$$+\uparrow \Sigma F_y = ma_y; \quad T - 50(9.81) = 50(6) \quad T = 790.5 \text{ N}$$

Equations of Equilibrium. Since the pulley is smooth, the tension is constant throughout entire cable. Referring to the FBD of the pulley shown in Fig. *b*,

$$\rightarrow \Sigma F_x = 0; \quad 790.5 \cos 30^\circ - B_x = 0 \quad B_x = 684.59 \text{ N}$$

$$+\uparrow \Sigma F_y = 0; \quad B_y - 790.5 - 790.5 \sin 30^\circ = 0 \quad B_y = 1185.75 \text{ N}$$

Consider the FBD of the cantilever beam shown in Fig. *c*,

$$\rightarrow \Sigma F_x = 0; \quad 684.59 - A_x = 0 \quad A_x = 684.59 \text{ N} = 685 \text{ N}$$

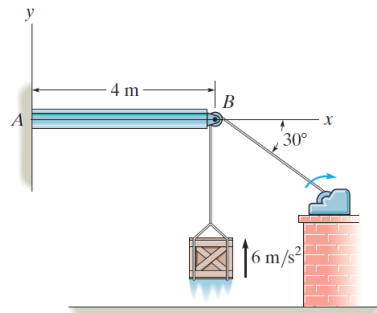
$$+\uparrow \Sigma F_y = 0; \quad A_y - 1185.75 = 0 \quad A_y = 1185.75 \text{ N} = 1.19 \text{ kN}$$

$$\zeta + \Sigma M_A = 0; \quad M_A - 1185.75(4) = 0 \quad M_A = 4743 \text{ N} \cdot \text{m} = 4.74 \text{ kN} \cdot \text{m}$$

Ans.

Ans.

Ans.



Chapter 13: Kinetics of a Particle

13.5: Normal and Tangential Coordinates



Objectives:

- To state Newton's Second Law of Motion and to define mass and weight.
- To analyze the accelerated motion of a particle using the equation of motion with different coordinate systems.
- To investigate central-force motion and apply it to problems in space mechanics.

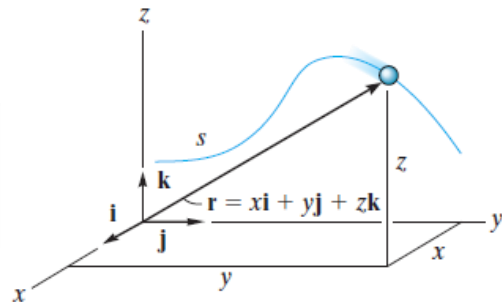
13.5: Equations of Motion: Normal and Tangential Coordinates

Equations of Motion – Rectangular

- If we use rectangular coordinates as an inertial frame, then we write the equations of motion along the coordinates as

$$\Sigma \mathbf{F}_x = m \mathbf{a}_x \quad \Rightarrow \quad \Sigma F_x = m\ddot{x}$$

$$\Sigma \mathbf{F}_y = m \mathbf{a}_y \quad \Rightarrow \quad \Sigma F_y = m\ddot{y}$$



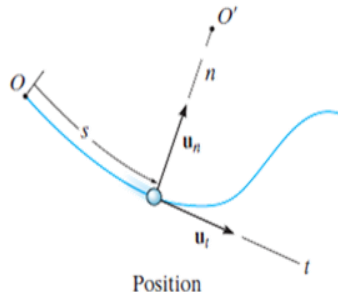
13.5: Equations of Motion: Normal and Tangential Coordinates

Equations of Motion – Normal-Tangential (n - t)

- If we use normal-tangential coordinates to describe the inertial frame, we write the equations of motion as

$$\begin{aligned}\Sigma \mathbf{F}_n &= m \mathbf{a}_n &\Rightarrow &\Sigma \mathbf{F}_n = m \frac{v^2}{\rho} \\ \Sigma \mathbf{F}_t &= m \mathbf{a}_t &\Rightarrow &\Sigma \mathbf{F}_t = m \dot{v}\end{aligned}$$

- The positive sense of
 - the normal coordinate must point **toward** the center of curvature C .
 - the tangential coordinate must point in the direction of motion along the path.



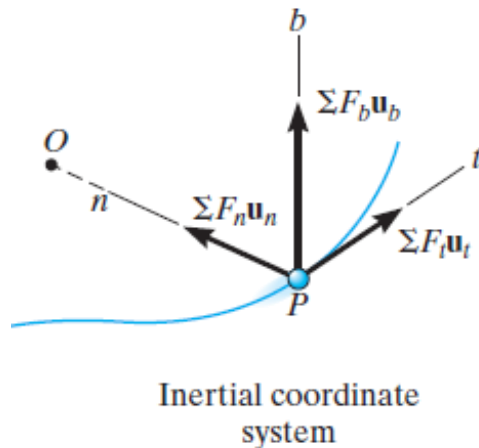
13.5: Equations of Motion: Normal and Tangential Coordinates

When a particle moves along a curved path which is known, the equation of motion for the particle may be written in the tangential, normal, and binormal directions.

$$\Sigma F_t = ma_t$$

$$\Sigma F_n = ma_n$$

$$\Sigma F_b = 0$$

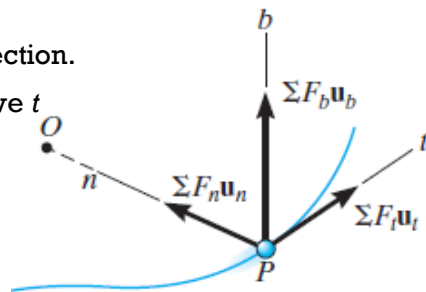


13.5: Equations of Motion: Normal and Tangential Coordinates

Procedure for Analysis

Free-Body Diagram.

- Establish the inertial t, n, b coordinate system at the particle and draw the particle's free-body diagram.
- The particle's normal acceleration $\mathbf{a}_n$ *always* acts in the positive n direction.
- If the tangential acceleration $\mathbf{a}_t$ is unknown, assume it acts in the positive t direction.
- There is no acceleration in the b direction.
- Identify the unknowns in the problem.



Inertial coordinate
system

13.5: Equations of Motion: Normal and Tangential Coordinates

Procedure for Analysis

Equations of Motion.

- Apply the equations of motion

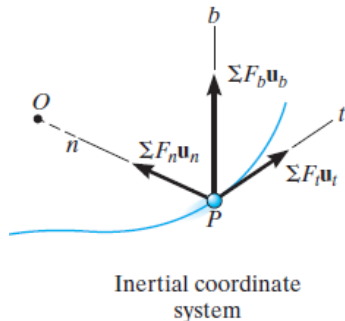
Kinematics.

- Formulate the tangential and normal components of acceleration; i.e., $a_t = dv/dt$ or $a_t = v dv/ds$ and $a_n = v^2/\rho$.
- If the path is defined as $y = f(x)$, the radius of curvature at the point where the particle is located can be obtained from $\rho = [1 + (dy/dx)^2]^{3/2} / |d^2y/dx^2|$.

$$\Sigma F_t = ma_t$$

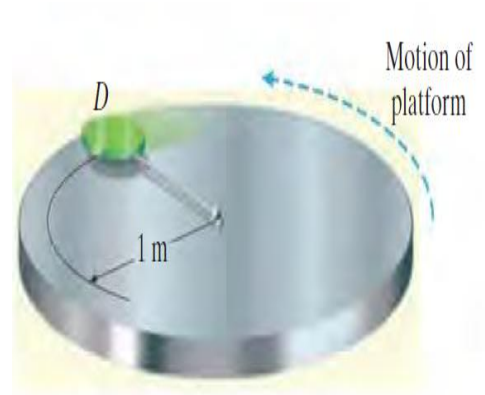
$$\Sigma F_n = ma_n$$

$$\Sigma F_b = 0$$



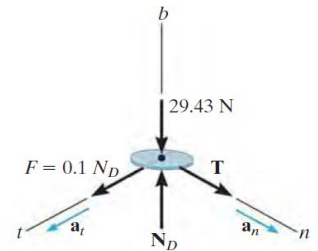
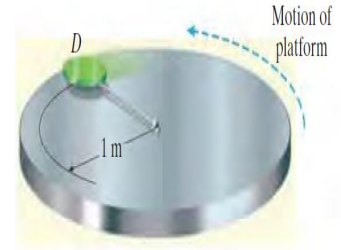
Ex1:

The 3-kg disk D is attached to the end of a cord. The other end of the cord is attached to a ball-and-socket joint located at the center of a platform. If the platform rotates rapidly, and the disk is placed on it and released from rest as shown, determine the time it takes for the disk to reach a speed great enough to break the cord. The maximum tension the cord can sustain is 100 N, and the coefficient of kinetic friction between the disk and the platform is $\mu_k = 0.1$.



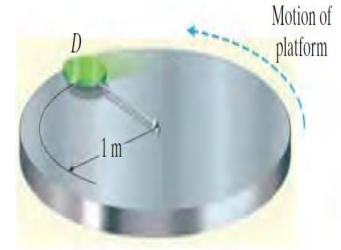
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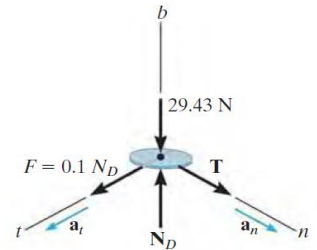


Equations of Motion.

$$\Sigma F_n = ma_n; \quad T = 3\left(\frac{v^2}{1}\right)$$

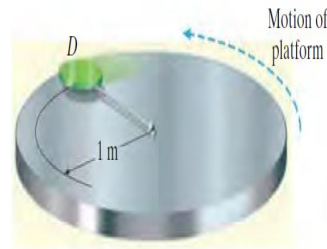
$$\Sigma F_t = ma_t; \quad 0.1N_D = 3a_t$$

$$\Sigma F_b = 0; \quad N_D - 29.43 = 0$$



Ex1:

The 3-kg disk D is attached to the end of a cord. The other end of the cord is attached to a ball-and-socket joint located at the center of a platform. If the platform rotates rapidly, and the disk is placed on it and released from rest as shown, determine the time it takes for the disk to reach a speed great enough to break the cord. The maximum tension the cord can sustain is 100 N, and the coefficient of kinetic friction between the disk and the platform is $\mu_k = 0.1$.



Equations of Motion.

$$\Sigma F_n = ma_n; \quad T = 3\left(\frac{v^2}{1}\right)$$

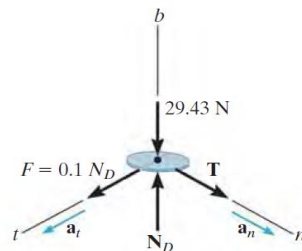
$$\Sigma F_t = ma_t; \quad 0.1N_D = 3a_t$$

$$\Sigma F_b = 0; \quad N_D - 29.43 = 0$$

$$N_D = 29.43 \text{ N}$$

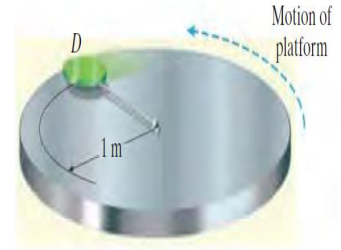
$$a_t = 0.981 \text{ m/s}^2$$

$$v_{cr} = 5.77 \text{ m/s}$$



Ex1:

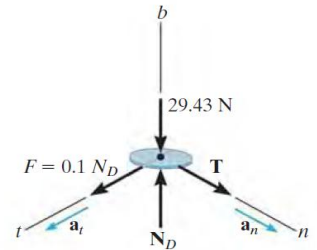
The 3-kg disk D is attached to the end of a cord. The other end of the cord is attached to a ball-and-socket joint located at the center of a platform. If the platform rotates rapidly, and the disk is placed on it and released from rest as shown, determine the time it takes for the disk to reach a speed great enough to break the cord. The maximum tension the cord can sustain is 100 N, and the coefficient of kinetic friction between the disk and the platform is $\mu_k = 0.1$.



$$v_{cr} = v_0 + a_t t$$

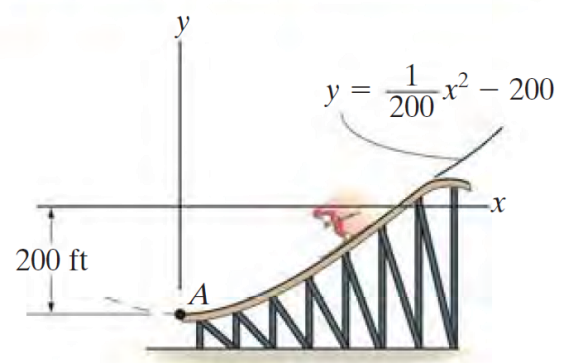
$$5.77 = 0 + (0.981)t$$

$$t = 5.89 \text{ s}$$



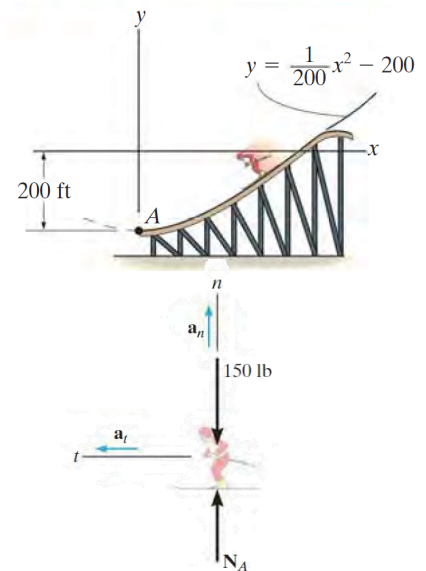
Ex2:

Design of the ski jump shown in the photo requires knowing the type of forces that will be exerted on the skier and her approximate trajectory. If in this case the jump can be approximated by the parabola shown, determine the normal force on the 150-lb skier the instant she arrives at the end of the jump, point *A*, where her velocity is 65 ft/s. Also, what is her acceleration at this point?



