

Chapter 1: UNITS AND DIMENSIONS

1- System of Units

There are three different systems of units that are most commonly used :

- Meter- kilogram- second or (MKS) or System International (SI) units.
- The laboratory system centimeter-gram- second or (CGS) system.
- The Foot- Pound- Second or (FPS) British system.

The system of SI is the most commonly used all over the world.

This system is based on powers of 10. In order to change from one unit to another in the metric system, it is a matter of multiplying or dividing by powers of 10. We can also, specify various units by the use of prefixes.

SI prefixes

Factor	Name	Symbol	Factor	Name	Symbol
10^{24}	Yotta	Y	10^{-1}	Deci	D
10^{21}	Zeta	Z	10^{-2}	Centi	C
10^{18}	Exa	E	10^{-3}	Milli	M
10^{15}	Peta	P	10^{-6}	Micro	μ
10^{12}	Tera	T	10^{-9}	Nano	N
10^9	Giga	G	10^{-12}	Pico	P
10^6	Mega	M	10^{-15}	Femto	F
10^3	Kilo	K	10^{-18}	Atto	A
10^2	Hector	H	10^{-21}	Zepto	Z
10^1	Deka	Da	10^{-24}	Yocto	Y

Mass is a measure of the amount of matter that an object contains. In SI the unit of mass is the Kilogram (10^3 grams).

Length is defined as a given number of wavelengths of the emitted light from a Cesium atom. In the SI system, the unit of length is the meter (m).

Time In system of SI, the unit of time is the second.

Electric current the unit is Ampere

Thermodynamic temperature the unit is Kelvin

a- Consistency of Units

◆ Principle of consistency of units:

- units on the left side of an equation must be the same as those on the right side of an equation
- dimensional homogeneity

When using units in Mathematics operations:

- ✓ Adding numbers with units, they must have the same units —also for subtraction
- ✓ Multiply or dividing numbers with units, also the units multiply and divide
- ✓ Never add or subtract terms with different dimensions

Examples

- | | |
|--|-------|
| 1- $12\text{ m (length)} \times 12\text{ m (length)} = 144\text{ m}^2\text{ (area)}$ | OK |
| 2- $3.0\text{ m Length} / 3\text{ s (time)} = 1.0\text{ m/s (velocity)}$ | OK |
| 3- $0.1\text{ g (mass)} / 10\text{ s (time)} = 0.001\text{ g / s (.....)}$ | OK |
| 4- $10\text{ Kg (mass)} + 10\text{ m (length)}$ | Wrong |

b- Units Conversion

We often need to change the units in which a physical quantity is expressed. We do so by a method called Chain- Link Method of conversion. In this method, we multiply the original measurement by a conversion factor (a ratio of units that is equal to one). This method can be used to convert from a unit to another.

Example 1

How many seconds are in 2 minutes?

$$2 \text{ min} = 2 \text{ min} \cdot \frac{60\text{s}}{1\text{min}} = 120\text{s}$$

Example 2

How many seconds are in 2.0 years?

$$2\text{yr} = 2.0\text{yr} \cdot \frac{365\text{days}}{1\text{yr}} \cdot \frac{24\text{hr}}{1\text{day}} \cdot \frac{60\text{min}}{1\text{hr}} \times \frac{60\text{s}}{1\text{min}} = 31536000$$

Example 3

The density of water is $1\text{g}/\text{cm}^3$. what is the density of water in MKS system?

We know that $1 \text{ g} = 10^{-3} \text{ Kg}$ and $1 \text{ cm}^3 = 10^{-6} \text{ m}^3$

$$\rho = 1\text{g} / \text{cm}^3 \times \left(\frac{10^{-3}\text{Kg} / \text{g}}{10^{-6}\text{m}^3 / \text{cm}^3} \right) = 10^3 \text{Kg} / \text{m}^3$$

Homework

- i. How many millimeters are in 20.0 meters?
- ii. How many square centimeters are in an area 6 km^2 ?
- iii. $120 \text{ Km/h} = \dots\dots\dots \text{m/s}$.
- iv. A car moves with a constant speed of 90 km/hr . Find this speed in terms of m/s , km/min .

2- Dimensions

The term Dimensional Analysis helps us to check the relationship of various quantities either correct or non correct and to: -

- i. Verify Physical Equations
- ii. Deduce Physical Equations
- iii. Deduce the dimensions of Physical Constants

a- Dimensional Analysis

Dimensions and units can be treated algebraically

Variable from Eq.	x	M	t	$V=(x_f-x_i)/t$	$a=(v_f-v_i)/t^2$	θ	Ω
<i>Dimension</i> [	$[L]$ <i>Lenght</i>	$[M]$ <i>Mass</i>	$[T]$ <i>Time</i>	$[L/T]$ <i>vilocity</i>	$[L/T^2]$ <i>Acceleration</i>	----- Plane angle	----- Solid angle
<i>Units</i>	Meter (m)	kilogram (kg)	Second (s)	Meter/second (m/s)	Meter/second ² (m/s ²)	Radian (rad)	Steradian (strad)

Example 4

Check dimensionally (correct or not) the following equation:

$$t = 2\pi \sqrt{\frac{l}{g}}$$

where t is the periodic time of a simple pendulum of length l and g is the acceleration due to gravity.

Solution:

The dimensions of the left hand side = [L H S] = [t] = [T] (1)

The dimensions of the right hand side = [R H S] = [l/g]^{1/2} = [L/LT⁻²] = [T].....(2)

From 1 and 2, **The equation is dimensionally correct**

Example 5:

Consider the following equation:

$$m \frac{v^2}{r} = G \frac{mM}{r^2}$$

where m and M are masses, r is a radius and v is a velocity. What are the dimensions of G ?

solution

From the above equation:- then $G = v^2 r / M$

Then the SI units of G is $(\text{m/s})^2 \text{ m/Kg}$, so it will be $\text{m}^3 / \text{Kg} \cdot \text{s}^2$

As a result of this the dimension s of $[G] = [\text{L}^3 \cdot \text{M}^{-1} \cdot \text{T}^{-2}]$

Exercices:

