

CHAPTER 3

Fluid Mechanics

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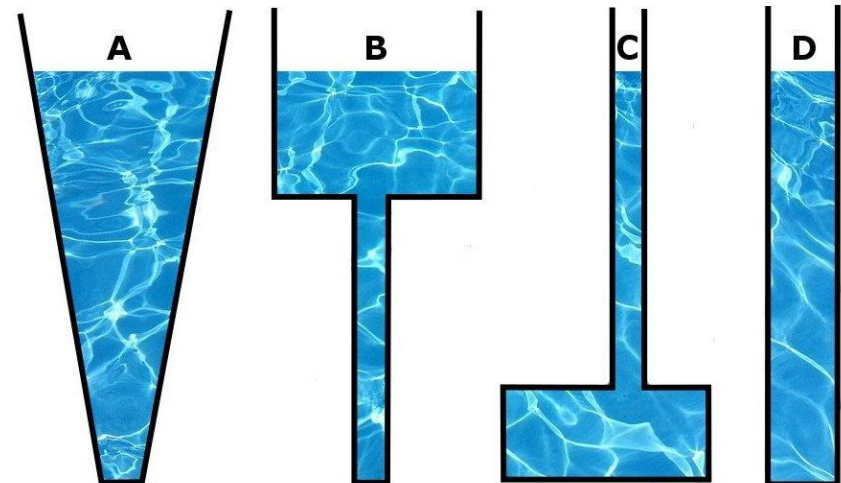
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1/ What is a fluid?

- A **fluid**, in contrast to a solid, is a substance that can **flow**.
- *Fluids conform to the boundaries of any container in which we put them.*
- *Liquids and gases are called fluids*



2/ Density

The *density* of a material is defined as its *mass per unit volume*.

$$\text{Kg/ m}^3 \leftarrow \rho = \frac{m}{V}$$

$\nearrow \text{Kg}$
 $\searrow \text{m}^3$

2/ Density

Example 1

Calculate the mass of a solid iron sphere that has a diameter of 2.8 cm. ($\rho_{\text{iron}} = 7860 \text{ Kg/m}^3$)

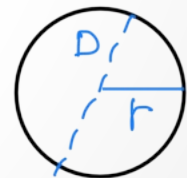
$$\rho = \frac{m}{V} \Rightarrow m = \rho \times V \Rightarrow m = \rho \times \frac{4}{3} \times \pi \times r^3$$

$$\Rightarrow m = 7860 \times \frac{4}{3} \times \pi \times (1.4 \times 10^{-2})^3 = 0.09 \text{ kg}$$

$$A = 4\pi r^2$$
$$V = \frac{4}{3}\pi r^3$$

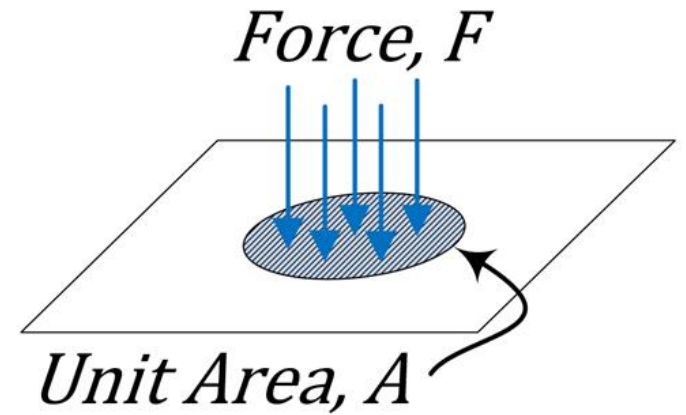


$$D = 2r$$
$$A = \pi r^2$$
$$C = 2\pi r$$



3/ Pressure

Pressure P is equal to the force F applied to a surface, divided by the area A .



$$N/m^2 \leftarrow P = \frac{F}{A} \rightarrow \begin{matrix} N \\ m^2 \end{matrix}$$

The SI unit of pressure is N/m^2 , this unit is given a special name: the **Pascal (Pa)**.

$$1 N/m^2 = 1 Pa$$

3/ Pressure

Remark 1

*Another unit of pressure is the **bar**.*

$$1 \text{ bar} = 10^5 \text{ Pa}$$

*Another unit of pressure is the **atmosphere**.*

$$1 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$$

*The **atmosphere** is the average pressure of the atmosphere at the sea level.*

3/ Pressure

Example 2

A living room has floor dimensions of 3.5 m and 4.2 m and a height of 2.4 m.
{ $P = 0.998 \text{ atm}$ and $\rho_{\text{air}} = 1.21 \text{ kg/m}^3$ }

- 1) What does the air in the room *weight*
- 2) What is *the magnitude of the atmosphere's force on the floor of the room?*

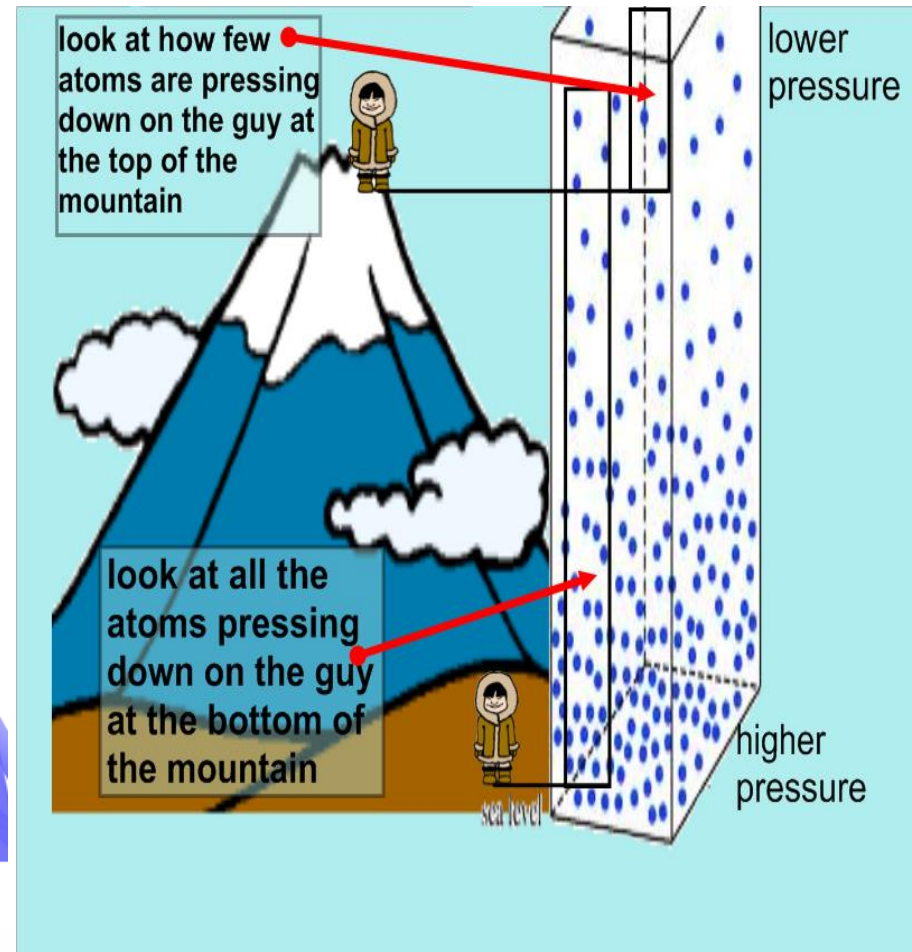
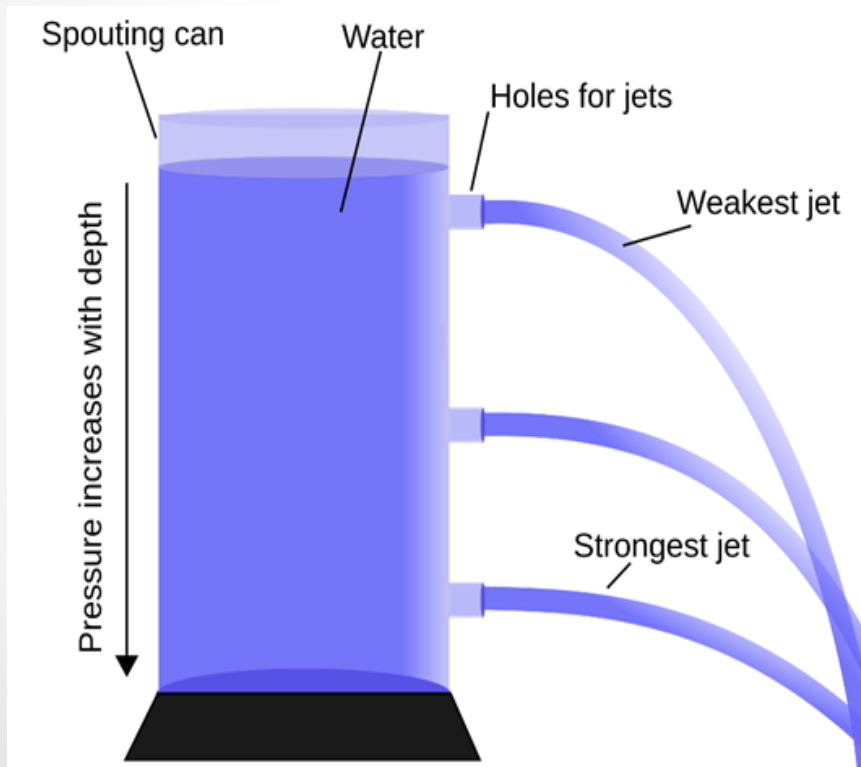
Solution