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Design of the ski jump shown in the photo requires knowing the type of forces that will be exerted on the skier and her approximate trajectory. If in this case the jump can be approximated by the parabola shown, determine the normal force on the 150-lb skier the instant she arrives at the end of the jump, point A , where her velocity is 65 ft/s. Also, what is her acceleration at this point?



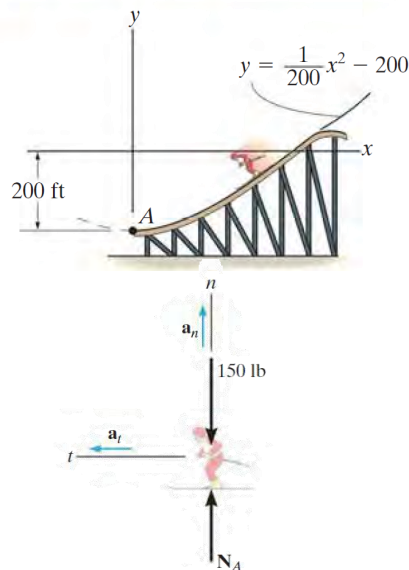
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Equations of Motion.

$$+\uparrow \Sigma F_n = ma_n; \quad N_A - 150 = \frac{150}{32.2} \left(\frac{(65)^2}{\rho} \right)$$

$$\leftarrow \Sigma F_t = ma_t; \quad 0 = \frac{150}{32.2} a_t$$

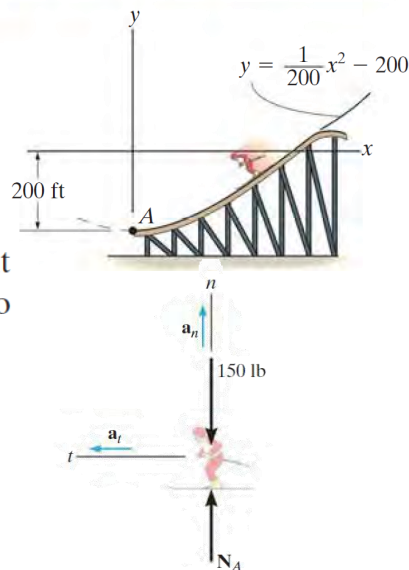


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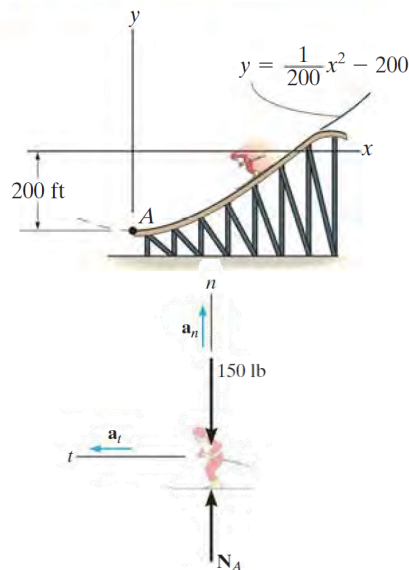
The radius of curvature ρ for the path must be determined at point A(0, -200 ft). Here $y = \frac{1}{200}x^2 - 200$, $dy/dx = \frac{1}{100}x$, $d^2y/dx^2 = \frac{1}{100}$, so that at $x = 0$,

$$\rho = \frac{[1 + (dy/dx)^2]^{3/2}}{|d^2y/dx^2|} \bigg|_{x=0} = \frac{[1 + (0)^2]^{3/2}}{|\frac{1}{100}|} = 100 \text{ ft}$$



Ex2:

Design of the ski jump shown in the photo requires knowing the type of forces that will be exerted on the skier and her approximate trajectory. If in this case the jump can be approximated by the parabola shown, determine the normal force on the 150-lb skier the instant she arrives at the end of the jump, point *A*, where her velocity is 65 ft/s. Also, what is her acceleration at this point?



$$+\uparrow \Sigma F_n = ma_n; \quad N_A - 150 = \frac{150}{32.2} \left(\frac{(65)^2}{\rho} \right)$$

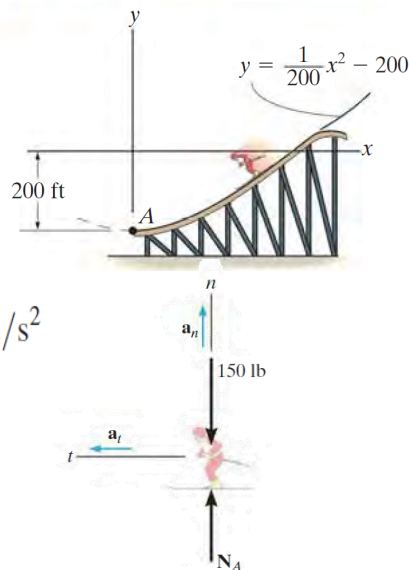
$$\leftarrow \Sigma F_t = ma_t; \quad 0 = \frac{150}{32.2} a_t$$

$$N_A = 347 \text{ lb}$$

$$a_t = 0$$

Ex2:

Design of the ski jump shown in the photo requires knowing the type of forces that will be exerted on the skier and her approximate trajectory. If in this case the jump can be approximated by the parabola shown, determine the normal force on the 150-lb skier the instant she arrives at the end of the jump, point A, where her velocity is 65 ft/s. Also, what is her acceleration at this point?

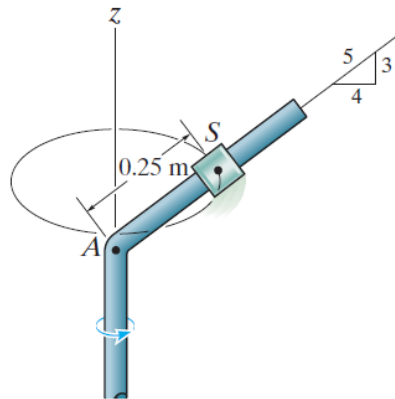


$$\begin{aligned} +\uparrow \Sigma F_n &= ma_n; & N_A - 150 &= \frac{150}{32.2} \left(\frac{(65)^2}{\rho} \right) & a_n &= \frac{v^2}{\rho} = \frac{(65)^2}{100} = 42.2 \text{ ft/s}^2 \\ \leftarrow \Sigma F_t &= ma_t; & 0 &= \frac{150}{32.2} a_t & a_A &= a_n = 42.2 \text{ ft/s}^2 \uparrow \\ N_A &= 347 \text{ lb} & a_t &= 0 \end{aligned}$$

Ex3:

The 2-kg spool S fits loosely on the inclined rod for which the coefficient of static friction is $\mu_s = 0.2$. If the spool is located 0.25 m from A , determine the minimum constant speed the spool can have so that it does not slip down the rod.

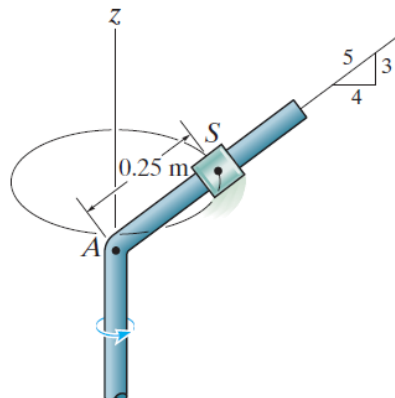
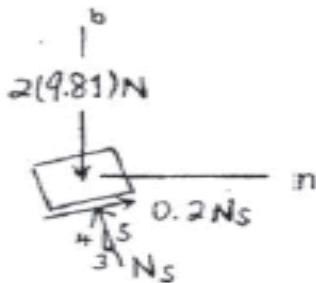
Solution



Ex3:

The 2-kg spool S fits loosely on the inclined rod for which the coefficient of static friction is $\mu_s = 0.2$. If the spool is located 0.25 m from A , determine the minimum constant speed the spool can have so that it does not slip down the rod.

Solution



Ex3:

The 2-kg spool S fits loosely on the inclined rod for which the coefficient of static friction is $\mu_s = 0.2$. If the spool is located 0.25 m from A, determine the minimum constant speed the spool can have so that it does not slip down the rod.

SOLUTION

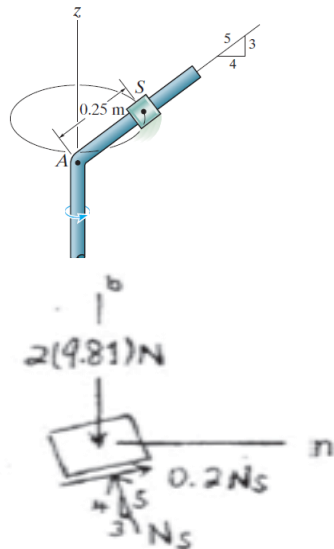
$$\rho = 0.25 \left(\frac{4}{5} \right) = 0.2 \text{ m}$$

$$\leftarrow \Sigma F_n = m a_n; \quad N_s \left(\frac{3}{5} \right) - 0.2 N_s \left(\frac{4}{5} \right) = 2 \left(\frac{v^2}{0.2} \right)$$

$$+\uparrow \Sigma F_b = m a_b; \quad N_s \left(\frac{4}{5} \right) + 0.2 N_s \left(\frac{3}{5} \right) - 2(9.81) = 0$$

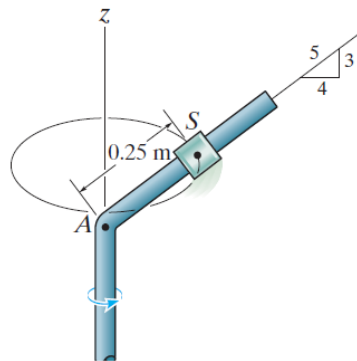
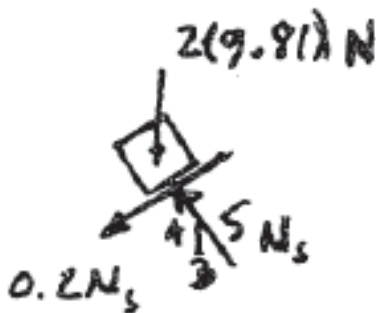
$$N_s = 21.3 \text{ N}$$

$$v = 0.969 \text{ m/s}$$



Ex4:

The 2-kg spool S fits loosely on the inclined rod for which the coefficient of static friction is $\mu_s = 0.2$. If the spool is located 0.25 m from A, determine the maximum constant speed the spool can have so that it does not slip up the rod.



Ex4:

The 2-kg spool S fits loosely on the inclined rod for which the coefficient of static friction is $\mu_s = 0.2$. If the spool is located 0.25 m from A, determine the maximum constant speed the spool can have so that it does not slip up the rod.