1- Choose the correct answer

1. which symbol representing the Dimension of the Length
a) (M) b) [M] c) (m) d) [m] e) [L]
2. The SI unit of velocity is
a) Kg m/s² b) N c) m/s² d) m/s (e) (a) and (b)
3. 1 Nano-meter = ... meter
a) 10⁻⁹ b) 10⁻⁶ c) 10⁹ d) 10⁶ e) 10¹²
4. 20 Mega-pixel = ... pixel
a) 2x10⁻⁹ b) 4x10⁻⁶ c) 2x10¹⁰ d) 2x 10⁷ e) 4x10¹²
5. If you have a box its dimension 2 meter length and 50 cm width and height 1 meter , then its volume is :
a) 1 m³ b) 1 m² c) 2 cm² d) 2 cm³ e) none
6. Which of the following is vector quantity(s)?
a) Speed b) density c) force d) mass
7. 1 Nano-meter = ... meter
a) 10⁻⁹ b) 10⁻⁶ c) 10⁹ d) 10⁶
8. 4 years =.....s
a) 1.1x10⁸ b) 1.26x10⁸ c) 1.44x10⁸ d) 1.77x10⁸

2- Which of the following quantities are scalars or vectors?

- 1- Volume

- 2- Acceleration
- 3- Pressure
- 4- Work

3- Match the following physical quantities with their respective units and dimensions:

Physical quantities	UNIT	Dimension
Velocity	Kg.m/s^2	$[\text{MLT}^{-2}]$
Acceleration	m/s	$[\text{LT}^{-1}]$
Volume	m	$[\text{L}^3]$
Force	m/s^2	$[\text{L}]$
Displacement	m^3	$[\text{LT}^{-2}]$

4- Complete the following sentences:

1. $1 \text{ kg} + 200 \text{ g} = \dots\dots\dots \text{ g}$
2. $1 \mu\text{m} = \dots\dots\dots \text{ m}$
3. $100 \text{ km/h} = \dots\dots\dots \text{ m/s}$
4. $12 \text{ m} \times 10 \text{ m}^2 = \dots\dots\dots$
5. $\text{kg} = \dots\dots\dots \text{ g}$.
6. $1 \text{ nm} = \dots\dots\dots \text{ m}$
7. $120 \text{ km/h} = \dots\dots\dots \text{ m/s}$

Chapter 2: VECTORS

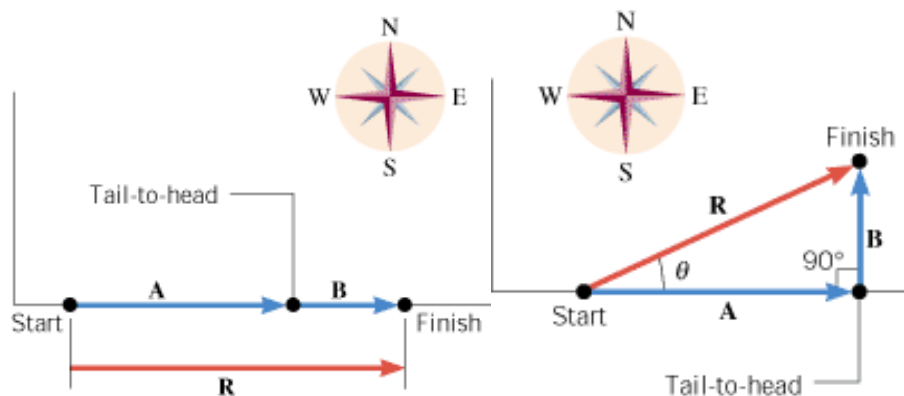
2.1 Vectors properties

The Vector has both magnitude and direction, and both these properties must be given in order to specify it. A quantity with magnitude but no direction is the Scalar.

2.1.1 Adding and subtracting Vectors Graphically

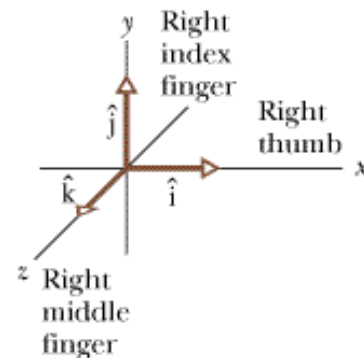
$$\vec{R} = \vec{a} + \vec{b}$$

General procedure for adding two vectors graphically:



2.1.2 Unit Vectors

The unit vectors are dimensionless vectors that point in the direction along a coordinate axis that is chosen to be positive



2.1.3 Vector Components:

The projection of a vector on an axis is called its component.

$$a_x = a \cos \theta$$

$$a_y = a \sin \theta$$

$$\vec{a} = \vec{a}_x + \vec{a}_y = a_x \hat{i} + a_y \hat{j}$$

2.1.4 Properties of vector components

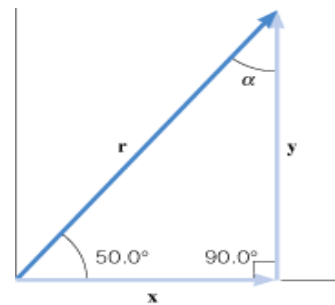
- It is possible for one of the components of a vector to be zero. This does not mean that the vector itself is zero, however. For a vector to be zero, every vector component must individually be zero.
- Two vectors are equal if, and only if, they have the same magnitude and direction

Components of vector

Example 1 Find the components displacement vector r has a magnitude of $r = 175$ m and points at an angle of 50° relative to the x axis in Figure. Find the x and y components of this vector.

$$x = r \cos \theta = 175(m) \cos(50^\circ) = 112.48m$$

$$y = r \sin \theta = 175(m) \sin(50^\circ) = 134.05m$$



Magnitude and direction of vector

Example 2 Find magnitude and direction

If $a_x = 3$ m & $a_y = 4$ m

Magnitude $|\vec{a}| = \sqrt{a_x^2 + a_y^2} \Rightarrow |\vec{a}| = \sqrt{3^2 + 4^2} = 5m$

Direction $\theta = \tan^{-1} \frac{a_y}{a_x} \Rightarrow \theta = \tan^{-1} \frac{4}{3} = 53^\circ$

2.1.5 Addition & subtraction of Vectors by Means of Components

For any two vectors $\vec{A}$ and $\vec{B}$. Then $\vec{C} = \vec{A} + \vec{B}$ so,