$$1) W = m \times g = \rho \times V \times g = 1.21 \times (3.5 \times 4.2 \times 2.4) \times 9.8 \\ = 4.18 \times 10^2 \text{ N}$$

$$2) P = \frac{F}{A} \Rightarrow F = P \times A = 0.998 \times 1.013 \times 10^5 \times (3.5 \times 4.2) \\ \cong 1.49 \times 10^6 \text{ N}$$

4/ Fluid at Rest

We want to find an expression for *hydrostatic pressure* as a function of *depth* or *altitude*.



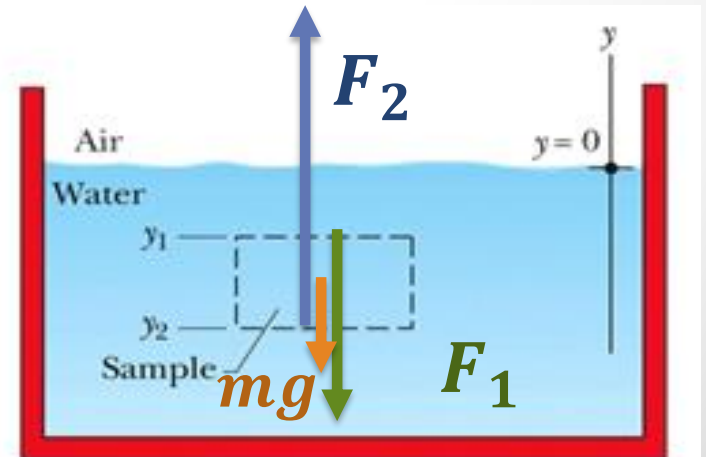
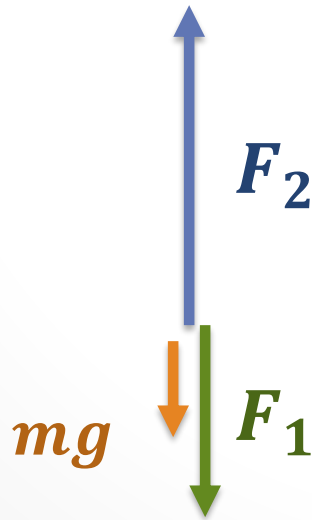
4/ Fluid at Rest

Three forces act on this sample of water..

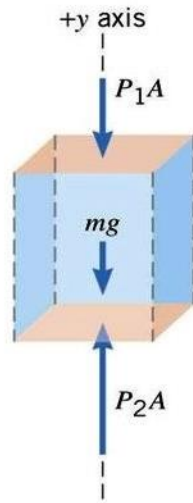
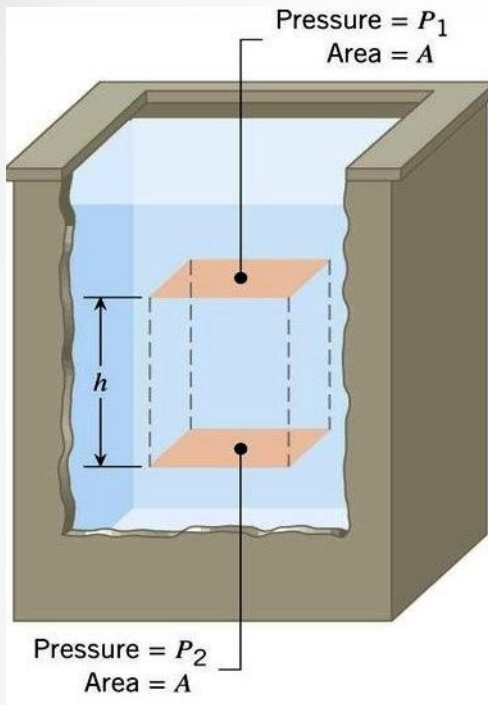
F_1 is due to the water pressure P_1 pushing on the top surface.

F_2 is due to the water pressure P_2 pushing on the bottom surface.

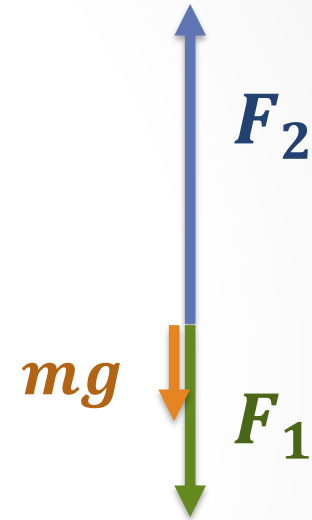
Gravity mg pulls downward on the sample. bottom surface.