SOLUTION

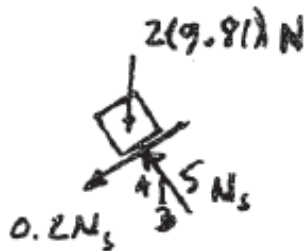
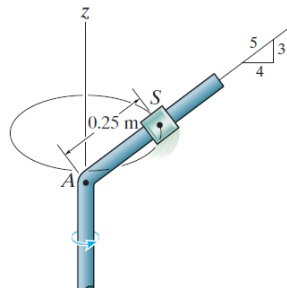
$$\rho = 0.25\left(\frac{4}{5}\right) = 0.2 \text{ m}$$

$$\leftarrow \Sigma F_n = m a_n; \quad N_s\left(\frac{3}{5}\right) + 0.2N_s\left(\frac{4}{5}\right) = 2\left(\frac{v^2}{0.2}\right)$$

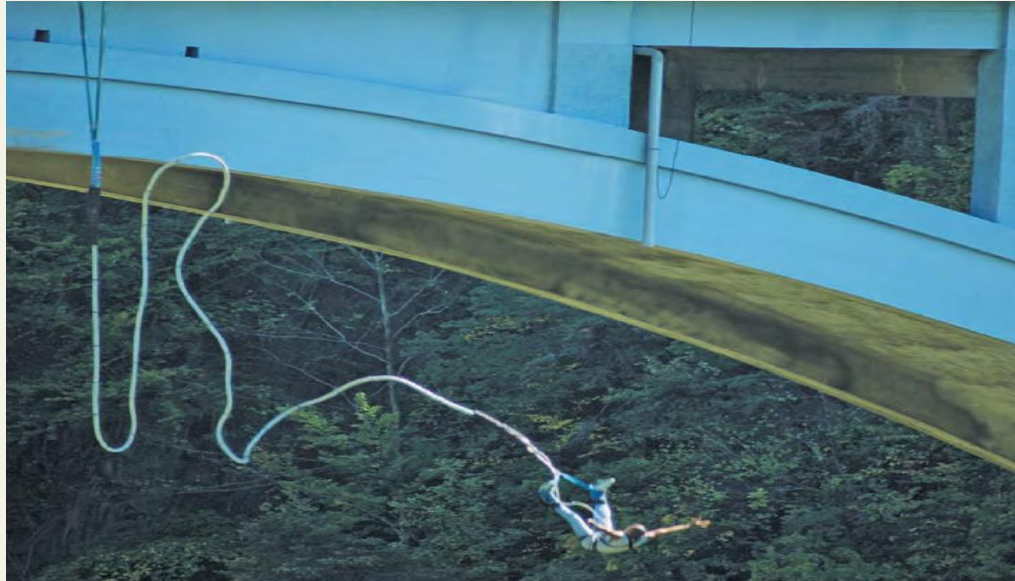
$$+\uparrow \Sigma F_b = m a_b; \quad N_s\left(\frac{4}{5}\right) - 0.2N_s\left(\frac{3}{5}\right) - 2(9.81) = 0$$

$$N_s = 28.85 \text{ N}$$

$$v = 1.48 \text{ m/s}$$



Chapter 14: Kinetics of a Particle Work and Energy



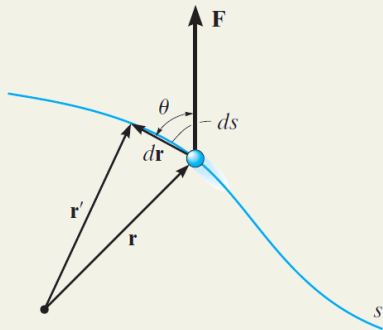
Objectives:

- To develop the principle of work and energy and apply it to solve problems that involve force, velocity, and displacement.
- To study problems that involve power and efficiency.
- To introduce the concept of a conservative force and apply the theorem of conservation of energy to solve kinetic problems..

14.1: The Work of a Force

In this chapter, we will analyze motion of a particle using the concepts of work and energy. The resulting equation will be useful for solving problems that involve force, velocity, and displacement. Before we do this, however, we must first define the **work of a force**. Specifically, a force $\mathbf{F}$ will do *work* on a particle only when the particle undergoes a *displacement in the direction of the force*.

If the force $\mathbf{F}$ causes the particle to move along the path s from position $\mathbf{r}$ to a new position $\mathbf{r}'$, the displacement is then $d\mathbf{r} = \mathbf{r} - \mathbf{r}'$. The magnitude of $d\mathbf{r}$ is ds , the length of the differential segment along the path. If the angle between the tails of $d\mathbf{r}$ and $\mathbf{F}$ is θ , then the work done by $\mathbf{F}$ is a scalar quantity, defined by:



Or simply

$$dU = F ds \cos(\theta)$$

$$dU = \mathbf{F} \cdot d\mathbf{r}$$

(dot product)

14.1: The Work of a Force

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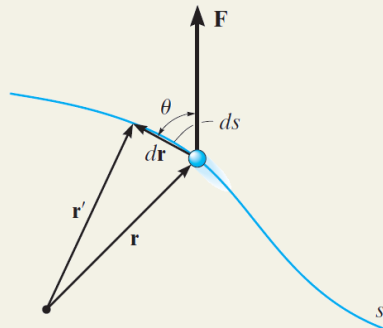
$$\begin{aligned} dU &= F ds \cos(\theta) \\ \text{Or simply } dU &= \mathbf{F} \cdot d\mathbf{r} \end{aligned} \quad (\text{dot product})$$

Note that:

if $0 \leq \theta < 90$	$\rightarrow$	work is positive
if $90 < \theta \leq 180$	$\rightarrow$	work is negative

Work is
zero

- if the force is **perpendicular** to displacement ($\cos(90) = 0$)
- if the force is applied at a fixed point (**no motion**)

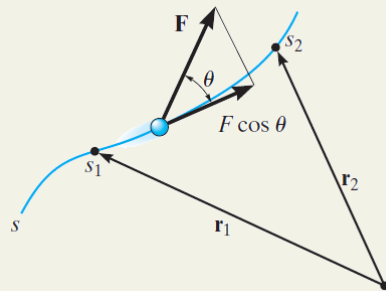


14.1: The Work of a Force

- Work of a Variable Force:

If the particle acted upon by the force $\mathbf{F}$ undergoes a finite displacement along its path from $\mathbf{r}_1$ to $\mathbf{r}_2$ or s_1 to s_2 , the work of force F is determined by integration. Provided $\mathbf{F}$ and $\mathbf{u}$ can be expressed as a function of position (s), then:

$$U_{1-2} = \int_{\mathbf{r}_1}^{\mathbf{r}_2} \mathbf{F} \cdot d\mathbf{r} = \int_{s_1}^{s_2} F \cos \theta ds$$

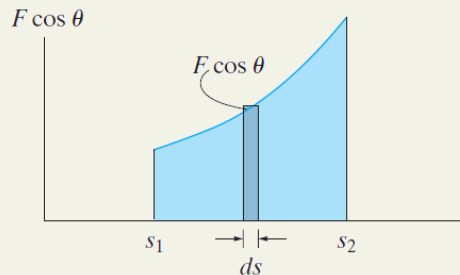
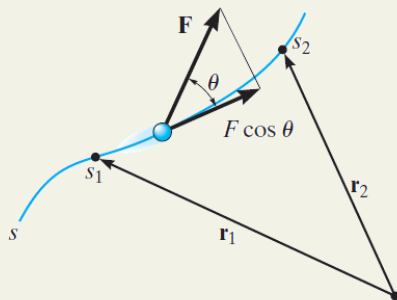


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14.1: The Work of a Force

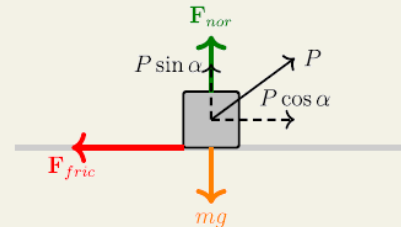
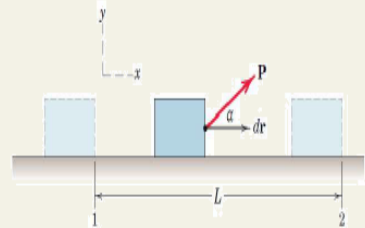
- Work of a Constant Force Moving Along a Straight Line:

1- NO Friction

$$dU_{1-2} = \int_{s_1}^{s_2} P \cos \alpha \cdot ds = \int_{s_1}^{s_2} P \cos \alpha \cdot ds = P \cos \alpha (s_2 - s_1) = P \cos \alpha L$$

2- With Friction

$$dU_{1-2} = P \cos \alpha L - F_{fric} L \quad \text{where} \quad F_{fric} = \mu_k F_{nor}$$



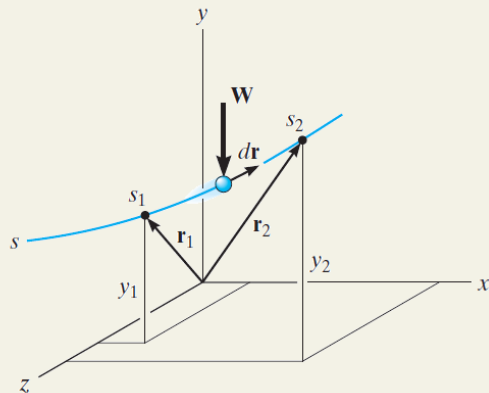
14.1: The Work of a Force

- Work of a Weight:

Consider a particle of weight W , which moves up along the path s from position s_1 to position s_2 . At an intermediate point, the displacement $d\mathbf{r} = dx\mathbf{i} + dy\mathbf{j} + dz\mathbf{k}$. Since $\mathbf{W} = -W\mathbf{j}$, we have:

$$\begin{aligned}U_{1-2} &= \int \mathbf{F} \cdot d\mathbf{r} = \int_{\mathbf{r}_1}^{\mathbf{r}_2} (-W\mathbf{j}) \cdot (dx\mathbf{i} + dy\mathbf{j} + dz\mathbf{k}) \\&= \int_{y_1}^{y_2} -W dy = -W(y_2 - y_1)\end{aligned}$$

$$U_{1-2} = -W \Delta y$$



14.1: The Work of a Force

- Work of a Spring Force:

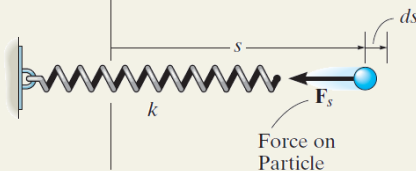
If an elastic spring is elongated a distance ds , then the work done by the force that acts on then attached particle is $dU = -F_s ds = -ks ds$.

The work is negative since F_s acts in the opposite sense to ds . If the particle displaces from s_1 to s_2 , the work of F_s is then:

$$U_{1-2} = \int_{s_1}^{s_2} F_s ds = \int_{s_1}^{s_2} -ks ds$$

$$U_{1-2} = -\left(\frac{1}{2}ks_2^2 - \frac{1}{2}ks_1^2\right)$$

$$U_{1-2} = \frac{1}{2} k (S_1^2 - S_2^2)$$



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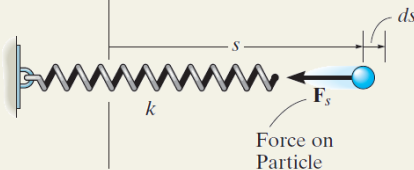
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14.2: Principle of Work and Energy

Procedure for Analysis:

Work (Free-Body Diagram).

- Establish the inertial coordinate system and draw a free-body diagram of the particle to account for all the forces that do work on the particle as it moves along its path.

Principle of Work and Energy.

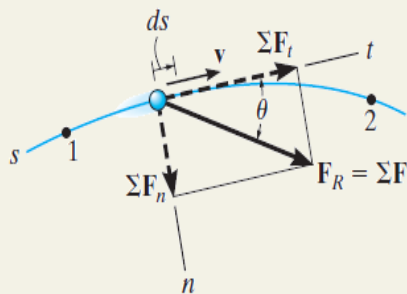
- Apply the principle of work and energy, $T_1 + \sum U_{1-2} = T_2$.
- The kinetic energy at the initial and final points is *always positive*.
- A force does work when it moves through a displacement in the direction of the force.
- Work is *positive* when the force component is in the *same sense of direction* as its displacement, otherwise it is negative.
- Forces that are functions of displacement must be integrated to obtain the work. Graphically, the work is equal to the area under the force-displacement curve.
- The work of a weight is the product of the weight magnitude and the vertical displacement, $U_w = \{\pm W\}y$. It is positive when the weight moves downwards.
- The work of a spring is of the form $U_s = \frac{1}{2} kx^2$, where k is the spring stiffness and s is the stretch or compression of the spring.