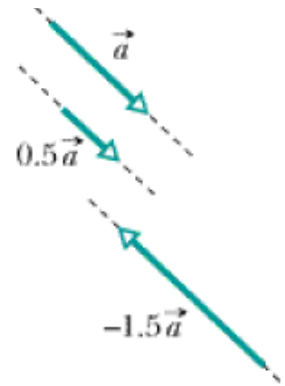
$$C_x = A_x + B_x \text{ and } C_y = A_y + B_y$$

$$C = \sqrt{C_x^2 + C_y^2}$$

$$\theta = \tan^{-1}(C_y / C_x)$$

Multiplying and Dividing a Vector by a Scalar 2.1

$$e\vec{V} = e(\vec{V}_x + \vec{V}_y) = e(V_x\hat{i} + V_y\hat{j}) = (eV_x)\hat{i} + (eV_y)\hat{j}$$



2-2 The Scalar Product of Vectors (dot product)

$$\vec{a} \cdot \vec{b} = ab \cos \phi,$$

$$\vec{a} \cdot \vec{b} = (a \cos \phi)(b) = (a)(b \cos \phi).$$

The dot product is a scalar.

- If the angle ϕ between two vectors is 0° , dot product is maximum
- If the angle ϕ between two vectors is 90° , dot product is zero

One of the most important features of the dot

product of vectors is, the commutative law:

$$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}.$$

$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z.$$

Note that : $\vec{i} \cdot \vec{i} = \vec{j} \cdot \vec{j} = \vec{k} \cdot \vec{k} = 1$ while $\vec{i} \cdot \vec{j} = \vec{j} \cdot \vec{k} = \vec{k} \cdot \vec{i} = 0$

2.4 The Vector Product (cross product)

For any two vectors $\vec{A}$ and $\vec{B}$, then $\vec{C} = \vec{A} \times \vec{B}$

Cross production is a vector $c = ab \sin \phi \hat{n}$

Property of vector cross product

- The order of the vector multiplication is important.

$$\vec{b} \times \vec{a} = -(\vec{a} \times \vec{b}).$$

$$\vec{a} \times \vec{b} = (a_y b_z - b_y a_z)\hat{i} + (a_z b_x - b_z a_x)\hat{j} + (a_x b_y - b_x a_y)\hat{k}.$$

- ❖ If two vectors are parallel or anti-parallel,
- ❖ If two vectors are perpendicular to each other, the magnitude of their cross product is maximum.

Note that:- 1- $\hat{i} \times \hat{j} = \hat{k}$, $\hat{j} \times \hat{k} = \hat{i}$ and $\hat{k} \times \hat{i} = \hat{j}$ & $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = \hat{0}$ (zero vector).

2- for two vectors $a = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{B} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ then

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}.$$

Exercises

- 1- A force of F of magnitude 10N makes an angle 30° with the horizontal axis (x-axis). Find its components?
- 2- A car moves 5 Km towards East and then 5 Km towards North. Find its total displacement and the direction of this displacement?
- 3- Find:- $\hat{i} \cdot \hat{i}$ and $\hat{k} \cdot \hat{k}$
- 4- Find :- $\hat{i} \cdot \hat{j}$ and $\hat{j} \cdot \hat{k}$
- 5- Find :- $\hat{i} \times \hat{i}$ and $\hat{i} \times \hat{j}$
- 6- If $\vec{D} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{E} = \hat{i} + 2\hat{j} - 3\hat{k}$, Find:-
 - a) $\vec{D} + \vec{E}$
 - b) $\vec{D} - \vec{E}$
 - c) $\vec{D} \cdot \vec{E}$
 - d) $\vec{D} \times \vec{E}$
 - e) Also find the angle between them.
- 7- A vector $\vec{E} = 3\hat{i} - 4\hat{j}$, Find the magnitude and direction of the vector E

8- If $\mathbf{A} = 2\mathbf{i} + \mathbf{j}$, $\mathbf{B} = \mathbf{i} - 4\mathbf{j} + \mathbf{k}$, and $\mathbf{C} = \mathbf{j} + \mathbf{k}$, Find: $\mathbf{D} = \mathbf{A} + \mathbf{B} + \mathbf{C}$

9- If: $\vec{A} = 3\hat{i} - 4\hat{j} + 4\hat{k}$ $\vec{B} = 2\hat{i} + 3\hat{j} - 7\hat{k}$ and $\vec{C} = -4\hat{i} + 2\hat{j} + 5\hat{k}$

calculate : (a) $\vec{A} + \vec{B} - \vec{C}$

(b) $\vec{A} \cdot \vec{B}$

(c) $(\vec{A} \times \vec{B}) \cdot \vec{C}$

10) If you are given the following three vectors

$$\vec{A} = 3\hat{i} - 4\hat{j} + 4\hat{k} \quad \vec{B} = 2\hat{i} + 3\hat{j} - 7\hat{k} \quad \vec{C} = 4\hat{i} + 2\hat{j} + 5\hat{k}$$

Calculate a) $\vec{A} + \vec{B} - \vec{C}$,

(b) $|\mathbf{A}|$ and $|\mathbf{B}|$ and $|\mathbf{C}|$

c) The angle between $\vec{A}$ and $\vec{B}$,

d) $\vec{A} \times \vec{C}$

11) if $\vec{A} = 2.0\hat{i} + 3.0\hat{j} + 4.0\hat{k}$ and $\vec{B} = 3.0\hat{i} + 2.0\hat{j} - 3.0\hat{k}$,

Find: 1) $\vec{A} + 2\vec{B} =$

2) $\vec{A} \cdot \vec{B} =$

3) The angle between $\vec{A}$ and $\vec{B}$

12) What is the angle between the two vectors:

$$\vec{a} = 3.0\vec{i} - 4.0\vec{j} \quad \text{and} \quad \vec{b} = -2.0\vec{i} + 3.0\vec{k}$$

13) Choose the correct answer

- For a given vector $\vec{A}$, $A_x=3\text{m}$ and $A_y=4\text{m}$, its direction θ (angle between $\vec{A}$ and the horizontal) is equal to :

- a) 0° c) 36.86° b) 60° a) 53.13°

- When the angle between two vectors $\vec{A}$ and $\vec{B}$ is 90° , the scalar product, $\vec{A} \cdot \vec{B}$ is: a) Minimum b) Maximum c) Zero d) None of these

- If $\vec{i}$ is the unit vector of the X- axis then

- a) $\vec{i} \times \vec{i} = -1$ b) $\vec{i} \times \vec{i} = 0$ c) $\vec{i} \times \vec{i} = 1$ d) $\vec{i} \times \vec{i} = 2$

Chapter 3: MOTIONS IN ONE AND TWO DIMENSIONS

3.1 Motion in One Dimension

Displacement: is the change in position,

$$\Delta \vec{x} = \vec{x}_f - \vec{x}_i$$

Where $\vec{x}_f$ is the final position and $\vec{x}_i$ is the initial position. The arrow indicates that displacement is a vector quantity:

Average Velocity : is the displacement over total time.