4/ Fluid at Rest



$$\begin{cases} F_2 = P_2 \times A \\ F_1 = P_1 \times A \\ m = \rho \times A \times h \end{cases}$$

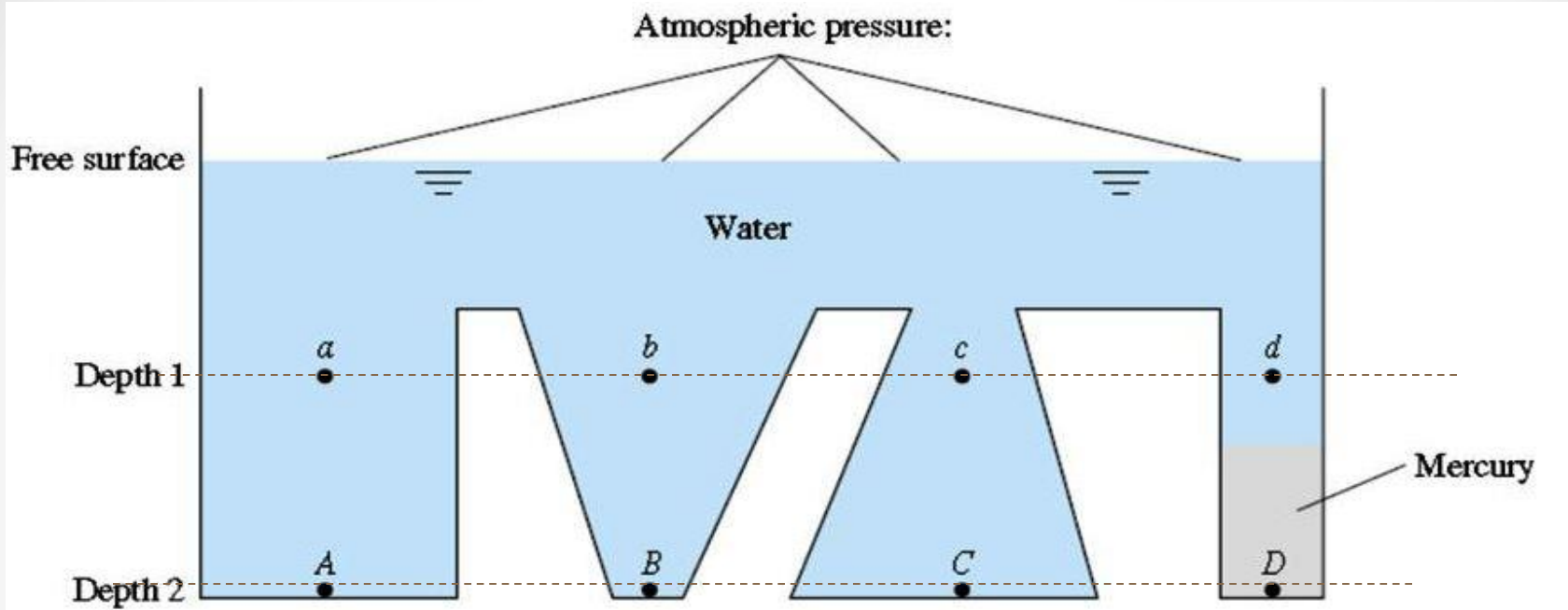


$$F_2 = F_1 + mg \Rightarrow P_2 \times \cancel{A} = P_1 \times \cancel{A} + \rho \times \cancel{A} \times h \times g$$

$$\Rightarrow P_2 = P_1 + \rho \times h \times g$$

$$P_2 - P_1 = \rho \times h \times g$$

Remark 2



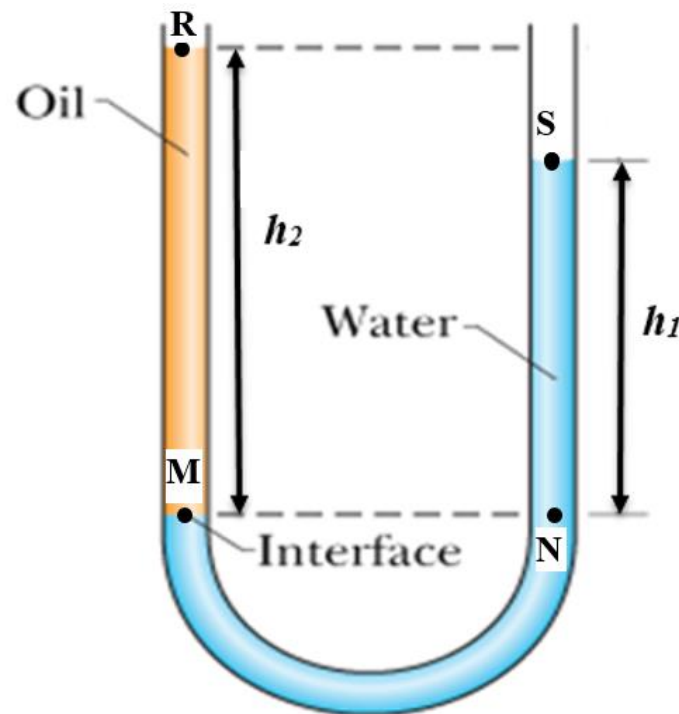
$$P_a = P_b = P_c = P_d \quad P_A = P_B = P_C \neq P_D$$

$$P_A > P_a ; P_B > P_b ; P_C > P_c$$

The pressure at a point in a fluid in static equilibrium depends on the depth of that point but not on any horizontal dimension of the fluid or its container.

Example 3

The U-tube in figure below contains two liquids in static equilibrium: Water of density ($\rho_w = 998 \text{ Kg/m}^3$) is in the right arm and oil of unknown density ρ_{oil} is in the left. Measurement gives $h_1 = 13.6 \text{ cm}$ and $h_2 = 14.8 \text{ cm}$. What is the density of the oil?



Solution

R and S are in contact with air, so :

$$P_R = P_S$$

M and N are in the same horizontal plane and the same liquid, so:

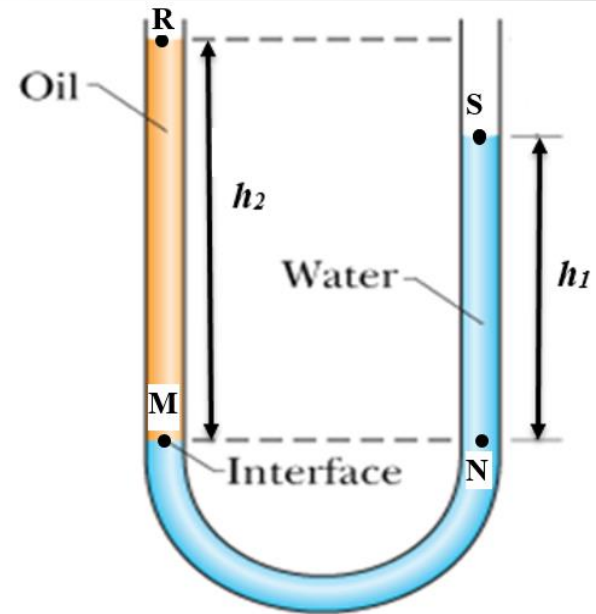
$$P_M = P_N$$

We deduce :

$$P_M - P_R = P_N - P_S \Rightarrow \rho_{oil} \times \cancel{g} \times h_2 = \rho_w \times \cancel{g} \times h_1$$