14.2: Principle of Work and Energy

If the particle has a mass m and is subjected to a system of external forces represented by the resultant $F_R = \sum F$, then the equation of motion for the particle in the tangential direction is $\sum F = ma_t$. Applying the kinematic equation $a_t = v dv/ds$ and integrating both sides, assuming initially that the particle has a position $s = s_1$ and a speed $v = v_1$, and later at $s = s_2$, $v = v_2$, we have:

$$\sum \int_{s_1}^{s_2} F_t ds = \int_{v_1}^{v_2} mv dv$$

$$\sum \int_{s_1}^{s_2} F_t ds = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$



14.2: Principle of Work and Energy

$$\Sigma \int_{s_1}^{s_2} F_t ds = \int_{v_1}^{v_2} mv dv$$

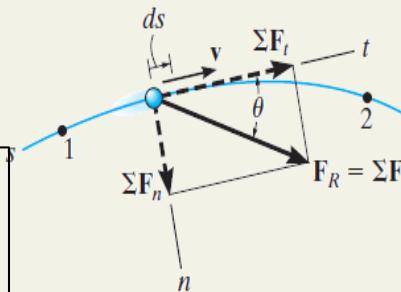
$$\Sigma \int_{s_1}^{s_2} F_t ds = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

$$\Sigma U_{1-2} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

$$T_1 + \Sigma U_{1-2} = T_2$$



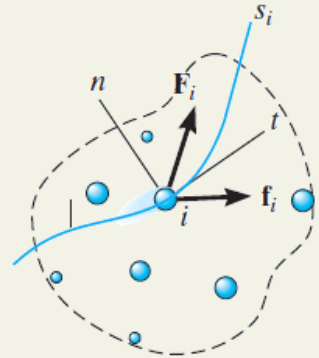
The particle's initial kinetic energy plus the work done by all the forces acting on the particle as it moves from its initial to its final position is equal to the particle's final kinetic energy.



14.3: Principle of Work and Energy for a System of Particles:

$$\Sigma T_1 + \Sigma U_{1-2} = \Sigma T_2$$

The initial kinetic energy of the system plus the work done by all the external and internal forces acting on the system is equal to the final kinetic energy of the system.

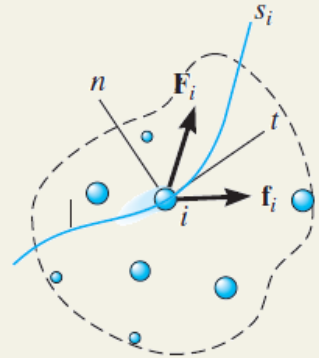


Inertial coordinate system

14.3: Principle of Work and Energy for a System of Particles:

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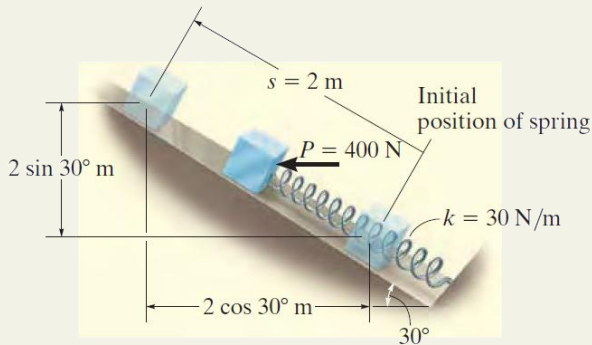
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Inertial coordinate system

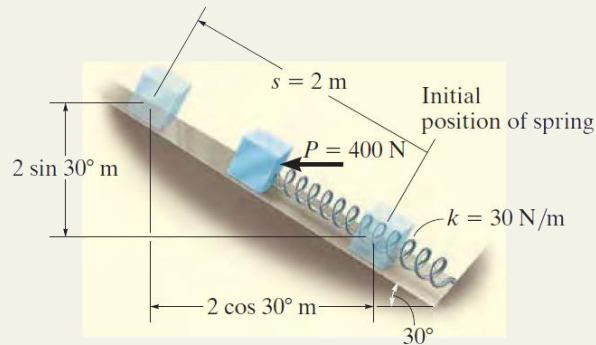
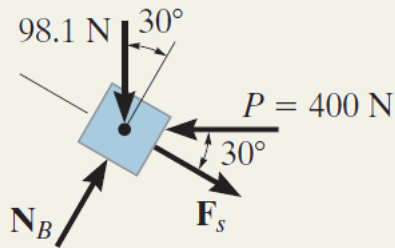
EX1:

The 10-kg block rests on the smooth incline. If the spring is originally stretched 0.5 m, determine the total work done by all the forces acting on the block when a horizontal force $P = 400$ N pushes the block up the plane $s = 2$ m.



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Horizontal Force P .

$$U_P = 400 \text{ N} (2 \text{ m} \cos 30^\circ) = 692.8 \text{ J}$$

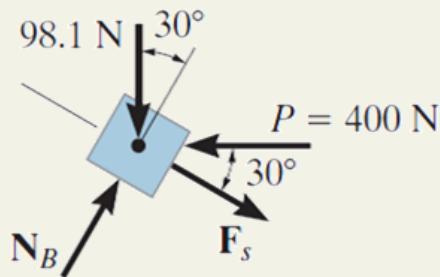
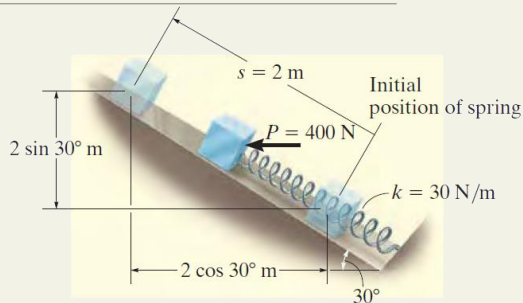
$$U_P = 400 \text{ N} \cos 30^\circ (2 \text{ m}) = 692.8 \text{ J}$$

Spring Force F_s .

$$s_1 = 0.5 \text{ m}$$

$$s_2 = 0.5 \text{ m} + 2 \text{ m} = 2.5 \text{ m}.$$

$$U_s = -\left[\frac{1}{2}(30 \text{ N/m})(2.5 \text{ m})^2 - \frac{1}{2}(30 \text{ N/m})(0.5 \text{ m})^2\right] = -90 \text{ J}$$



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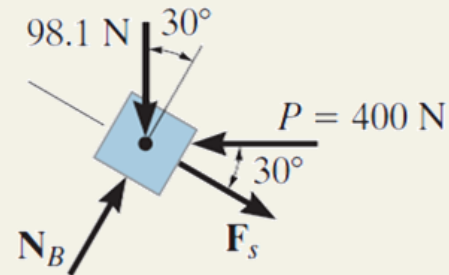
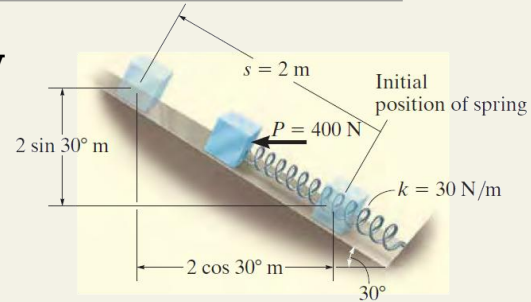
Weight W :

$$U_W = -(98.1 \text{ N})(2 \text{ m} \sin 30^\circ) = -98.1 \text{ J}$$

Normal Force N_B . This force does no work since it is always perpendicular to the displacement

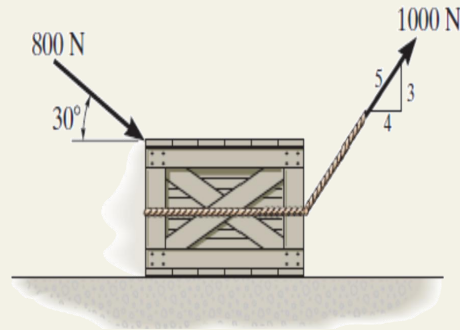
Total Work:

$$U_T = 692.8 \text{ J} - 90 \text{ J} - 98.1 \text{ J} = 505 \text{ J}$$



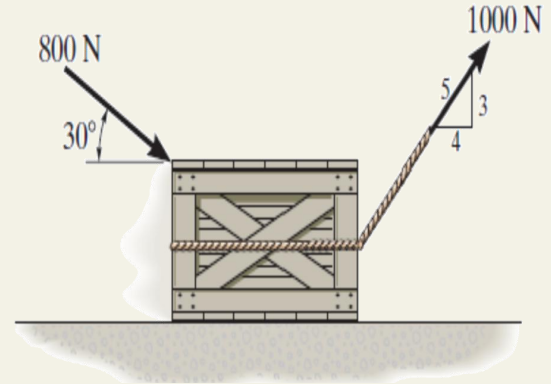
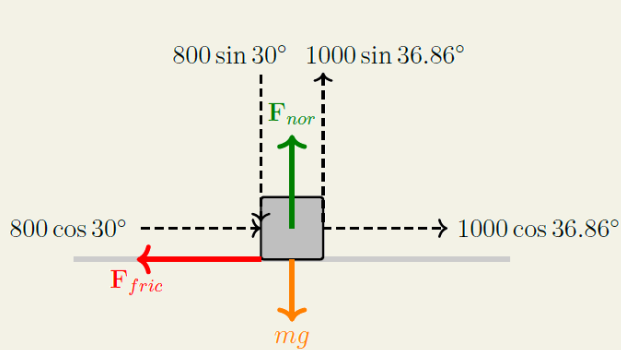
EX2:

The crate, which has a mass of $m = 100$ (kg), is subjected to the action of the two forces. If it is originally at rest, determine the distance (s) it slides to attain a speed of $v = 6$ (m/s). The coefficient of kinetic friction between the crate and the surface is $\mu_k = 0.2$.



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$$F_{fric} = \mu_k F_{nor} = 0.2 F_{nor}$$

along y -axis

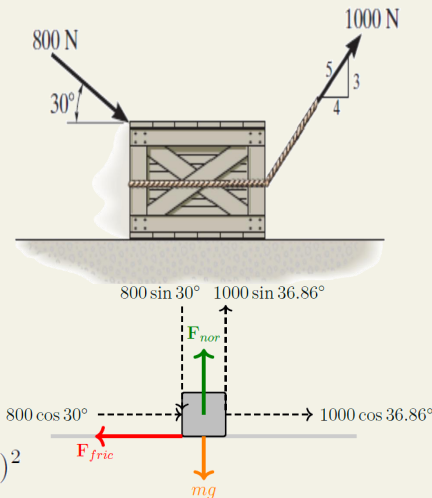
$$-mg + F_{nor} - 800 \sin 30^\circ + 1000 \sin 36.86^\circ = m \cancel{a_t}^0$$

$$F_{nor} = 100 \times 9.81 + 800 \sin 30^\circ - 1000 \sin 36.86^\circ = 781.138 \text{ (N)}$$

$$F_{fric} = \mu_k F_{nor} = 0.2 \times 781.138 = 156.228 \text{ (N)}$$

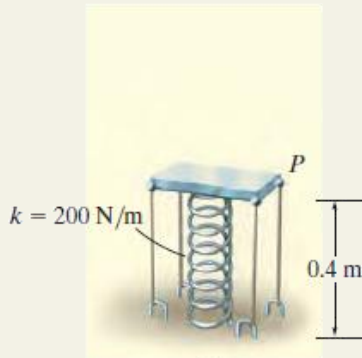
$$T_1 + \sum U_{1-2} = T_2 \Rightarrow 800 \cos 30^\circ(s) + 1000 \cos 36.86^\circ(s) - 156.228(s) = \frac{1}{2} 100(6)^2$$

$$s = 1.35 \text{ (m)}$$

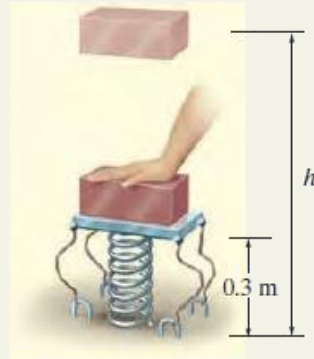


EX3:

The platform P , has negligible mass and is tied down so that the 0.4-m-long cords keep a 1-m-long spring compressed 0.6 m when *nothing* is on the platform. If a 2-kg block is placed on the platform and released from rest after the platform is pushed down 0.1 m, determine the maximum height h the block rises in the air, measured from the ground.



(a)



(b)

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Principle of Work and Energy.

$$T_1 + \Sigma U_{1-2} = T_2$$

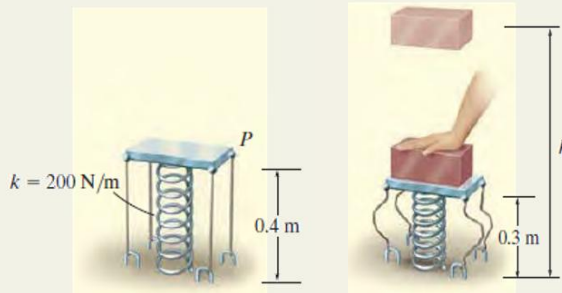
$$\frac{1}{2}mv_1^2 + \left\{ -\left(\frac{1}{2}ks_2^2 - \frac{1}{2}ks_1^2\right) - W \Delta y \right\} = \frac{1}{2}mv_2^2$$

$$0 + \left\{ -\left[\frac{1}{2}(200 \text{ N/m})(0.6 \text{ m})^2 - \frac{1}{2}(200 \text{ N/m})(0.7 \text{ m})^2\right] - (19.62 \text{ N})[h - (0.3 \text{ m})] \right\} = 0$$

Solving yields

$$h = 0.963 \text{ m}$$

Ans.