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} = \frac{\text{change in displacement}}{\text{change in time}}$$

$$v = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t}$$

Instantaneous Velocity: is

- Instantaneous velocity gives more information than average velocity.
- The magnitude of the velocity (either average or instantaneous) is referred to as the speed.

Average acceleration : is the change in velocity over the change in time:

$$\vec{a} \equiv \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$

Instantaneous acceleration: is calculated by taking shorter and shorter time intervals, i.e. taking $\Delta t \rightarrow 0$:

$$\vec{a} \equiv \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t}$$

- When velocity and acceleration are in the same direction, speed increases with time. When velocity and acceleration are in opposite directions, speed decreases with time.

Example 1:

The velocity of a particle moving along the x axis varies in time according to the expression $v=(40 - 5t^2)$ m/s, where t is in seconds.

- 1) Find the average acceleration in the time interval $t= 0$ to $t= 2.0$ s.
- 2) Determine the acceleration at $t =2.0$ s

Solution:

- 1) The average acceleration is given by:

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i} = \frac{v(t=2s) - v(t=0s)}{2-0} = \frac{(40 - 5 \times 2^2) - (40 - 5 \times 0)}{2} = \frac{-20}{2} = -10 \text{ m/s}^2 \text{ By}$$

differentiating, the instantaneous acceleration $a = \frac{dv}{dt} = \frac{d}{dt}(40 - 5t^2) = -10t$

$$\text{At } t = 2s, a(t=2s) = -20 \text{ m/s}^2$$

Thus, the particle is slowing down.

Example 2: The position, x (in m), of an object that moves along a straight line is changing with time, t (in sec), as follows:

$$x = 16 - 12t + 2t^2$$

Find:

- 1) What is the instantaneous velocity at $t = 0$ and $+4$?
- 2) What is the instantaneous acceleration at $t = 0$ and $+4$?
- 3) What is the average velocity between $t = 0$ and $t = +6$?

Solution:

- 1) By differentiating, $v(0) = -12 \text{ m/s}$, and $v(4) = 4 \text{ m/s}$
- 2) $a(0) = a(4) = 4 \text{ m/s}^2$.
- 3) The average velocity between two times is

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{x(6) - x(0)}{6 - (0)} = \frac{16 - 16}{6} = 0 \text{ m/s}$$

3.2 One Dimensional Motion with Constant Acceleration

$$v = v_0 + at$$

$$x = v_0 t + \frac{1}{2} at^2$$

$$v^2 = v_0^2 + 2ax$$

3.2.2 Freely Falling Bodies

A freely falling object is an object that moves under the influence of gravity only. Where, $|\vec{a}| \equiv g = 9.8 \text{ m/s}^2$ and $x \rightarrow y$.

Equations of Motion for the 1-d vertical motion of an object in free fall:

$$v = v_0 - gt$$

$$y = v_0 t - \frac{1}{2} gt^2$$

$$v^2 = v_0^2 - 2gy$$

Note: Since the acceleration due to gravity is the same for any object, a heavy object does not fall faster than a light object.

Example 3:

A golf ball is released from rest from the top of a very tall building. Calculate the position and the velocity of the ball after 1.0 s and 3.0 s.

Solution:

Choose the origin at ($y=0$, $t=0$) at the starting point of the ball and take upward as positive. Then $y_0=0$, $v_0=0$ and $a=-g=-9.8\text{m/s}^2$. The position and the velocity at time t become:

$$y_f - y_0 = -\frac{1}{2}gt^2 + v_0t \Rightarrow y_f = -\frac{1}{2}gt^2 = -\frac{1}{2}(9.8)t^2 \quad \text{and} \quad v_f - v_0 = -gt \Rightarrow v_f = -9.8t$$

a) at $t=1\text{s}$: $y_f = -4.9\text{m}$; at $t=2\text{s}$: $y_f = -19.6\text{m}$; at $t=3\text{s}$: $y_f = -44.1\text{m}$

b) at $t=1\text{s}$: $v_f = -9.8\text{m/s}$; at $t=2\text{s}$: $v_f = -19.6\text{m/s}$; at $t=3\text{s}$: $v_f = -29.4\text{m/s}$

Example 4:

A ball is thrown vertically upward from the ground with an initial speed of 15.0 m/s . The free fall acceleration $a=-g=-9.8\text{m/s}^2$.

- 1) How long does it take the ball to reach its maximum altitude?
- 2) What is its maximum altitude?
- 3) Determine the velocity and acceleration of the ball at $t = 2\text{ s}$.

Solution:

$v_0 = 15\text{m/s}$, $a = -g = -9.8\text{m/s}^2$ and $y_0 = 0\text{m}$ (the ball is thrown upward from the ground) We choose y axis positive in the upward direction.