$$\Rightarrow \rho_{oil} \times h_2 = \rho_w \times h_1$$

$$\Rightarrow \rho_{oil} = \frac{\rho_w \times h_1}{h_2} = 917 \text{ Kg/m}^3$$



Remark 3: Measuring pressure

The Mercury Barometer is a device used to measure the pressure of the atmosphere

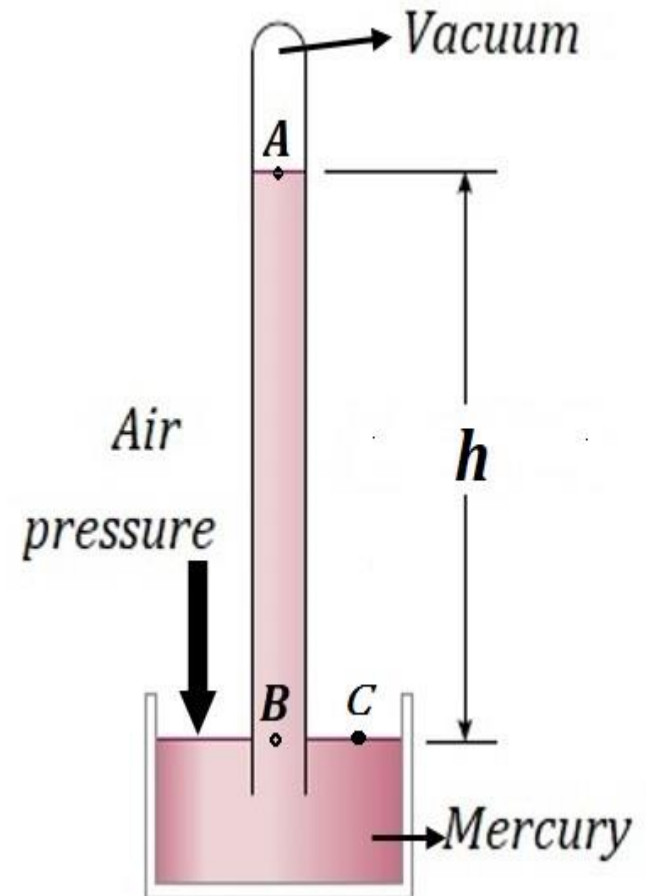
$$P_B - P_A = \rho_{Hg} \times g \times h$$

$$P_B = P_C = P_{atm}$$

$$P_A = 0$$

Therefore

$$P_{atm} = \rho_{Hg} \times g \times h$$



Remark 4: Gauge pressure

$$P_A - P_1 = \rho \times g \times h$$

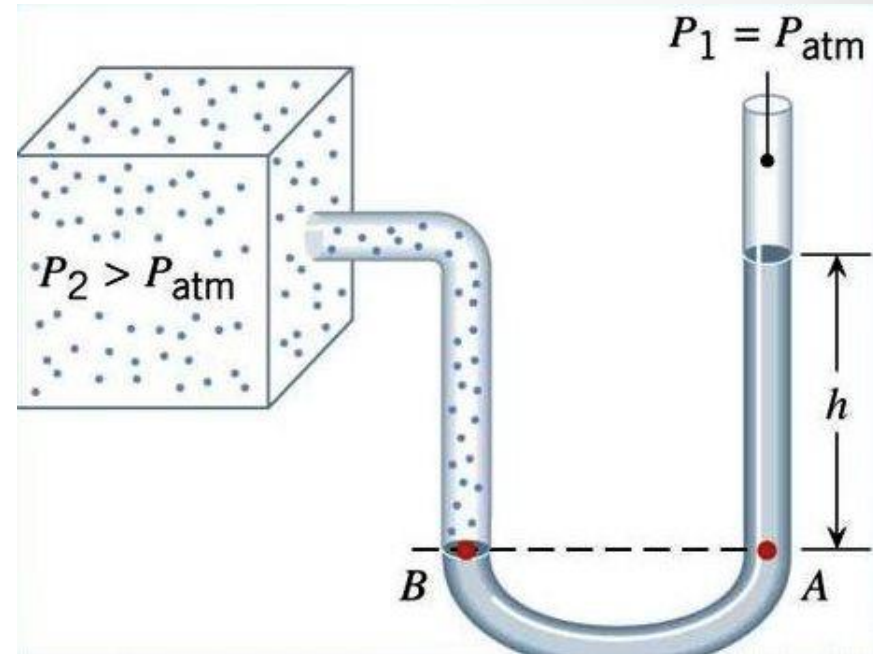
$$P_2 = P_B = P_A$$

$$P_2 \text{ Absolute Pressure}$$

Therefore

$$P_2 - P_1 = \rho \times g \times h$$

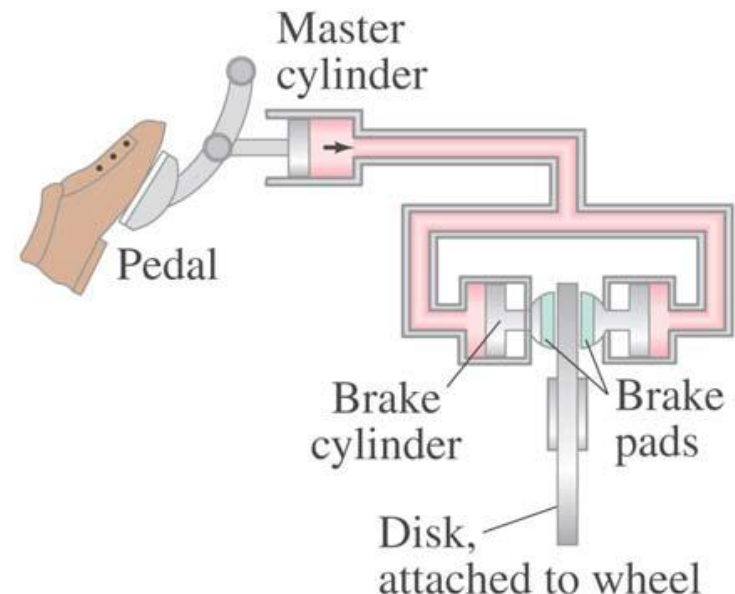
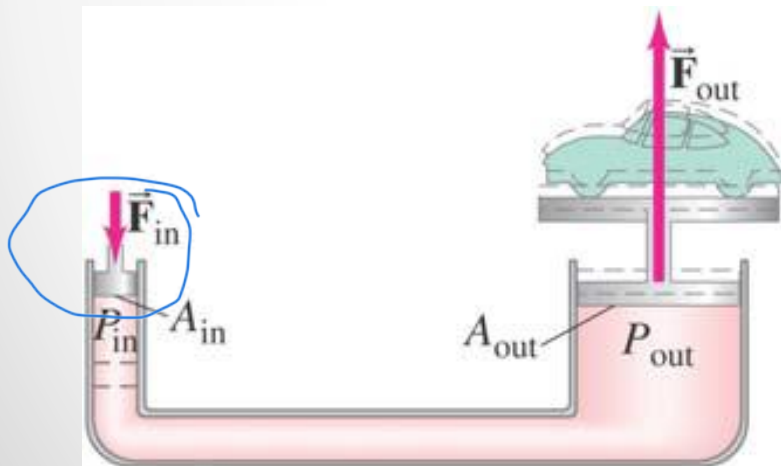
Gauge Pressure P_g



5/ Pascal's principle

The change in the pressure applied to a fluid is transmitted undiminished to every point of the fluid and to the walls of the container.

This principle is used, for example, in hydraulic lifts and hydraulic brakes.



Example 4

In a car lift used in a service station, compressed air exerts a force on a small piston that has a circular cross section and a radius of **5 cm**. This pressure is transmitted by a liquid to a piston that has a radius of **15 cm**.