EX4:

When the driver applies the brakes of a light truck traveling 40 km/h, it skids 3 m before stopping. How far will the truck skid if it is traveling 80 km/h when the brakes are applied?



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SOLUTION

$$40 \text{ km/h} = \frac{40(10^3)}{3600} = 11.11 \text{ m/s} \quad 80 \text{ km/h} = 22.22 \text{ m/s}$$

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}m(11.11)^2 - \mu_k mg(3) = 0$$

$$\mu_k g = 20.576$$

$$T_1 + \Sigma U_{1-2} = T_2$$

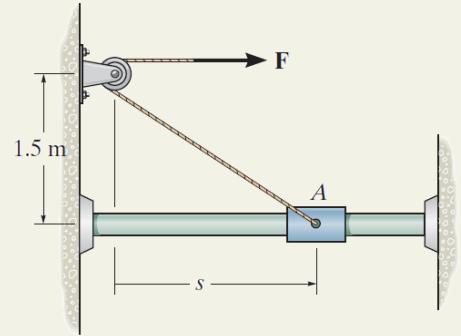
$$\frac{1}{2}m(22.22)^2 - (20.576)m(d) = 0$$

$$d = 12 \text{ m}$$



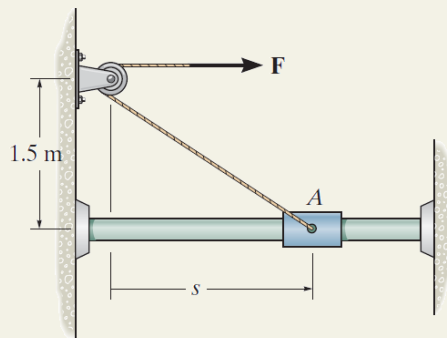
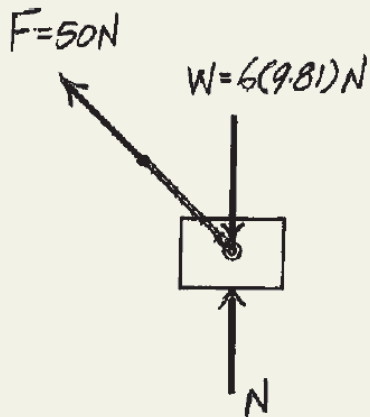
EX5:

The force of $F = 50 \text{ N}$ is applied to the cord when $s = 2 \text{ m}$. If the 6-kg collar is originally at rest, determine its velocity at $s = 0$. Neglect friction.



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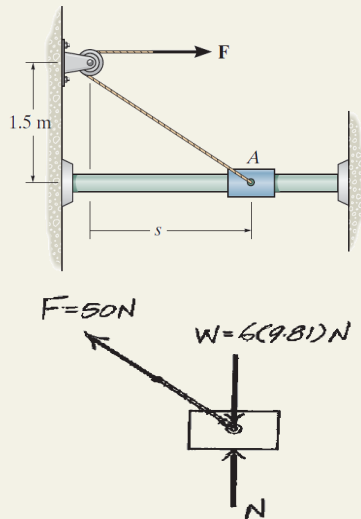
$$U_F = 50(1.00) = 50.0 \text{ J}$$

Principle of Work And Energy. Applying Eq. 14-7,

$$T_1 + \Sigma U_{1-2} = T_2$$

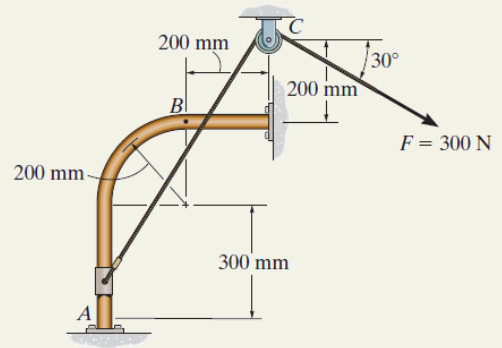
$$0 + 50 = \frac{1}{2}(6)v^2$$

$$v = 4.082 \text{ m/s} = 4.08 \text{ m/s}$$



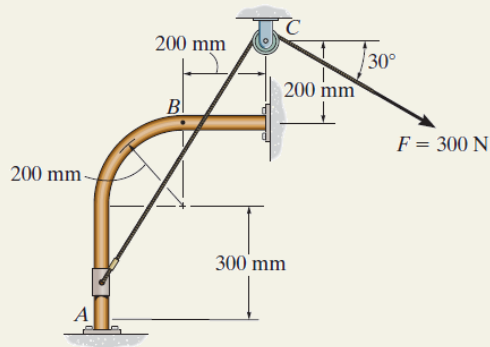
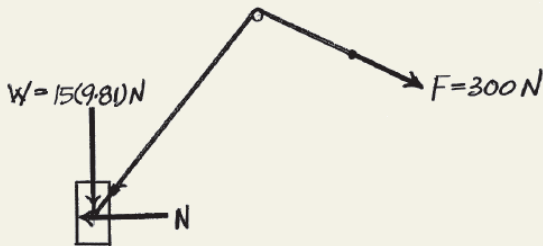
EX5:

If the cord is subjected to a constant force of $F = 300 \text{ N}$ and the 15-kg smooth collar starts from rest at A , determine the velocity of the collar when it reaches point B . Neglect the size of the pulley.



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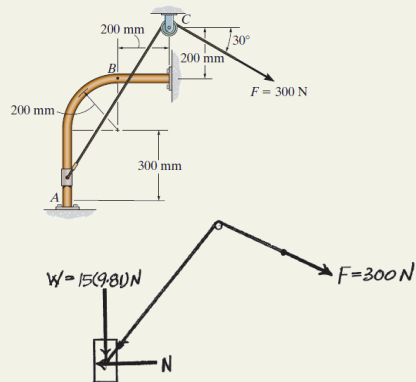
Principle of Work and Energy: only $\mathbf{N}$ does no work since it always acts perpendicular to the motion. When the collar moves from position A to B $\mathbf{W}$ displaces vertically upward a distance: $h = 0.3 + 0.2 = 0.5 \text{ m}$

while force F displaces a distance of $s = AC - BC = \sqrt{0.7^2 + 0.4^2} - \sqrt{0.2^2 + 0.2^2} = 0.5234 \text{ m}$. Here, the work of $\mathbf{F}$ is positive, whereas $\mathbf{W}$ does negative work.

$$T_A + \sum U_{A-B} = T_B$$

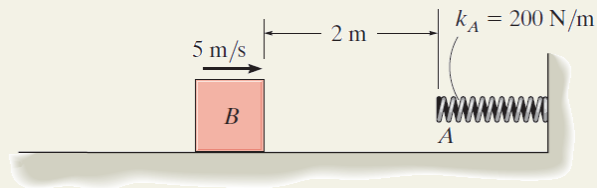
$$0 + 300(0.5234) + [-15(9.81)(0.5)] = \frac{1}{2} (15)v_B^2$$

$$v_B = 3.335 \text{ m/s} = 3.34 \text{ m/s}$$



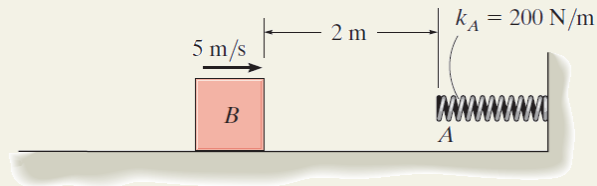
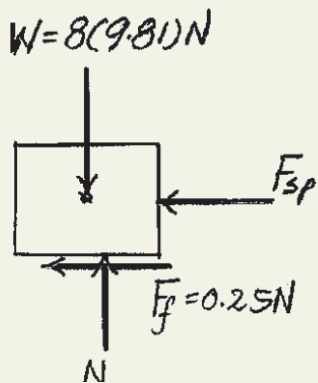
EX6:

The 8-kg block is moving with an initial speed of 5 m/s. If the coefficient of kinetic friction between the block and plane is $\mu_k = 0.25$, determine the compression in the spring when the block momentarily stops.



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$$+\uparrow \Sigma F_y = 0; \quad N - 8(9.81) = 0 \quad N = 78.48 \text{ N}$$

Thus, the friction is $F_f = \mu_k N = 0.25(78.48) = 19.62 \text{ N}$ and $F_{sp} = kx = 200x$.

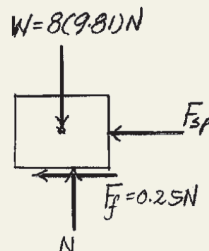
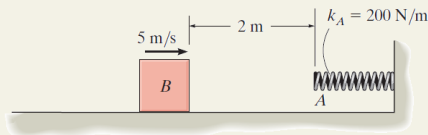
$$U_{F_{sp}} = - \int_0^x 200x \, dx = -100x^2$$

$$U_{F_f} = -19.62(x + 2)$$

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(8)(5^2) + (-100x^2) + [-19.62(x + 2)] = 0$$

$$100x^2 + 19.62x - 60.76 = 0 \quad x = 0.6875 \text{ m} = 0.688 \text{ m}$$



EX7:

The 5-lb cylinder is falling from A with a speed $v_A = 10 \text{ ft/s}$ onto the platform. Determine the maximum displacement of the platform, caused by the collision. The spring has an unstretched length of 1.75 ft and is originally kept in compression by the 1-ft long cables attached to the platform. Neglect the mass of the platform and spring and any energy lost during the collision.

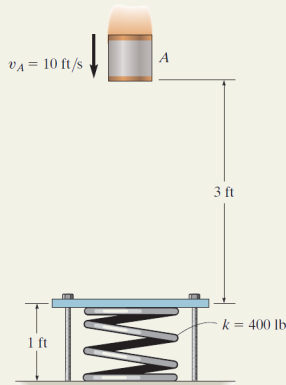
$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2} \left(\frac{5}{32.2} \right) (10^2) + 5(3 + s) - \left[\frac{1}{2} (400) (0.75 + s)^2 - \frac{1}{2} (400) (0.75)^2 \right] = 0$$

$$200 s^2 + 295 s - 22.76 = 0$$

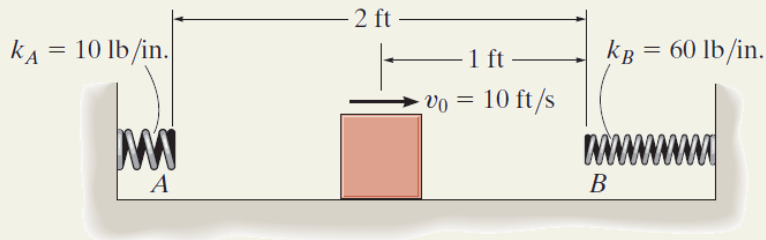
$$s = 0.0735 \text{ ft} < 1 \text{ ft}$$

$$s = 0.0735 \text{ ft}$$



EX8:

The 25-lb block has an initial speed of $v_0 = 10$ ft/s when it is midway between springs A and B. After striking spring B, it rebounds and slides across the horizontal plane toward spring A, etc. If the coefficient of kinetic friction between the plane and the block is $\mu_k = 0.4$, determine the total distance traveled by the block before it comes to rest.



EX8:

The 25-lb block has an initial speed of $v_0 = 10$ ft/s when it is midway between springs A and B. After striking spring B, it rebounds and slides across the horizontal plane toward spring A, etc. If the coefficient of kinetic friction between the plane and the block is $\mu_k = 0.4$, determine the total distance traveled by the block before it comes to rest.

$$T_1 + \sum U_{1-2} = T_2$$

$$\frac{1}{2} \left(\frac{25}{32.2} \right) (10)^2 - 10(1 + s_1) - \frac{1}{2} (60)s_1^2 = 0$$

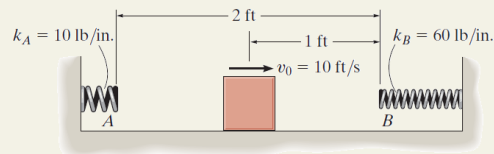
$$s_1 = 0.8275 \text{ ft}$$

Assume the block bounces back and stops without striking spring A

$$T_2 + \sum U_{2-3} = T_3$$

$$0 + \frac{1}{2} (60)(0.8275^2) - 10(0.8275 + s_2) = 0$$

$$s_2 = 1.227 \text{ ft}$$



Chapter 15: Kinetics of a Particle Impulse and Momentum



Objectives:

- To develop the principle of linear impulse and momentum for a particle and apply it to solve problems that involve force, velocity and time.
- To study the conservation of linear momentum for particles.
- To analyze the mechanics of impact.
- To introduce the concept of angular impulse and momentum.
- To solve problems involving steady fluid streams and propulsion with variable mass.