- 1) The time taken by the ball to reach its maximum altitude is:

$$v = v_0 - gt \Rightarrow t = \frac{v_0 - v}{g} = \frac{15 - 0}{9.8} = 1.53\text{s}$$

- 2) The maximum altitude ($v=0\text{m/s}$) is given by the equation

$$v^2 - v_0^2 = -2g(y - y_0) \Rightarrow y_{\max} = \frac{v^2 - v_0^2}{-2g} = \frac{0 - (15)^2}{-2 \times 9.8} = 11.48\text{m}$$

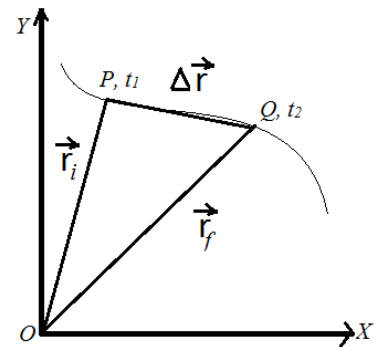
- 3) The velocity of the ball at $t=2\text{s}$: $v = v_0 - gt = 15 - 9.8 \times 2 = -4.6\text{m/s}$

The acceleration of the ball at $t=2\text{s}$ is equals $a = -g = -9.8\text{m/s}^2$

3.3 Motion in Two Dimensions

3.3.1 Displacement, Velocity and Acceleration in 2-Dimensions

In the figure, an object is at position $\vec{r}_i$ at time t_i (point P). Some time later, t_f , the object is at position $\vec{r}_f$ (point Q). The displacement vector of the object is given by:



$$\Delta \vec{r} = \vec{r}_f - \vec{r}_i$$

The average velocity:

$$\vec{v}_{av} = \frac{\Delta \vec{r}}{\Delta t}$$

The instantaneous velocity $\vec{v}$

$$\vec{v} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

Average Acceleration:

$$\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t}$$

Instantaneous Acceleration:

$$\vec{a} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d}{dt} \left(\frac{d\vec{r}}{dt} \right) = \frac{d^2 \vec{r}}{dt^2}$$

Example 5:

A golf ball is hit off a tee at the edge of a cliff. Its x and y coordinates as functions of time are given by the following expressions:

$$x = (18.0 \text{ m/s})t \quad \text{and} \quad y = (4.0 \text{ m/s})t - (4.9 \text{ m/s}^2)t^2$$

- (a)** Write a vector expression for the ball's position as a function of time.
- (b)** Find expressions for the velocity vector $\vec{v}$ and the acceleration $\vec{a}$.
- (c)** write expressions for the position, the velocity, and the acceleration of the golf ball, all at $t = 3.0 \text{ s}$.

Solution:

$$\text{a) } \vec{r} = 18.0t\hat{i} + (4.0t - 4.9t^2)\hat{j}$$

$$\text{b) } \vec{v} = (18.0 \text{ m/s})\hat{i} + [4.0 \text{ m} - (9.8 \text{ m/s}^2)t]\hat{j}, \quad \vec{a} = -(9.8 \text{ m/s}^2)\hat{j}$$

$$\text{c) } \vec{r}(3.0\text{s}) = (54\text{m})\hat{i} - (32.1\text{m})\hat{j}$$

$$\vec{v}(3.0\text{s}) = (18.0\text{m/s})\hat{i} - (25.4\text{m/s})\hat{j}, \quad \vec{a}(3.0\text{s}) = -(9.8\text{m/s}^2)\hat{j}$$

3.3.2. Projectile Motion

An example on the 2 - dimensional motion.

The only force present is the force due to gravity.

The magnitude of the acceleration due to gravity is $|\vec{a}| = g = 9.8 \text{ m/s}^2$.

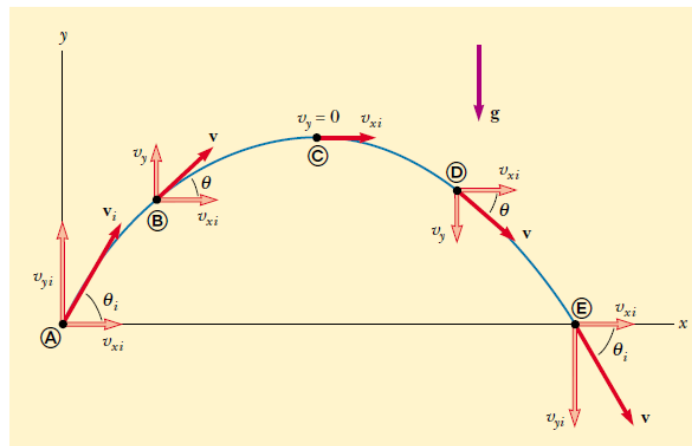


Figure of projectile motion path

Horizontal Range and Maximum Height of a Projectile:

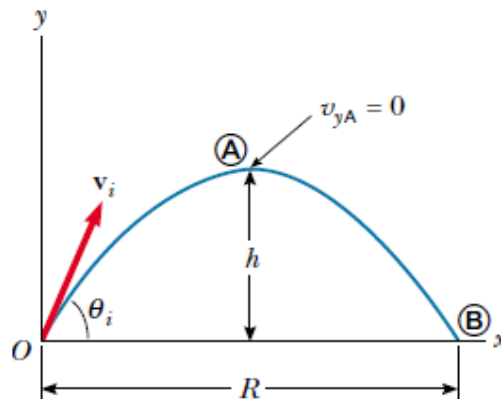


Figure of h and R

The time taken to reach point A: $t_A = \left(\frac{v_0 \sin(\theta_0)}{g} \right)$

The maximum height: $h = \frac{v_0^2 \sin^2(\theta_0)}{2g}$

The horizontal range R $R = \frac{v_0^2 \sin(2\theta_0)}{g}$

The maximum value of R is $R_{\max} = \frac{v_0^2}{g}$

Example 6:

A long-jumper leaves the ground at an angle of 20.0° above the horizontal and at a speed of 11.0 m/s.

- 1) How far does he jump in the horizontal direction (R)
- 2) What is the maximum height reached?

Solution:

$$v_0 = 11.0 \text{ m/s}, \quad \theta_0 = 20^\circ$$

$$\text{The range: } R = \frac{v_0^2 \sin(2\theta_0)}{g} = \frac{11^2 \sin(40)}{9.8} = 7.94 \text{ m}$$

2) We find the maximum height reached by using equation (3.15):

$$h = y_{\max} = \frac{v_0^2 \sin^2(\theta_0)}{2g} = \frac{(11.0 \times \sin(20^\circ))^2}{(2 \times 9.8)} = 0.722 \text{ m}$$

Exercises:

علي الطالب التدرب علي إعادة حل الأمثلة المذكورة أعلاه

Q1: Choose the correct answer:

1- The average speed of a moving particle is given by:

- a) $2\left(\frac{\Delta x}{\Delta t}\right)$ b) $\frac{\Delta x}{\Delta t}$ c) $\frac{\Delta t}{\Delta x}$ d) None