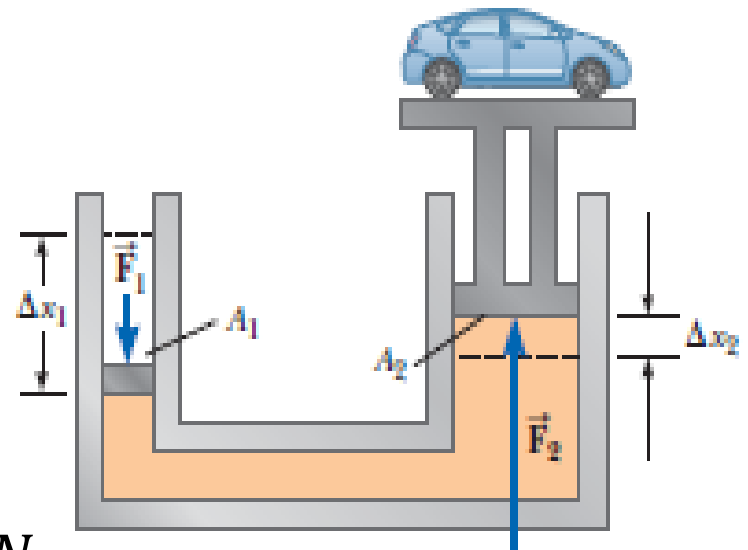
- 1) What force must the compressed air exert to lift a car weighting **13 050 N**?
- 2) What air pressure produces this force?

Solution $p_1 = p_2$

$$1) \frac{F_1}{A_1} = \frac{F_2}{A_2} \Rightarrow F_1 = \frac{A_1 \times F_2}{A_2} = \frac{\pi \times r_1^2 \times F_2}{\pi \times r_2^2}$$

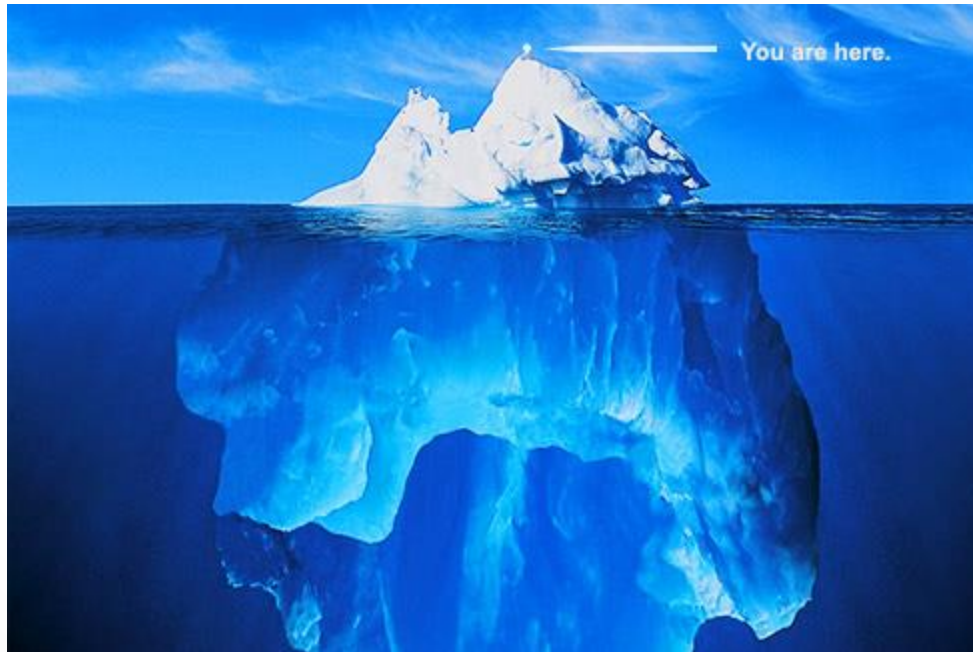
$$\Rightarrow F_1 = 1450 \text{ N}$$

$$2) P_1 = \frac{F_1}{A_1} = \frac{F_1}{\pi \times r_1^2} = 1.85 \times 10^5 \text{ N}$$



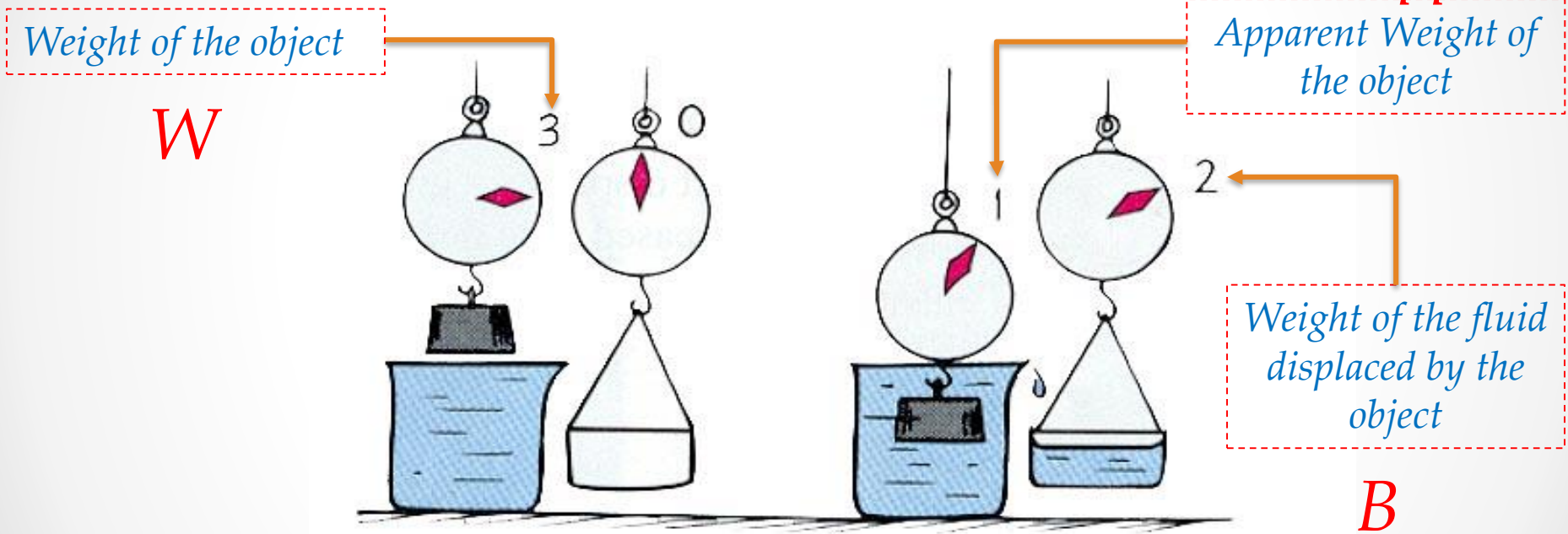
6/ Buoyant Forces

The upward force exerted by a fluid on any immersed object is called a buoyant force.



Archimedes's Principle

The magnitude of the buoyant force always equals the weight of the fluid displaced by the object.



$$B = W - W_{app} = \text{Weight of the fluid displaced by the object}$$

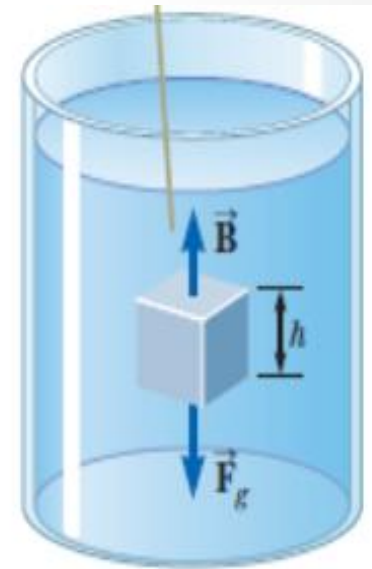
6/ Buoyant Forces

Case 1: Totally Submerged Object

$$\begin{cases} \rho_{flui} < \rho_{obj} \\ V_{disp} = V_{obj} \end{cases}$$

$$\begin{aligned} B &= (P_{Bot} - P_{top}) \times A \\ &= \rho_{flui} \times g \times h \times A \\ &= \rho_{flui} \times g \times V_{obj} \end{aligned}$$

$$\begin{aligned} F_g &= M \times g \\ &= \rho_{obj} \times V_{obj} \times g \end{aligned}$$