15.1: Principle of Linear Impulse and Momentum

In this section we will integrate the equation of motion with respect to time and thereby obtain the principle of impulse and momentum. The resulting equation will be useful for solving problems involving **force, velocity, and time**.

$$\Sigma \mathbf{F} = m\mathbf{a} = m \frac{d\mathbf{v}}{dt}$$

where $\mathbf{a}$ and $\mathbf{v}$ are both measured from an inertial frame of reference. Rearranging the terms and integrating between the limits $\mathbf{v} = \mathbf{v}_1$ at $t = t_1$ and $\mathbf{v} = \mathbf{v}_2$ at $t = t_2$, we have

$$\Sigma \int_{t_1}^{t_2} \mathbf{F} dt = m \int_{\mathbf{v}_1}^{\mathbf{v}_2} d\mathbf{v} \quad \longrightarrow \quad \Sigma \int_{t_1}^{t_2} \mathbf{F} dt = m\mathbf{v}_2 - m\mathbf{v}_1 \quad \longrightarrow \quad \boxed{\Sigma \int_{t_1}^{t_2} \mathbf{F} dt = m\mathbf{v}_2 - m\mathbf{v}_1}$$

15.1: Principle of Linear Impulse and Momentum

This equation is referred to as the principle of linear impulse and momentum.

$$\Sigma \int_{t_1}^{t_2} \mathbf{F} dt = m\mathbf{v}_2 - m\mathbf{v}_1$$

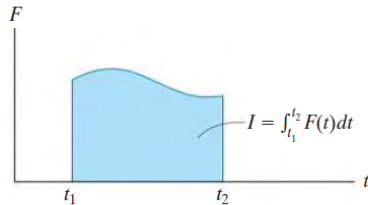
It provides a direct means of obtaining the particle's final velocity $\mathbf{v}_2$ after a specified time period when the particle's initial velocity is known and the forces acting on the particle are either constant or can be expressed as functions of time.

15.1: Principle of Linear Impulse and Momentum

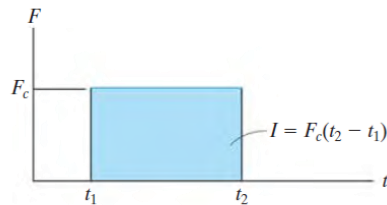
$$\Sigma \int_{t_1}^{t_2} \mathbf{F} dt = m\mathbf{v}_2 - m\mathbf{v}_1$$

Linear Momentum. $\mathbf{L} = m\mathbf{v}$ $\longrightarrow$ particle's linear momentum. (kg.m/s, or slug .ft/s.)
Since m is a positive scalar, the linear-momentum vector has the same direction as $\mathbf{v}$.

Linear Impulse. The integral $\mathbf{I} = \int_{t_1}^{t_2} \mathbf{F} dt$ $\longrightarrow$ the *linear impulse*. (N . s or lb . s)
This term is a vector quantity, the impulse acts in the same direction as the force.



Variable Force



Constant Force

15.2: Principle of Linear Impulse and Momentum

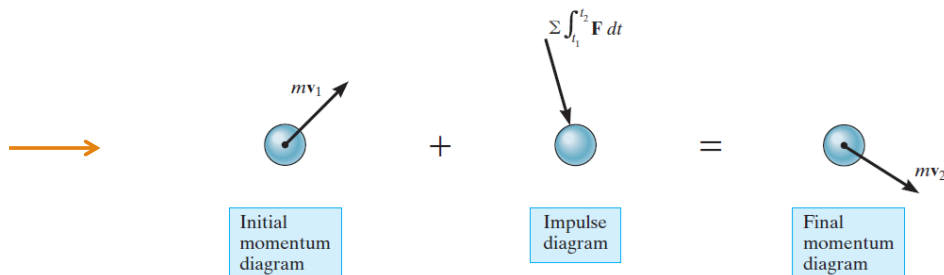
Principle of Linear Impulse and Momentum: The initial momentum of the particle at time t_1 plus the sum of all the impulses applied to the particle from t_1 to t_2 is equivalent to the final momentum of the particle at time t_2

$$m\mathbf{v}_1 + \Sigma \int_{t_1}^{t_2} \mathbf{F} dt = m\mathbf{v}_2$$

$$m(v_x)_1 + \Sigma \int_{t_1}^{t_2} F_x dt = m(v_x)_2$$

$$m(v_y)_1 + \Sigma \int_{t_1}^{t_2} F_y dt = m(v_y)_2$$

$$m(v_z)_1 + \Sigma \int_{t_1}^{t_2} F_z dt = m(v_z)_2$$



15.1: Principle of Linear Impulse and Momentum

Procedure for Analysis:

Free-Body Diagram.

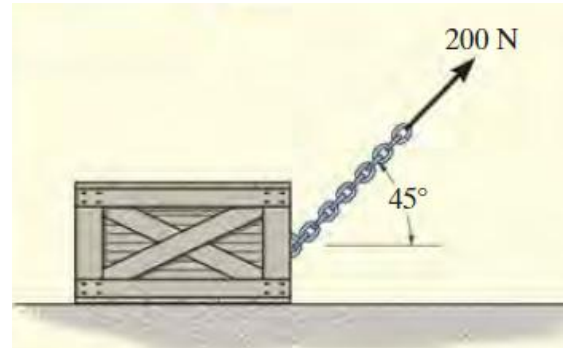
- Establish the x , y , z inertial frame of reference and draw the particle's free-body diagram to account for all the forces that produce impulses on the particle.
- The direction and sense of the particle's initial and final velocities should be established.
- If a vector is unknown, assume that the sense of its components is in the direction of the positive inertial coordinate(s).

Principle of Impulse and Momentum.

- In accordance with the established coordinate system, apply the principle of linear impulse and momentum,
- If motion occurs in the x - y plane, the two-scalar component equations can be formulated by either resolving the vector components of F from the free-body diagram, or by using the data on the impulse and momentum diagrams.
- Realize that every force acting on the particle's free-body diagram will create an impulse, even though some of these forces will do no work.
- Forces that are functions of time must be integrated to obtain the impulse. Graphically, the impulse is equal to the area under the force-time curve.

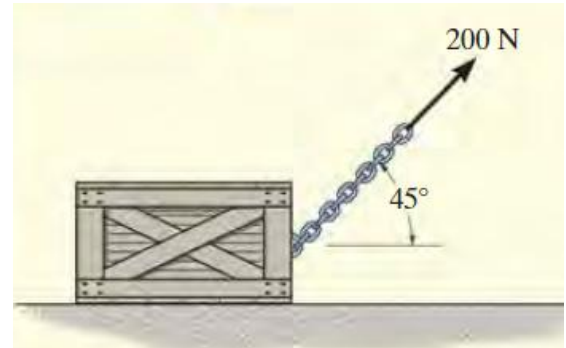
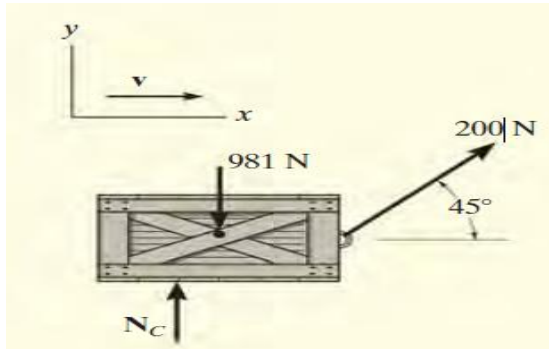
Ex: 1

The 100-kg crate shown is originally at rest on the smooth horizontal surface. If a towing force of 200 N, acting at an angle of 45° , is applied for 10 s, determine the final velocity and the normal force which the surface exerts on the crate during this time interval.



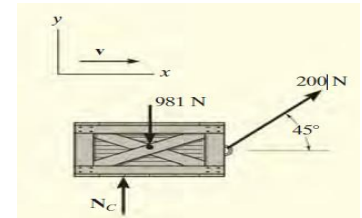
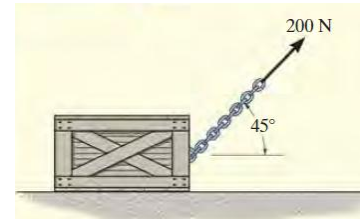
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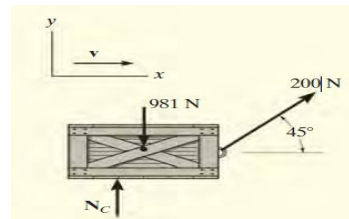
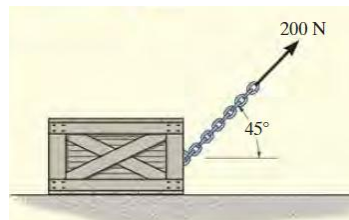
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$$(\rightarrow) \quad m(v_x)_1 + \Sigma \int_{t_1}^{t_2} F_x dt = m(v_x)_2$$

$$0 + 200 \text{ N} \cos 45^\circ (10 \text{ s}) = (100 \text{ kg})v_2$$
$$v_2 = 14.1 \text{ m/s}$$

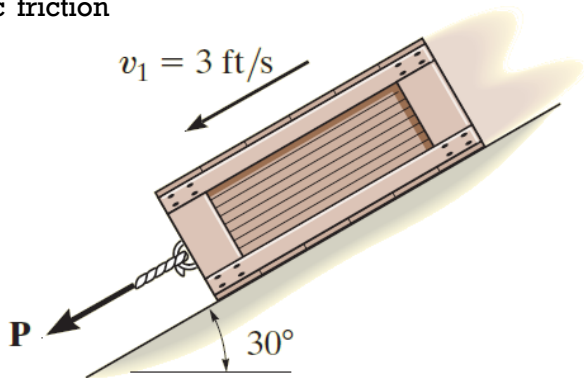
$$(+\uparrow) \quad m(v_y)_1 + \Sigma \int_{t_1}^{t_2} F_y dt = m(v_y)_2$$

$$0 + N_C(10 \text{ s}) - 981 \text{ N}(10 \text{ s}) + 200 \text{ N} \sin 45^\circ (10 \text{ s}) = 0$$
$$N_C = 840 \text{ N}$$



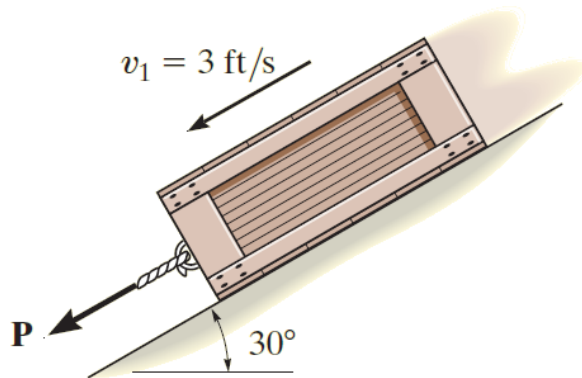
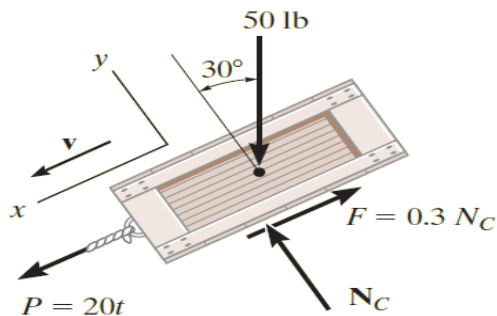
Ex:2

The 50-lb crate shown in is acted upon by a force having a variable magnitude $P = (20t)$ lb, where t is in seconds. Determine the crate's velocity 2 s after P has been applied. The initial velocity is $v_1 = 3$ ft/s down the plane, and the coefficient of kinetic friction between the crate and the plane is $\mu_k = 0.3$.