2- A ball is thrown vertically upwards, at maximum high the velocity is

- a) Minimum b) 0 c) maximum d) v_0

3- In the one dimensional motion with constant acceleration (a), the final velocity given by:

- a) $v_0 - at$ b) $v_0 + at$ c) $v_0 + at^2$ d) $v_0 - at^2$

4- The vertical component (v_y) of the projectile at the maximum height is: (a) zero (b) maximum (c) minimum (d) negative

Q2 complete the following equations

(A) = $v_0 + at$, (B) $x = \dots\dots + \frac{1}{2} at^2$,

(C) $v^2 = \dots\dots + 2ax$ (D) $\dots\dots = \frac{dv}{dt}$, (E) $v = \frac{d\dots\dots}{dt}$

Q3: The position x (m) of an object as a function of time is given by:

$$x = 5t + t^2$$

1- calculate x of the object at $t = 5$ s.

$$X_{(t=5s)} =$$

2- Calculate the instantaneous velocity v_{inst} at $t = 5s$

$$V_{inst (t=5s)} =$$

3- Calculate the instantaneous acceleration a_{inst} at $t = 5s$

$$a_{inst (t=5s)} =$$

Chapter 4: NEWTON'S LAWS OF MOTION

4-1 Force and The fundamental forces of nature:

4-1-1 *Force*: is the cause of motion in classical mechanics.

Force is a vector.

- There are two kinds of forces:
 - Contact Forces - involve physical contact between objects.
Examples: the force involved in kicking a ball, pulling a wagon, compressing a spring, etc.
 - Field forces - don't involve physical contact between objects.
Examples: the gravitational force and the electromagnetic force.

Contact Forces

Frictional Force

Tension Force

Normal Force

Air Resistance Force

Spring Force

Field force

Gravitational Force

Electrical Force

Magnetic Force

Drawing force vectors Use a vector arrow to indicate the magnitude and direction of the force.

Case I (pull directed 30° above horizontal), we have: $\vec{F}_{net} = \sum \vec{F} = m\vec{a}$

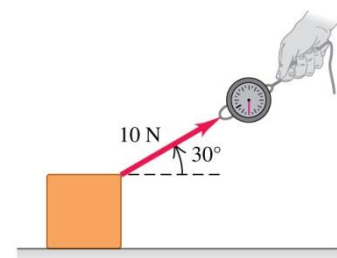
For x-axis : $F_x = \sum F_x = ma_x$, $F \cos 30^\circ = m a$

(a) A 10-N pull directed 30° above the horizontal

If the object moves with a constant speed, the above equation becomes $F = 0$

For y-axis: $F_y = \sum F_y = ma_y$. There is no motion along y-axis, so that $a_y = 0$.

$N + F \sin \theta = mg$, or $N = mg - F \sin \theta$.



Case II (push directed 45° above horizontal).

For x-axis: $F_x = \sum F_x = ma_x$, $F \cos \theta = F \cos 30^\circ = ma$

For y-axis: $F_y = \sum F_y = ma_y$. $a_y = 0$,

so $N - F \sin \theta = mg$, or $N = mg + F \sin \theta$.

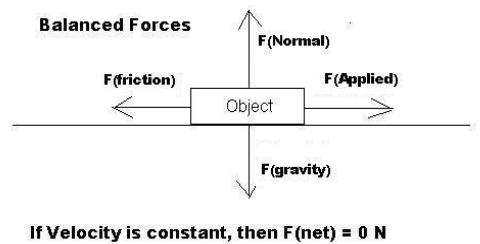
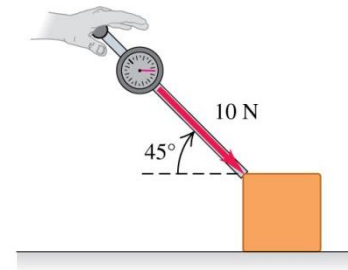
Case III: An object moving along a rough surface with a force F parallel to the surface

For x-axis: $F_x = \sum F_x = ma_x$, $F - f_k = ma$, f_k is the kinetic friction force .

If the object moves with a constant speed: $F = f_k$

For y-axis: $F_y = \sum F_y = ma_y$, $a_y = 0$, $N = mg$.

(b) A 10-N push directed 45° below the horizontal



4-2 Newton's First Law (Law of inertia)

Newton's First Law States: an object at rest stays at rest, an object in motion stays in motion with a constant velocity, if there is no net external force between the object and the environment. In equation form we can write:

$$\sum \vec{F} = 0 \Rightarrow \vec{a} = 0 \quad (\text{condition of equilibrium})$$

4-3 Newton's Second Law (Force law)

If $\sum \vec{F} \neq 0$ (i.e. there is a net external force acting on an object) then,

$$\sum \vec{F} = m\vec{a}$$

- $\vec{a}$ is inversely proportional to m .
- The SI unit of force is the Newton (N).

Definition: *Weight* ($\vec{w}$) is the force exerted on an object by a gravitational field.

From Newton's second law, $w = mg$

4-4 Newton's Third Law (action reaction Law)

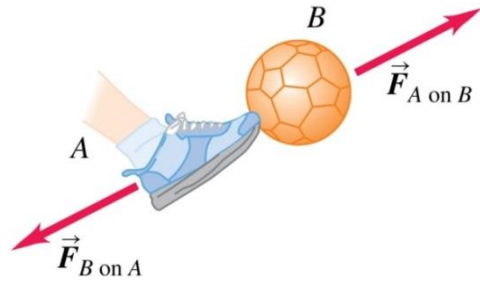
Idea: For every action, there is an equal and opposite reaction. When two bodies interact:

$$\vec{F}_{2 \text{ on } 1} = - \vec{F}_{1 \text{ on } 2}$$

Where $\vec{F}_{2 \text{ on } 1}$ is the force exerted on body 1 by body 2 and $\vec{F}_{1 \text{ on } 2}$ is the force exerted on body 2 by body 1.

For example: If you exert a force on a body, the body always exerts a force (the “reaction”) back upon you.