6/ Buoyant Forces

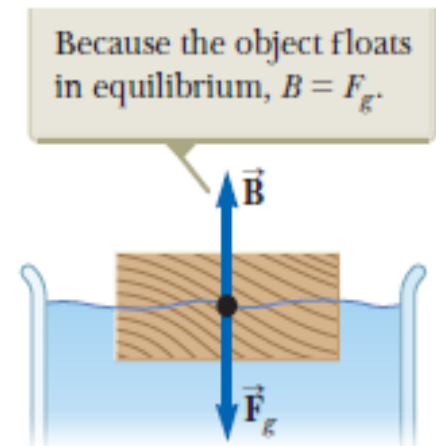
Case 2: Floating (partely submerged)Object

$$\rho_{obj} < \rho_{flui}$$

$$\mathbf{B} = \mathbf{F}_g \Rightarrow \rho_{flui} \times \cancel{g} \times V_{disp} = \rho_{obj} \times V_{obj} \times \cancel{g}$$

$$\Rightarrow \rho_{flui} \times V_{disp} = \rho_{obj} \times V_{obj}$$

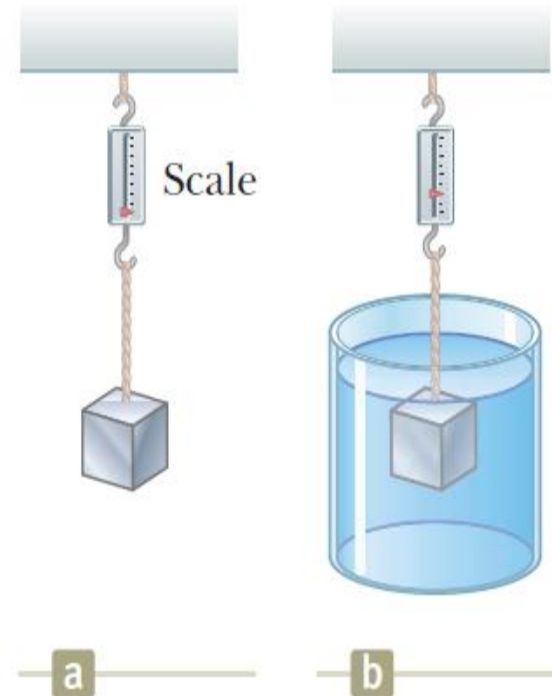
$$\Rightarrow \frac{V_{disp}}{V_{obj}} = \frac{\rho_{obj}}{\rho_{flui}}$$



Example 5

The weight of a solid object is $W = 5\text{ N}$. When this object is suspended from a spring scale and submerged in water, the scale reads $C = 3.5\text{ N}$. ($\rho_w = 998\text{ Kg/m}^3$)

- 1) Find the magnitude of the buoyant force B
- 2) Calculate the mass of this object
- 3) Find the volume of the object
- 4) deduce the density of the object



Solution

1) Find the magnitude of the buoyant force B

$$B = W - W_{app} = 5 - 3.5 = 1.5 \text{ N}$$

2) Calculate the mass of this object

$$W = m_{obj} \times g \Rightarrow m_{obj} = \frac{W}{g} = 0.51 \text{ Kg}$$

3) Find the volume of the object

$$\text{Submerged Object : } B = \rho_{flui} \times g \times V_{obj} \Rightarrow V_{obj} = \frac{B}{\rho_{flui} \times g}$$

$$\Rightarrow V_{obj} = 1.53 \times 10^{-4} \text{ m}^3$$

4) Deduce the density of the object

$$\rho_{obj} = \frac{m_{obj}}{V_{obj}} = 3333 \text{ Kg/m}^3$$

Example 6

A block of density $\rho = 800 \text{ Kg/m}^3$ floats face down in a fluid of density $\rho_f = 1200 \text{ Kg/m}^3$. The block has height $H=6 \text{ cm}$. By what depth h is the block submerged?

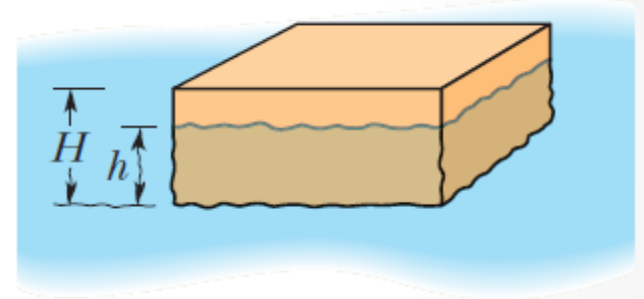
Solution

For floating object

$$\rho_{flui} \times V_{disp} = \rho_{obj} \times V_{obj}$$

$$\rho_{flui} \times \cancel{A} \times h = \rho_{obj} \times \cancel{A} \times H$$

$$\rho_{flui} \times h = \rho_{obj} \times H \Rightarrow h = \frac{\rho_{obj} \times H}{\rho_{flui}} = 4 \text{ cm}$$



7/ Fluid Dynamics

In our model of *ideal fluid* flow, we make the following four assumptions

Ideal fluids

The fluid is non-viscous

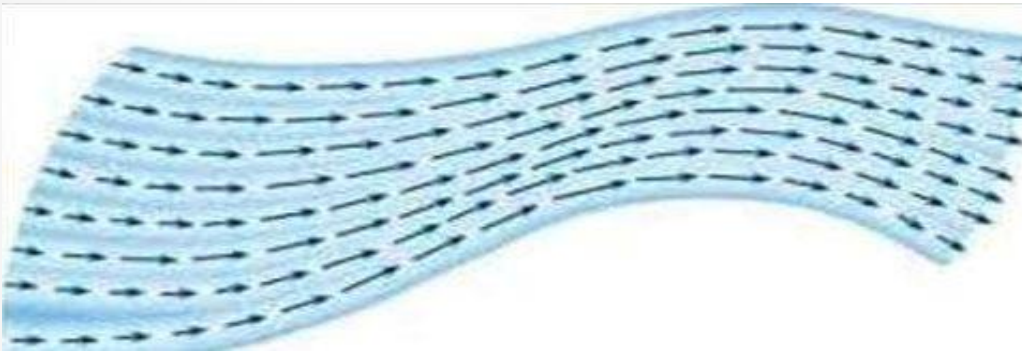
The flow is steady

The fluid is incompressible

The flow is irrotational

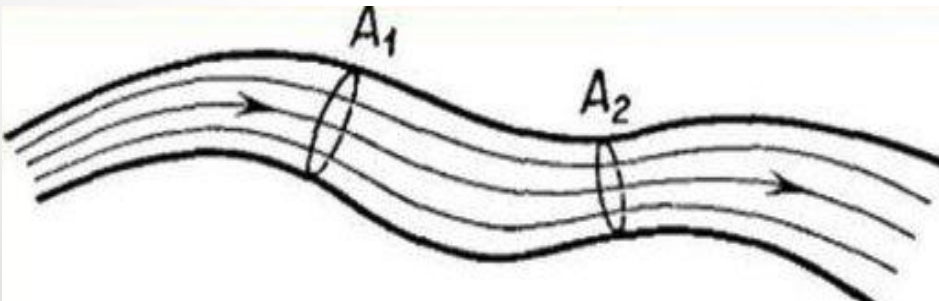
7/ Fluid Dynamics

Steady flow: the velocity of the moving ideal fluid at any fixed point does not change with time



V is constant

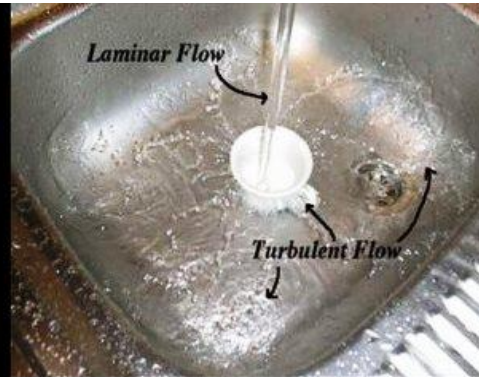
Incompressible flow: The ideal fluid density has a constant value.