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$$\begin{aligned}
 (+\swarrow) \quad m(v_x)_1 + \sum \int_{t_1}^{t_2} F_x dt &= m(v_x)_2 \\
 \frac{50 \text{ lb}}{32.2 \text{ ft/s}^2} (3 \text{ ft/s}) + \int_0^{2 \text{ s}} 20t dt - 0.3N_C(2 \text{ s}) + (50 \text{ lb}) \sin 30^\circ(2 \text{ s}) &= \frac{50 \text{ lb}}{32.2 \text{ ft/s}^2} v_2 \\
 4.658 + 40 - 0.6N_C + 50 &= 1.553v_2
 \end{aligned}$$

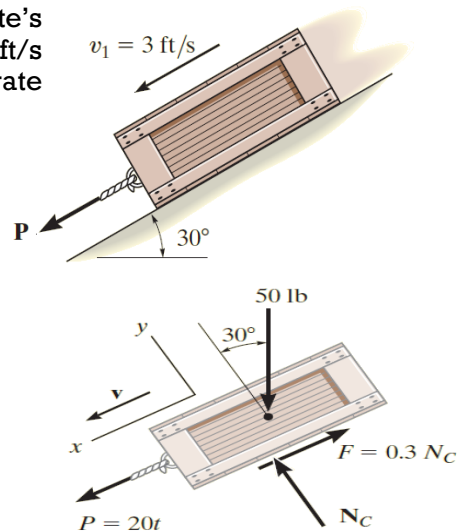
The equation of equilibrium can be applied in the y direction.

$$+\Uparrow \sum F_y = 0; \quad N_C - 50 \cos 30^\circ \text{ lb} = 0$$

Solving,

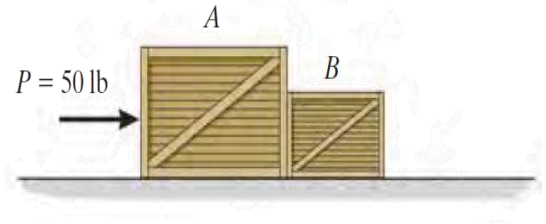
$$N_C = 43.30 \text{ lb}$$

$$v_2 = 44.2 \text{ ft/s} \swarrow$$



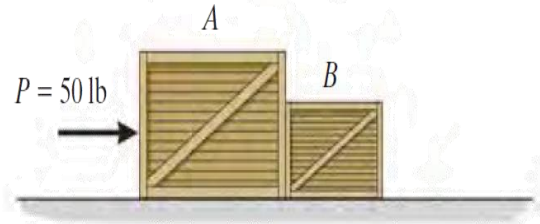
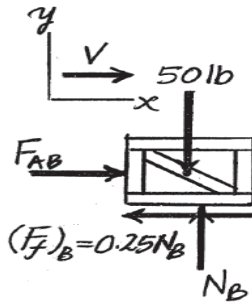
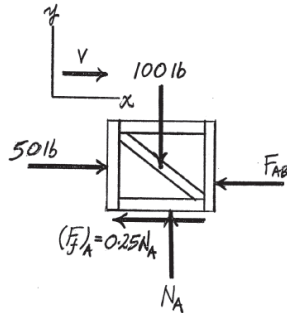
Ex:3

Crates A and B weigh 100 lb and 50 lb, respectively. If they start from rest, determine their speed when $t = 5$ s. Also, find the force exerted by crate A on crate B during the motion. The coefficient of kinetic friction between the crates and the ground is $\mu_k = 0.25$.



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Principle of Impulse and Momentum: Referring to Fig. a,

$$(+\uparrow) \quad m(v_1)_y + \sum \int_{t_1}^{t_2} F_y dt = m(v_2)_y$$

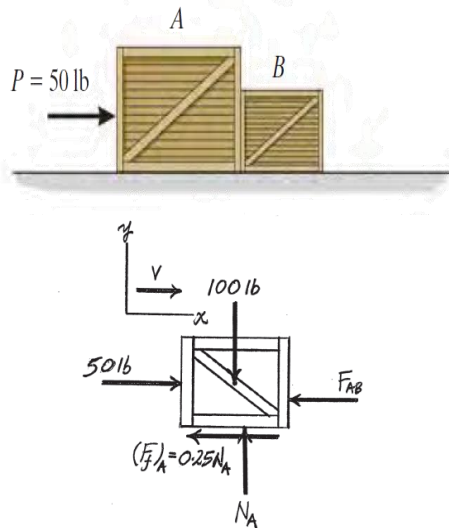
$$\frac{100}{32.2}(0) + N_A(5) - 100(5) = \frac{100}{32.2}(0)$$

$$N_A = 100 \text{ lb}$$

$$(\rightarrow) \quad m(v_1)_x + \sum \int_{t_1}^{t_2} F_x dt = m(v_2)_x$$

$$\frac{100}{32.2}(0) + 50(5) - 0.25(100)(5) - F_{AB}(5) = \frac{100}{32.2}v$$

$$v = 40.25 - 1.61F_{AB}$$



Ex:3

Crates A and B weigh 100 lb and 50 lb, respectively. If they start from rest, determine their speed when $t = 5$ s. Also, find the force exerted by crate A on crate B during the motion. The coefficient of kinetic friction between the crates and the ground is $\mu_k = 0.25$.

By considering Fig. b,

$$(+\uparrow) \quad m(v_1)_y + \Sigma \int_{t_1}^{t_2} F_y dt = m(v_2)_y$$

$$\frac{50}{32.2}(0) + N_B(5) - 50(5) = \frac{50}{32.2}(0)$$

$$N_B = 50 \text{ lb}$$

$$(\rightarrow) \quad m(v_1)_x + \Sigma \int_{t_1}^{t_2} F_x dt = m(v_2)_x$$

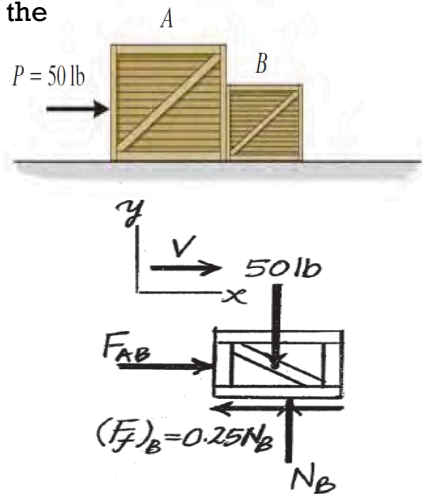
$$\frac{50}{32.2}(0) + F_{AB}(5) - 0.25(50)(5) = \frac{50}{32.2}v$$

$$v = 3.22 F_{AB} - 40.25$$

Solving Eqs. (1) and (2) yields

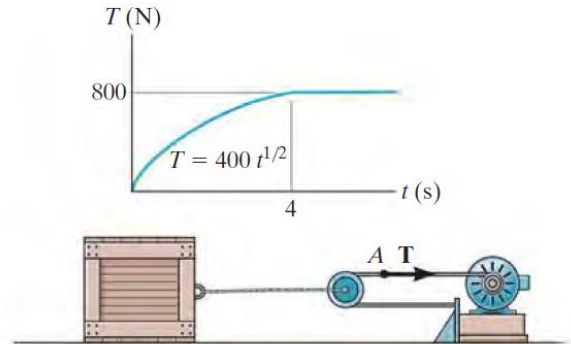
$$F_{AB} = 16.67 \text{ lb} = 16.7 \text{ lb}$$

$$v = 13.42 \text{ ft/s} = 13.4 \text{ ft/s}$$



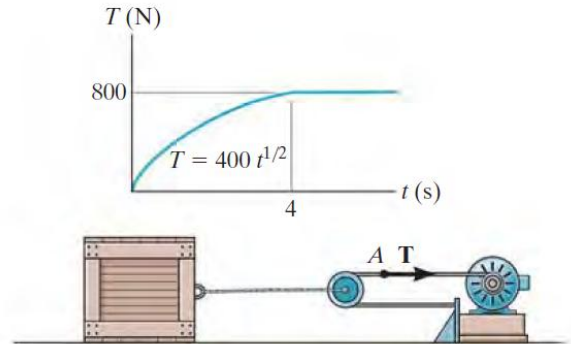
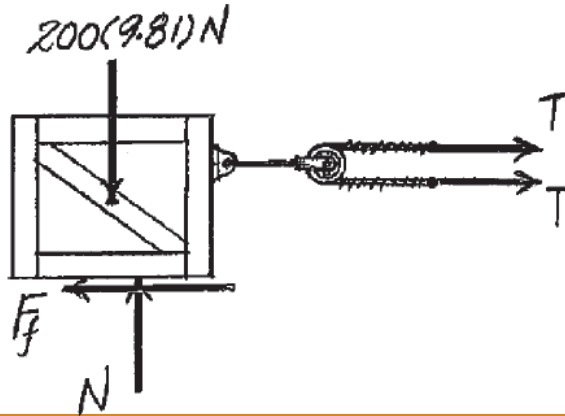
Ex:4

The 200-kg crate rests on the ground for which the coefficients of static and kinetic friction are $\mu_s = 0.5$ and $\mu_k = 0.4$, respectively. The winch delivers a horizontal towing force T to its cable at A which varies as shown in the graph. Determine the speed of the crate when $t = 4$ s. Originally the tension in the cable is zero.



Ex:4

The 200-kg crate rests on the ground for which the coefficients of static and kinetic friction are $\mu_s = 0.5$ and $\mu_k = 0.4$, respectively. The winch delivers a horizontal towing force T to its cable at A which varies as shown in the graph. Determine the speed of the crate when $t = 4$ s. Originally the tension in the cable is zero.



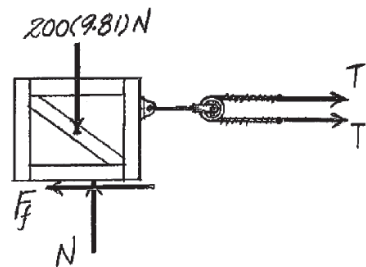
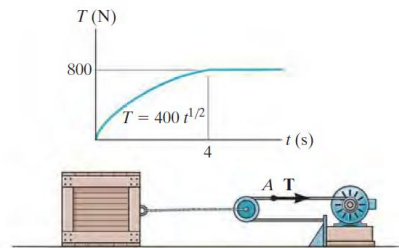
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Equilibrium. The time required to move the crate can be determined by considering the equilibrium of the crate. Since the crate is required to be on the verge of sliding, $F_f = \mu_s N = 0.5 N$. Referring to the FBD of the crate, Fig. *a*,

$$+\uparrow \Sigma F_y = 0; \quad N - 200(9.81) = 0 \quad N = 1962 \text{ N}$$

$$+\rightarrow \Sigma F_x = 0; \quad 2(400t^{1/2}) - 0.5(1962) = 0 \quad t = 1.5037 \text{ s}$$



Ex:4

The 200-kg crate rests on the ground for which the coefficients of static and kinetic friction are $\mu_s = 0.5$ and $\mu_k = 0.4$, respectively. The winch delivers a horizontal towing force T to its cable at A which varies as shown in the graph. Determine the speed of the crate when $t = 4$ s. Originally the tension in the cable is zero.

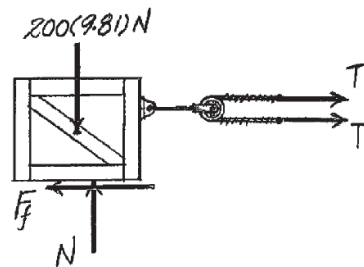
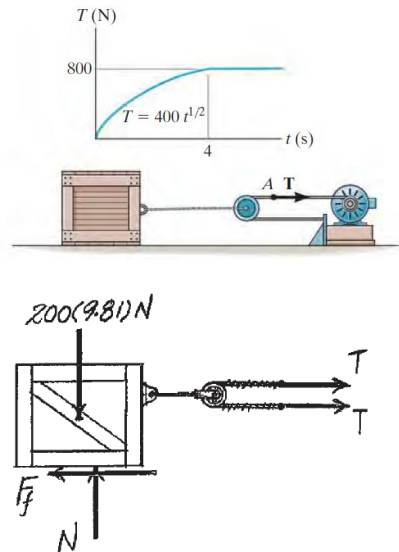
Principle of Impulse and Momentum. Since the crate is sliding, $F_f = \mu_k N = 0.4(1962) = 784.8$ N. Referring to the FBD of the crate, Fig. *a*

$$(\rightarrow) \quad m(v_x)_1 + \Sigma \int_{t_1}^{t_2} F_x dt = m(v_x)_2$$

$$0 + 2 \int_{1.5037 \text{ s}}^{4 \text{ s}} 400t^{1/2} dt - 784.8(4 - 1.5037) = 200v$$

$$v = 6.621 \text{ m/s} = 6.62 \text{ m/s}$$

Ans.