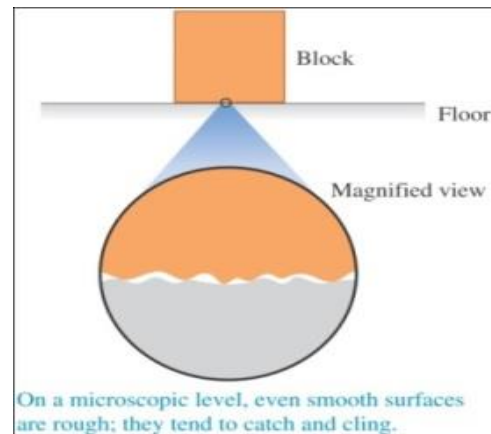
- Figure shows “an action-reaction pair.”



4-5 Frictions

Frictional forces (Interaction or contact forces). As shown in Fig.

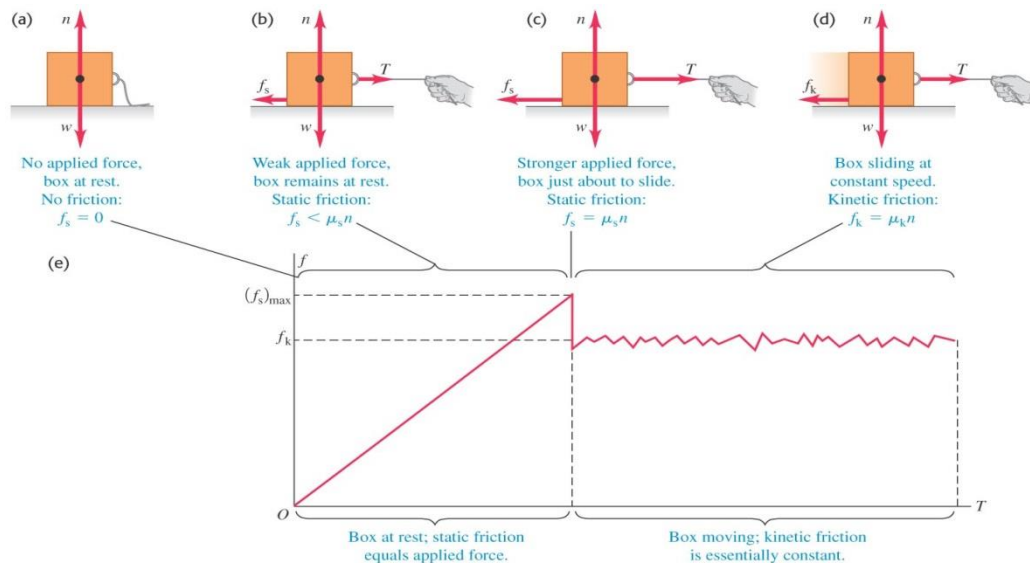
- When a body rests or slides on a surface, the friction force is parallel to the surface.
- Friction between two surfaces arises from interactions between molecules on the surfaces.



Also, the contact force between the surface and the body is normal to the surface and called normal force " F_N ".

Kinetic and static friction

- Kinetic friction acts when a body slides over a surface. $f_k = \mu_k F_N$.
- Static friction acts when there is no relative motion between bodies. $\therefore f_s \leq \mu_s F_N$.



4-6 Experimental facts about friction

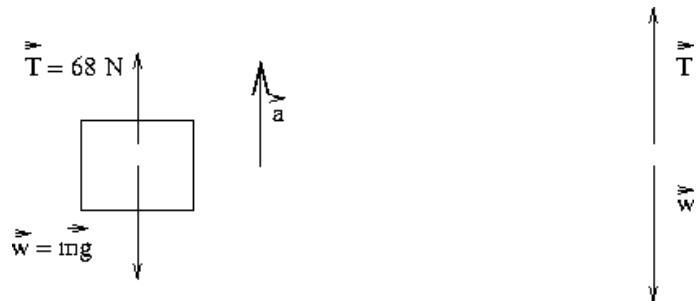
1. $f_s \leq \mu_s F_N$ where μ_s is the coefficient of static friction and N is the magnitude of the normal force. Equality holds when the object is on the point of slipping: $f_s(\max) = \mu_s F_N$.
2. $f_k = \mu_k F_N$ where μ_k is the coefficient of kinetic friction and is approximately constant for any given pair of materials.
3. Values of μ_s and μ_k depend on the nature of the surfaces that are in contact. Usually $\mu_k < \mu_s$. Examples: rubber on concrete $\mu_s = 1.0$, $\mu_s = 0.8$; waxed wood on wet snow $\mu_s = 0.14$, $\mu_k = 0.10$.
4. The direction of the force of friction is opposite to the direction the object wants to move.
5. μ_k and μ_s are nearly independent of the area of contact between the two surfaces.
6. μ_k is nearly independent of the velocity of the object under consideration.

Problem 1

A box of mass 5.0 kg is pulled vertically upwards by a force of 68 N applied to a rope attached to the box. **Find a)** the acceleration of the box and **b)** the vertical velocity of the box after 2 seconds.

Solution:

a) From Newton's 2nd Law:



$$\begin{aligned}
 ma &= T - mg \\
 \Rightarrow a &= \frac{T}{m} - g \\
 &= \frac{68 \text{ N}}{5 \text{ Kg}} - 9.8 (\text{m} / \text{s}^2) = 3.8 (\text{m} / \text{s}^2)
 \end{aligned}$$

b) Since a is constant

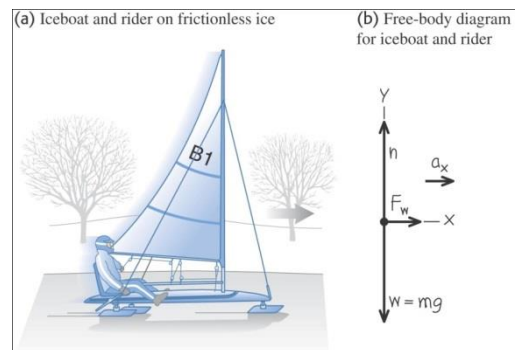
$$\begin{aligned}
 v &= v_0 + at \\
 &= 0 + 3.8(2) = 7.6 \text{ m/s.}
 \end{aligned}$$

Problem 2: An Iceboat is at rest. A wind is blowing after 4s from its release, so it attained a velocity of 6 m/s. If the wind exerts a constant horizontal force on the boat. **Find** the horizontal force F_w which exerts on the boat if its mass = 200 kg. as shown in Fig.