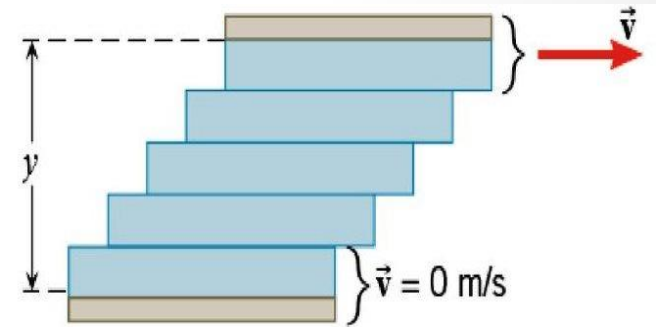
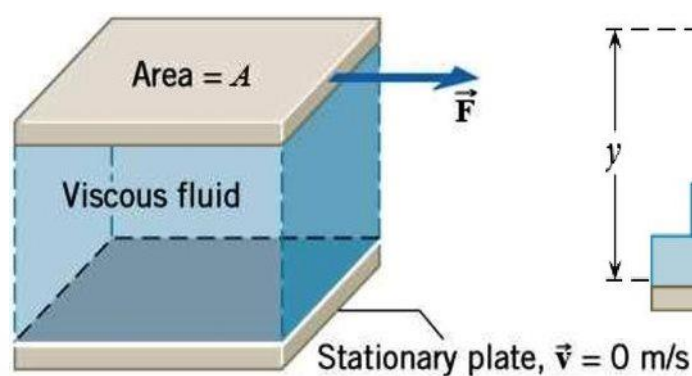
ρ is constant

7/ Fluid Dynamics

The flow is irrotational : the fluid moves without turbulence



The fluid is non-viscous: Viscosity is, roughly, resistance to flow..

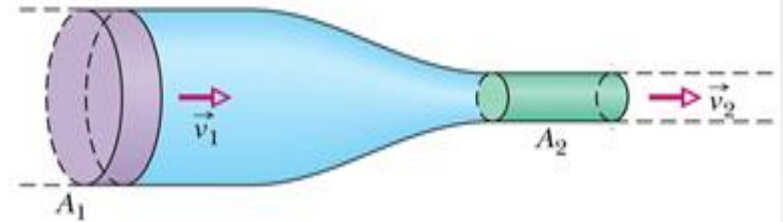


The Equation of Continuity

Conservation of *Mass* gives:

$$A_1 \times V_1 = A_2 \times V_2$$

This expression is called *the equation of continuity* for fluids



Remark 5

$$R_V = A \times V = \text{constant}$$

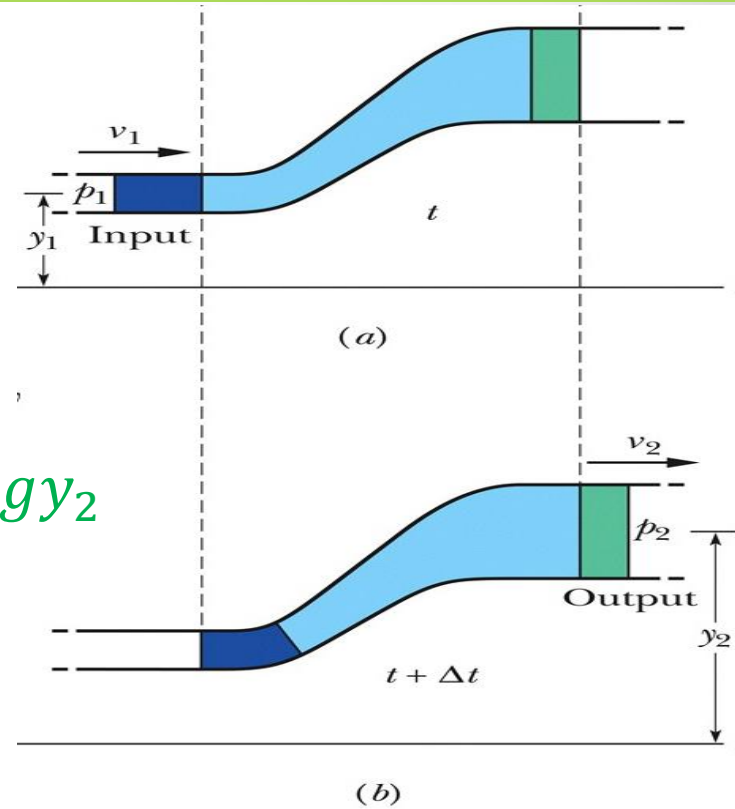
R_V : *Volume flow rate* (the unit of R_V is m^3/s)

$R_m = \rho \times R_V$: *Masse flow rate* (the unit of R_m is kg/s)

8/ Bernoulli's Equation

Conservation of *Energy* gives

$$P_1 + \frac{1}{2}\rho V_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho g y_2$$



This result is summarized in *Bernoulli's equation*

Remark 6

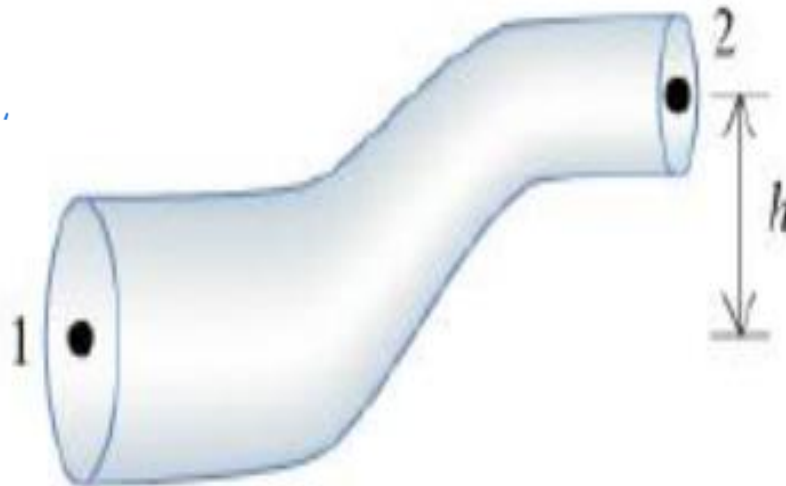
If $y_1 = y_2$

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$$

Large speed means small pressure

Example 7

Water circulates throughout a house in a hot-water heating system. If the water is pumped at a speed of 0.5 m/s through a 4 cm diameter pipe in the basement under a pressure of 3 atm , what will be the flow speed and pressure in a 2.6 cm diameter pipe on the second 5 m above? Assume the pipes do not divide into branches.