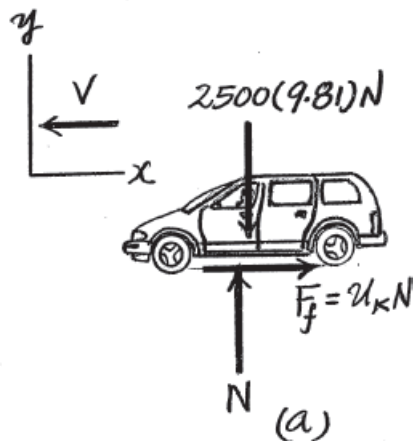
Ex:5

The 2.5-Mg van is traveling with a speed of 100 km/h when the brakes are applied and all four wheels lock. If the speed decreases to 40 km/h in 5 s, determine the coefficient of kinetic friction between the tires and the road.



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Principle of Impulse and Momentum: The initial and final speeds of the van are

$$v_1 = \left[100(10^3) \frac{\text{m}}{\text{h}} \right] \left[\frac{1 \text{ h}}{3600 \text{ s}} \right] = 27.78 \text{ m/s} \quad \text{and} \quad v_2 = \left[40(10^3) \frac{\text{m}}{\text{h}} \right] \left[\frac{1 \text{ h}}{3600 \text{ s}} \right] = 11.11 \text{ m/s}.$$

Referring to Fig. a,

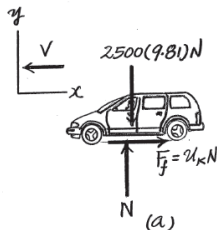
$$\begin{aligned} (+\uparrow) \quad m(v_1)_y + \sum \int_{t_1}^{t_2} F_y dt &= m(v_2)_y \\ 2500(0) + N(5) - 2500(9.81)(5) &= 2500(0) \end{aligned}$$

$$N = 24\,525 \text{ N}$$

$$\begin{aligned} (\leftarrow) \quad m(v_1)_x + \sum \int_{t_1}^{t_2} F_x dt &= m(v_2)_x \\ 2500(27.78) + [-\mu_k(24\,525)(5)] &= 2500(11.1) \end{aligned}$$

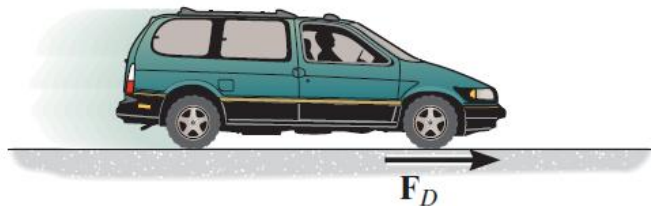
$$\mu_k = 0.340$$

Ans.



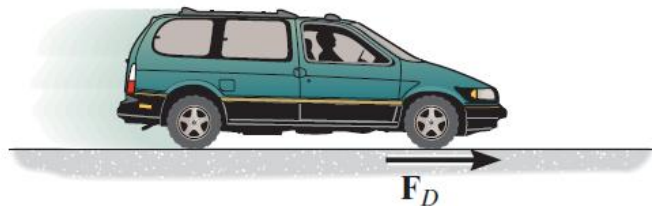
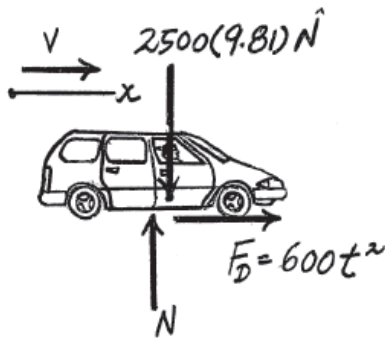
Ex:6

For a short period of time, the frictional driving force acting on the wheels of the 2.5-Mg van is $F_D = (600t^2)$ N, where t is in seconds. If the van has a speed of 20 km/h when $t = 0$, determine its speed when $t = 5$ s.



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For a short period of time, the frictional driving force acting on the wheels of the 2.5-Mg van is $F_D = (600t^2)$ N, where t is in seconds. If the van has a speed of 20 km/h when $t = 0$, determine its speed when $t = 5$ s.



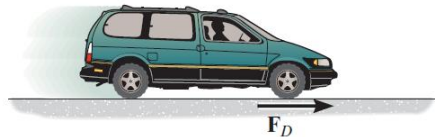
Ex:6

For a short period of time, the frictional driving force acting on the wheels of the 2.5-Mg van is $F_D = (600t^2) \text{ N}$, where t is in seconds. If the van has a speed of 20 km/h when $t = 0$, determine its speed when $t = 5 \text{ s}$.

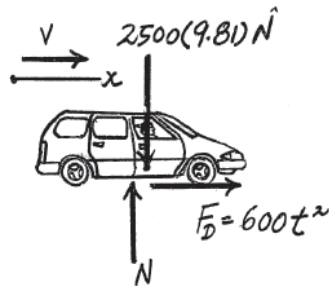
Principle of Impulse and Momentum: The initial speed of the van is $v_1 = \left[20(10^3) \frac{\text{m}}{\text{h}} \right]$

$\left[\frac{1 \text{ h}}{3600 \text{ s}} \right] = 5.556 \text{ m/s}$. Referring to the free-body diagram of the van shown in Fig. *a*,

$$\begin{aligned} (\rightarrow) \quad m(v_1)_x + \Sigma \int_{t_1}^{t_2} F_x dt &= m(v_2)_x \\ 2500(5.556) + \int_0^{5\text{s}} 600t^2 dt &= 2500 v_2 \\ v_2 &= 15.6 \text{ m/s} \end{aligned}$$

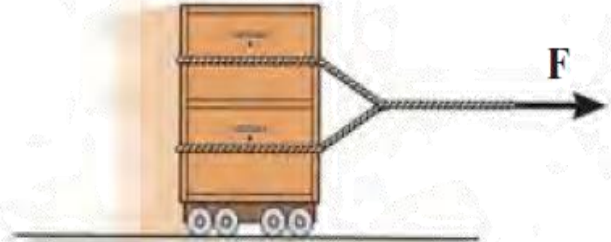
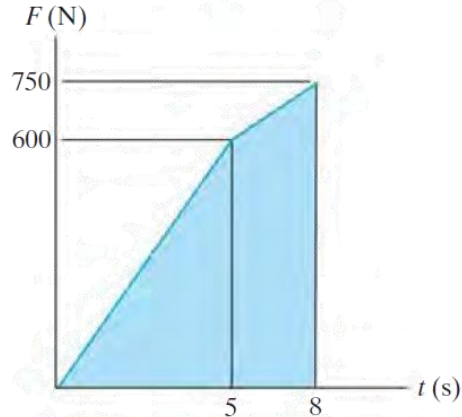


Ans.



Ex:7

The towing force acting on the 400-kg safe varies as shown on the graph. Determine its speed, starting from rest, when $t = 8$ s. How far has it traveled during this time?



Ex: 7

The towing force acting on the 400-kg safe varies as shown on the graph. Determine its speed, starting from rest, when $t = 8$ s. How far has it traveled during this time?

Principle of Impulse and Momentum. The FBD of the safe is shown in Fig. *a*.

For $0 \leq t < 5$ s, $F = \frac{600}{5}t = 120t$.

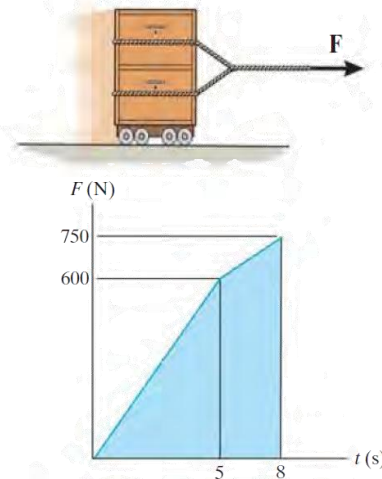
$$(\rightarrow) \quad m(v_x)_1 + \Sigma \int_{t_1}^{t_2} F_x dt = m(v_x)_2$$

$$0 + \int_0^t 120t dt = 400v$$

$$v = \{0.15t^2\} \text{ m/s}$$

At $t = 5$ s,

$$v = 0.15(5^2) = 3.75 \text{ m/s}$$



Ex: 7

The towing force acting on the 400-kg safe varies as shown on the graph. Determine its speed, starting from rest, when $t = 8$ s. How far has it traveled during this time?

For $5 \text{ s} < t \leq 8 \text{ s}$, $\frac{F - 600}{t - 5} = \frac{750 - 600}{8 - 5}$, $F = 50t + 350$. Here,

$(v_x)_1 = 3.75 \text{ m/s}$ and $t_1 = 5 \text{ s}$.

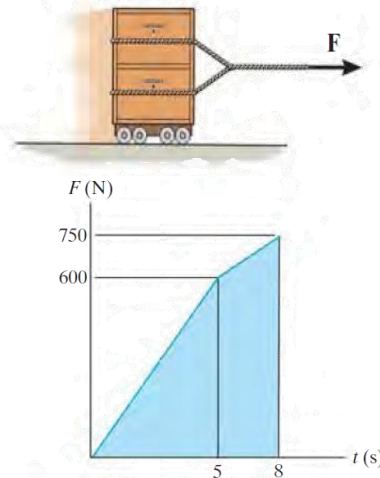
$$(\pm) \quad m(v_x)_1 + \Sigma \int_{t_1}^{t_2} F_x dt = m(v_x)_2$$

$$400(3.75) + \int_{5 \text{ s}}^t (50t + 350) dt = 400v$$

$$v = \{0.0625t^2 + 0.875t - 2.1875\} \text{ m/s}$$

At $t = 8 \text{ s}$,

$$v = 0.0625(8^2) + 0.875(8) - 2.1875 = 8.8125 \text{ m/s} = 8.81 \text{ m/s} \quad \text{Ans.}$$



Ex: 7

The towing force acting on the 400-kg safe varies as shown on the graph. Determine its speed, starting from rest, when $t = 8$ s. How far has it traveled during this time?

Kinematics. The displacement of the safe can be determined by integrating $ds = v dt$. For $0 \leq t < 5$ s, the initial condition is $s = 0$ at $t = 0$.

$$\int_0^s ds = \int_0^t 0.15t^2 dt$$

$$s = \{0.05t^3\} \text{ m}$$

At $t = 5$ s,

$$s = 0.05(5^3) = 6.25 \text{ m}$$

For $5 \text{ s} < t \leq 8$ s, the initial condition is $s = 6.25$ m at $t = 5$ s.

$$\int_{6.25 \text{ m}}^s ds = \int_{5 \text{ s}}^t (0.0625t^2 + 0.875t - 2.1875) dt$$

$$s - 6.25 = (0.02083t^3 + 0.4375t^2 - 2.1875t) \Big|_{5 \text{ s}}^t$$

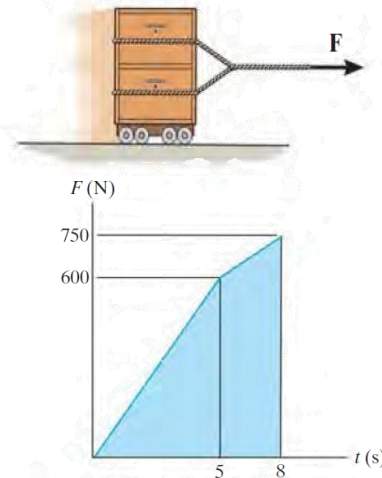
$$s = \{0.02083t^3 + 0.4375t^2 - 2.1875t + 3.6458\} \text{ m}$$

At $t = 8$ s,

$$s = 0.02083(8^3) + 0.4375(8^2) - 2.1875(8) + 3.6458$$

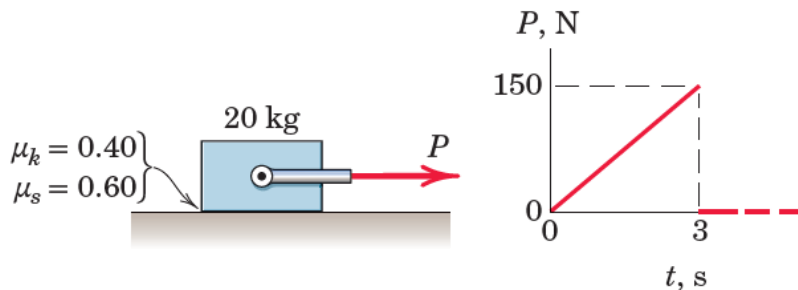
$$= 24.8125 \text{ m} = 24.8 \text{ m}$$

Ans.



Ex:8

The initially stationary 20-kg block is subjected to the time-varying horizontal force whose magnitude P is shown in the plot. Note that the force is zero for all times greater than 3 s. Determine the time t_s at which the block comes to rest.



Ex:9

The initially stationary 20-kg block is subjected to the time-varying horizontal force whose magnitude P is shown in the plot. Note that the force is zero for all times greater than 3 s. Determine the time t at which the block comes to rest.