Solution

From the given information, we have:

$v_0 = 0 \text{ m/s}$, after $t = 4 \text{ s}$, the iceboat velocity is $v = 6 \text{ m/s}$



Then from the equation: $v = v_0 + a_x t$ in one dimensional motion

So, $6 = 0 + a_x (4)$ and so $a_x = 6/4 = 1.5 \text{ m/s}^2$

From Newton's Second Law, we have $\sum \vec{F} = ma$

In this problem, $\sum \vec{F}_y = 0$ & $\sum \vec{F}_x = ma_x = \vec{F}_w$ and so, $\vec{F}_w = 200(\text{Kg})(1.5 \text{ m} / \text{s}^2) = 300 \text{ N}$

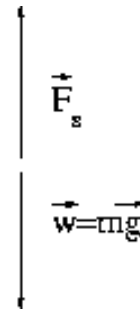
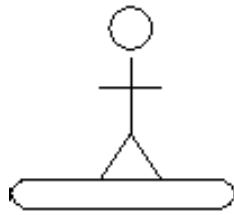
Problem 3: A student of mass 50 kg tests Newton's laws by standing on a bathroom scale in an elevator. Assume that the scale reads in Newton's. **Find** the scale reading when the elevator is a) accelerating upward at 0.5 m/s^2 , b)

going up at a constant speed of 3.0 m/s and c) going up but decelerating at 1.0 m/s².

Solution:

From the 2nd Law:

$$F_s - mg = ma \Leftrightarrow F_s = m(g + a)$$



a) $F_s = 50(9.8 + 0.5) = 515 \text{ N}$

b) $F_s = 50(9.8 + 0) = 490 \text{ N}$

c) $F_s = 50(9.8 - 1.0) = 440 \text{ N}$

Problem 4: A 3kg block starts from rest at the top of 30° incline and slides a distance of 2m down the incline in 1.5s. Find (a) the acceleration of the block, (b) the coefficient of kinetic friction between the block and the plane, (c) the friction force acting on the block, and (d) the speed of the block after it has slid 2m.

Solution:

Given $m = 3\text{kg}$, $\theta = 30^\circ$, $x = 2\text{m}$, $t = 1.5\text{s}$

$$x = \frac{1}{2}at^2 \Rightarrow 2 = \frac{1}{2}a(1.5)^2$$

$$\Rightarrow a = 1.78\text{m/s}^2$$

$$mg \sin 30 - f = ma \Rightarrow f = m(g \sin 30 - a)$$

$$\Rightarrow f = 9.37\text{N}$$

$$F_N - mg \cos 30 = 0 \Rightarrow N = mg \cos 30$$

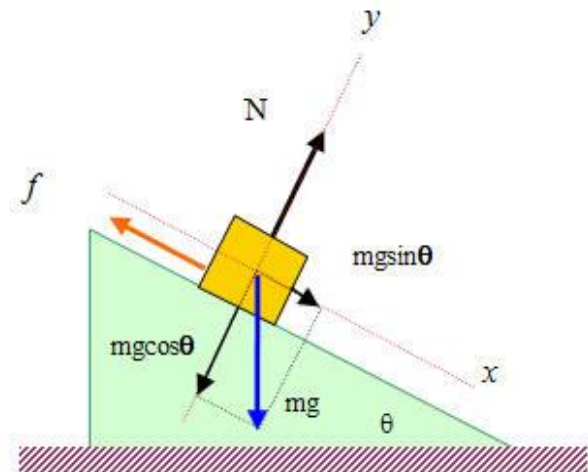
$$\therefore F_N = 25.46 \text{ N}$$

$$\therefore \mu_k = f / F_N = 0.368$$

$$v^2 = v_o^2 + 2a(x - x_o)$$

$$v^2 = 0 + 2(1.78)(2) = 7.11$$

$$\therefore v = 2.67\text{m/s}$$



Figure

Exercises

(يجب حل جميع المسائل المحلوقة أعلاه)

■ Complete the following:

a) The coefficient of kinetic friction $\mu_k = \dots / \dots$ and its SI unit is

■ Choose the correct answer:

1. Two perpendicular forces $\vec{F}_1$ and $\vec{F}_2$ are applied to an object, with magnitudes $F_1 = 20 \text{ N}$ and $F_2 = 15 \text{ N}$. The resultant force magnitude in Newtons is:

- A) 25 B) 50 C) 75 D) 100

2. The first condition of equilibrium is

- A) $\sum F_{ext} > 0$ B) $\sum F_{ext} < 0$ C) $\sum \vec{F}_{ext} = \vec{0}$ D) $\sum \vec{F}_{ext} \neq \vec{0}$

3. The maximum range of a projectile is given by:

- A) $\frac{2v_0^2}{g}$ B) $\frac{v_0^2}{g}$ C) $\frac{2g}{v_0^2}$ D) $\frac{g}{2v_0^2}$

4. A body of mass 10 kg is moving along an incline of angle 30° with the horizontal axis. The normal force on this body is

- A) 98 N B) 85 N C) 49 N D) 29 N

5. A body is moving with constant speed, then:

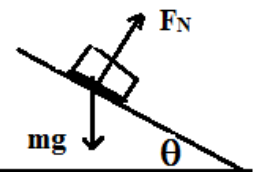
- A) $F = f_s$ B) $F = f_k$ C) $F > f_s$ D) $F > f_k$

6. The coefficients of kinetic friction (μ_k) and of the static friction (μ_s), then:

- a) $\mu_k \geq \mu_s$ b) $\mu_k < \mu_s$ and dimensionless c) $\mu_k \geq \mu_s$ and have dimensions

7. If a box slides on inclined plane of an angle θ , the normal force F_N is equal to:

- a) $mg \cos\theta$ b) mg c) $mg \sin\theta$ d) $mg \tan\theta$



■ Matching between A, B and C:

Column A	Column B	Column C
a) Newton's first law	1. $\Sigma F = ma$	i. Always opposite to the direction of motion
b) Newton's second law	2. $F_{1 \text{ on } 2} = - F_{2 \text{ on } 1}$	ii. Depends on the value of g
c) Newton's third law	3. mg	iii. Law of inertia
d) weight =	4. $\Sigma F = 0$	vi. Force law
e) force of kinetic friction	5. $= \mu_k F_N$	v. Action - reaction law