Solution

Starting from *equation of continuity*

$$A_1 \times V_1 = A_2 \times V_2 \Rightarrow V_2 = \frac{A_1 \times V_1}{A_2} = 1.19 \text{ m.s}^{-1}$$

Starting from ***Bernoulli's equation***

$$P_1 + \frac{1}{2} \rho V_1^2 + \rho g y_1 = P_2 + \frac{1}{2} \rho V_2^2 + \rho g y_2$$

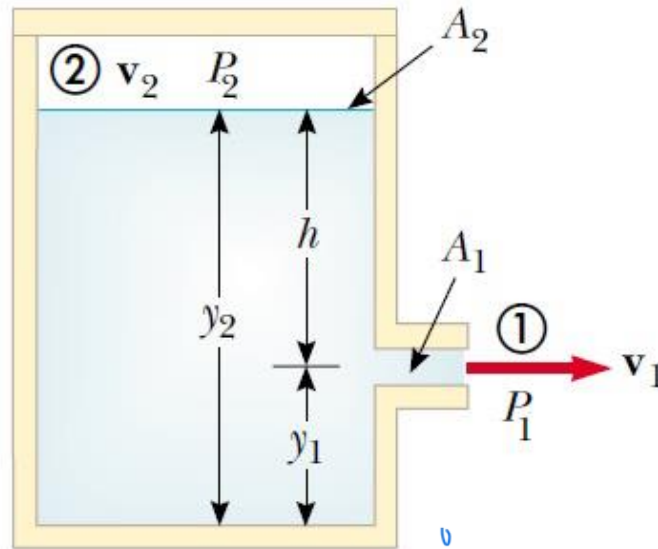
$$P_2 = P_1 + \frac{1}{2} \rho V_1^2 - \frac{1}{2} \rho V_2^2 + \rho g y_1 - \rho g y_2$$

$$P_2 = P_1 + \frac{1}{2} \rho (V_1^2 - V_2^2) + \rho g (y_1 - y_2)$$

$$P_2 = P_1 + \frac{1}{2} \rho (V_1^2 - V_2^2) - \rho g h = 2.5 \times 10^5 \text{ Pa}$$

Example 8

A closed tank containing a liquid with a density $\rho = 980 \text{ Kg/m}^3$ has a hole in its side. The hole is located at $h = 15.65 \text{ m}$ below the water level and open to the atmosphere ($P_1 = P_{atm} = 1.013 \times 10^5 \text{ Pa}$). We can make the following approximations: $P_2 = 0$ and $V_2 = 0 \text{ m/s}$. Calculate the speed V_1 of the water leaking from the hole



Solution

Starting from ***Bernoulli's equation***

$$P_1 + \frac{1}{2}\rho V_1^2 + \rho g y_1 = \cancel{P_2} + \cancel{\frac{1}{2}\rho V_2^2} + \rho g y_2$$

$$P_1 + \frac{1}{2}\rho V_1^2 + \rho g y_1 = \rho g y_2$$

$$\frac{1}{2}\rho V_1^2 = \rho g y_2 - \rho g y_1 - P_1$$

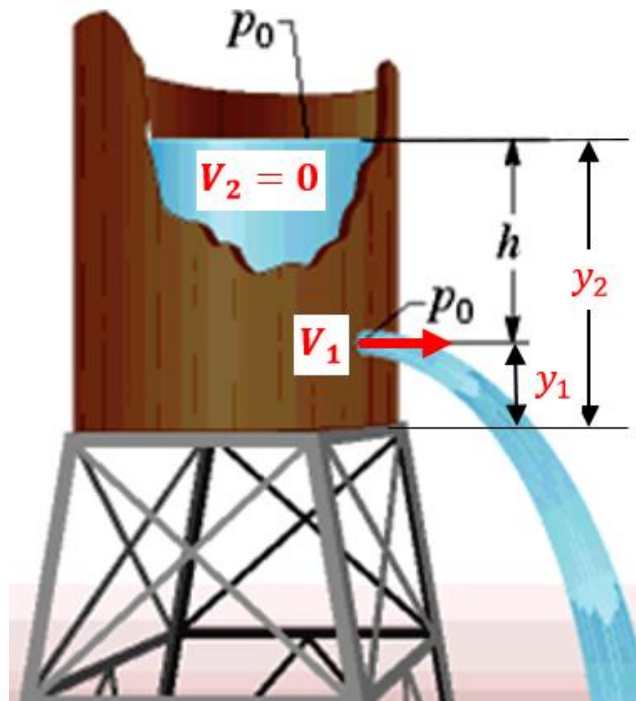
$$\frac{1}{2}\rho V_1^2 = \rho g(y_2 - y_1) - P_1 = \rho g h - P_1$$

$$V_1^2 = 2 g h - \frac{2P_1}{\rho}$$

$$V_1 = \sqrt{2 g h - \frac{2P_1}{\rho}} = 10 \text{ m/s}$$

Example 9

An Open tank containing a liquid has a hole in its side. The hole is located at $h = 11.48 \text{ m}$ below the water level and open to the atmosphere ($P_1 = P_2 = P_0$). We can make the following approximation: $V_2 = 0 \text{ m/s}$. Calculate the speed V_1 of the water leaking from the hole



Solution

Starting from ***Bernoulli's equation***

$$\cancel{P_1} + \frac{1}{2} \rho V_1^2 + \rho g y_1 = \cancel{P_2} + \frac{1}{2} \rho \cancel{V_2^2} + \rho g y_2$$

$$\cancel{\frac{1}{2} \rho V_1^2} + \cancel{\rho g y_1} = \cancel{\rho g y_2}$$

$$\frac{1}{2} V_1^2 = g y_2 - g y_1$$

$$\frac{1}{2} V_1^2 = g(y_2 - y_1) = gh$$

$$V_1^2 = 2 gh$$

$$V_1 = \sqrt{2 gh} = 15 \text{ m/s}$$

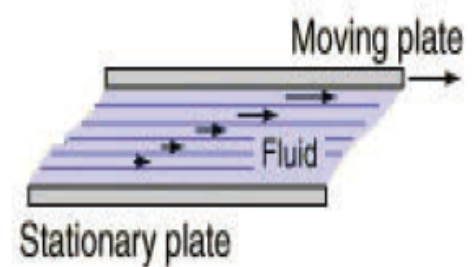
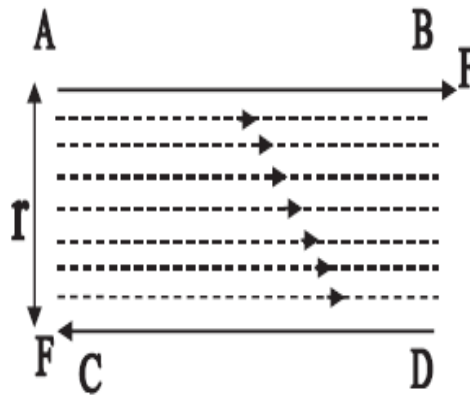
Remark 7: Coefficient of Viscosity η

Consider a layer AB of a liquid moving with a velocity V with respect to a parallel layer CD at a distance r from it.

Consider that a **force F** acting on an **area A** is required to produce this motion. An equal force will, therefore, act on it in the backward direction due to **viscosity**.

The magnitude of the backward force F is given by

$$F = \eta \times \frac{A \times V}{r}$$



Remark 8: Surface Tension

Surface tension is the energy required to increase the surface area of a liquid by a unit amount.

The surface tension is defined as the ratio of the magnitude of the surface tension force to the length along which the force acts:

$$\gamma = \frac{F}{L} \text{ (N} \cdot \text{m}^{-1}\text{)}$$

